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Abstract

In this paper we suggest ways to characterize the transmission mechanisms of volatility between markets by making use of a new Markov Switching bivariate model where the state of one variable feeds into the transition probability of the state of the other. The comparison between this model and other Markov Switching models allows us to derive statistical tests stressing the role of one market relative to another (contagion, interdependence, comovement, independence, Granger causality). We estimate the model on the weekly high–low range of several Asian markets, with a specific interest in the role of Hong Kong.

KEY WORDS: Markov Switching, multiple chains, volatility, transmission mechanisms, comovements

JEL classification: C32 C52 C53

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1 Introduction

The diffusion of international investments and capital movements across borders has marked the evolution of financial markets and has changed the profile of correlations among assets denominated in different currencies which are exchanged in geographically separated markets. A single market volatility reacts to innovations in other markets as a result of financial integration.

Mechanisms of transmission of shocks across variables in an econometric model have received a great deal of attention in the literature. A stream of research in the financial literature has dealt with spillovers of volatility from one market to another, focussing on shocks to volatility in a GARCH framework (Engle *et al.*, 1990). In recent times, several studies have focused on some notable financial crises (especially, Mexico, Russia, East Asia, Argentina) with the intention of analyzing the sources of the crisis: a recurring question is whether the crises originated in one region and spilled over to other regions (contagion) or whether they are the result of an interdependent reaction to some common shock.

In discussing the presence and the extension of contagion effects, several authors have concentrated on different aspects, and hence different definitions of contagion: the World Bank site on Financial Crises[1] provides a broad definition of cross-country transmission of shocks which may take place during both "good" and "bad" times, whereas more restrictive definitions are centered around a specific situation of crisis and the consequent increase in the level of interdependence across countries.

From an empirical point of view, methodologies vary considerably: according to the taxonomy in Pericoli and Sbracia (2003) one can recognize models where the period of the crisis is known and some explanation for its inception is sought. In a Probit/Logit model the crisis is translated into a binary variable and contagion is tantamount to the statistical significance of a dummy variable flagging an existing crisis in another market; in a Leading Indicators model one examines the predictive value of variables linked to economic fundamentals or to foreign markets; in the line of Forbes and Rigobon (2002) one would detect a correlation breakdown in correspondence to the known dates of the crisis.

A different line of research is characterized by volatility spillovers which characterize the structure of interrelationships across markets: the GARCH models put forth by Engle *et al.* (1990) allow to see whether conditional variances are affected by additional information in the form of squared innovations occurring in

[1] http://www1.worldbank.org/economicpolicy/managing%20volatility/contagion/index.html

other markets. This basic idea gets more involved if one considers that volatility clustering may be characterized by the presence of regimes alternating between low and high levels of unconditional volatility. In this respect, a further category of models which has received considerable attention relates to Markov Switching models (MS; diffused in the econometric literature by Hamilton, 1989, and adapted to switching volatility by Hamilton and Susmel, 1994, with a switching ARCH model, SWARCH): in the context of financial crises the presence of sudden switches ruled by a Markov chain can be accommodated for the variance equation, as in Edwards and Susmel (2001) and (2003), who suggest a bivariate version of the SWARCH model for interest rates. In these models, the idea of crisis and contagion translates into a sudden change in the volatility of stock returns or interest rates measured in a pair of countries and of their correlation. The MS model provides a framework in which regimes are associated with the various combinations of low and high volatility in each country. The interesting feature of their approach is that one country is *ex ante* considered the *originator* of the crisis (dominant market) and the correlation coefficient is made dependent on the state of such originator country. Contagion is had when the correlation coefficients significantly change value across states.

In our approach we pursue the idea that transmission mechanisms operate in the presence of volatility regimes. To this end, we choose to focus on the mean[2] of an observable volatility proxy measured on different markets, namely the weekly range (log of the ratio of the highest recorded to the lowest recorded values, cf. Alizadeh et al., 2002). We adopt a new version of the Markov Switching model called the Multi Chain MS model (MCMS, Otranto, 2005),[3] where asymmetries of behavior can be considered by making the transition probability of each market dependent on the state of the other markets. We prefer not to pursue modelling second moments in view of the computational difficulties which characterize switching models in this context. Modelling the mean allows us also to make use of the nonparametric Bayesian approach by Otranto and Gallo (2002) as a preliminary step to detect the number of different states of nature exhibited by the series involved.

In this context, we will study market characterizations relying on the following definitions of contagion, interdependence and comovement. *Contagion* is seen as a situation in which a switch in regime of a dominating market leads to a change

[2]Cf. also the application of a MS approach for the mean equation to a measure of exchange market pressure in Fratzscher (2003).

[3]The idea of using dependent Markov chains in a switching framework is also used by Anas et al. (2004) to study transmission mechanisms for the business cycle.

in regime in the dominated market (with a lag). *Interdependence* is seen as a situation in which a switch in regime of one of the markets leads a change in regime of the other markets (in other terms, the same market could be dominated and dominant). Finally, *comovement* is represented by contemporaneous change in regimes. As detailed in what follows, the various hypotheses corresponding to the different market features can be tested within the context of MCMS models.

In the next two sections the multivariate models used and their interpretation will be explained; in section 4 the methodology exposed will be applied to analyze the characteristics of the Asian markets in the period 1993-2004, including the East Asian crisis of 1997. Concluding remarks follow.

2 Multivariate MS

The presence of multiple regimes can be acknowledged using a popular multivariate model introduced by Hamilton (1990) where parameters are made dependent on a hidden state process ruled by a Markov chain: such a model, the multivariate Markov Switching Model (MS), considers an n-dimensional vector $\boldsymbol{y}_t \equiv (y_{1t}, \ldots, y_{nt})'$, which is assumed to follow a VAR(p) with time-varying parameters:

$$\begin{aligned} \boldsymbol{y}_t &= \boldsymbol{\mu}(s_t) + \sum_{i=1}^{p} \boldsymbol{\Phi}_i(s_t)\boldsymbol{y}_{t-i} + \boldsymbol{\epsilon}_t \\ \boldsymbol{\epsilon}_t &\sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}(s_t)) \end{aligned} \tag{1}$$

where the parameters for the mean equation $\boldsymbol{\mu}(s_t)$ and $\boldsymbol{\Phi}_i(s_t)$, $i = 1, \ldots, p$, as well as the variances and covariances of the error terms $\boldsymbol{\epsilon}_t$ in the matrix $\boldsymbol{\Sigma}(s_t)$ all depend upon the state variable s_t which can assume a number q of values (corresponding to different regimes). The transition probability matrix $\boldsymbol{P}$ contains the probabilities of being in a generic state j at time t given that the state at time $t-1$ was i, namely, for a generic element

$$p_{ij} = Pr(s_t = j | s_{t-1} = i), \; i, j = 1, \ldots, q$$

The properties of this model are well known by now and need not be discussed here: we refer to Hamilton (1994) for the estimation, filtering and smoothing procedures for this model. For this model it is crucial to keep in mind that all variables in the process $\boldsymbol{y}$ depend on the same state variable s_t, and as such they are subject to a common switching.

Such a model is of limited use in deciding whether there is contagion or interdependence, in that it can only signal the common switch of all the variables analyzed from one state to another. In this respect, this model is a good candidate to represent common contemporaneous changes across markets, which we have defined as *comovement*. For the same reasons, it is going to be misleading in cases in which variables are ruled by different states which may be temporally dependent on one another (mutually or in one direction only) or even independent.

3 The Multi–Chain Markov Switching Model

The idea behind a Multi–Chain Markov Switching model (MCMS), as suggested by Otranto (2005), is to consider a multivariate process in which the switching mechanism across regimes makes the state for one variable be dependent on the lagged states of all variables. This case could be considered as representative of the situation of *interdependence*, because the change in the state of each variable can be transmitted to all the others with a certain probability. As a special case, one can consider a process in which one variable is assumed to be dominant on the others and the switching dynamics intrinsically asymmetric: a particular state for one variable alters the probability of other variables to change states, but not vice versa. This feature is suitable to describe transmission mechanisms occurring in financial crises, but also to any relationship where a leading variable is present (in Otranto, 2005, new orders are assumed to be leading the turnover at the aggregate level) thus representing the case of *contagion*. Finally, the reciprocal dependence on the state of the other variables could turn out to be not significant, representing the case in which markets are ruled by *independent* state variables.

To fix ideas, let us consider a bivariate case with two latent states for each variable: the dynamics of the two variables are thus subject to state dependence. The transition from one (multi–) state to another is ruled by a Markov chain obtained by letting the transition probabilities for one variable be a function of the (lagged) state of both variables.

In formal terms, as before, $\boldsymbol{y}_t$ is assumed to follow a VAR(p) process (note that $\boldsymbol{s}_t$ is now a vector):

$$\boldsymbol{y}_t \;=\; \boldsymbol{\mu}(\boldsymbol{s}_t) + \sum_{i=1}^{p} \boldsymbol{\Phi}_i(\boldsymbol{s}_t)\boldsymbol{y}_{t-i} + \boldsymbol{\epsilon}_t$$

$$\boldsymbol{\epsilon}_t \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}(\boldsymbol{s}_t)) \tag{2}$$

where the parameters for the mean equation $\boldsymbol{\mu}(\boldsymbol{s}_t)$ and $\boldsymbol{\Phi}_i(\boldsymbol{s}_t)$, $i = 1, \ldots, p$, as well as the variances and covariances of the error terms $\boldsymbol{\epsilon}_t$ in the matrix $\boldsymbol{\Sigma}(\boldsymbol{s}_t)$ all depend upon the state vector $\boldsymbol{s}_t \equiv (s_{1t}, \ldots, s_{nt})'$ with s_{jt} representing the state associated with variable y_{jt}. Each state can assume a number q of regimes (in principle these could be different across states). The difference with respect to the classical multivariate MS models is that $y_{1,t}$ and $y_{2,t}$ depend on separate but potentially related state variables.

To illustrate how the asymmetric behavior of the variables can be embedded in the model, let us consider the transition probability matrix $\boldsymbol{P}$ with generic element representing

$$\boldsymbol{P} = \{\Pr[\boldsymbol{s}_t|\boldsymbol{s}_{t-1}]\}.$$

If we consider, for simplicity, the case $n = q = 2$, the state vector $\boldsymbol{s}_t$ can assume four different values $\{(0,0), (0,1), (1,0), (1,1)\}$ and the matrix $\boldsymbol{P}$ is a 4×4 matrix. Let us suppose that, conditional on (s_{1t-1}, s_{2t-1}), the states s_{1t} and s_{2t} are independent, so that:

$$\Pr[s_{1t}, s_{2t}|s_{1t-1}, s_{2t-1}] = \Pr[s_{1t}|s_{1t-1}, s_{2t-1}] \Pr[s_{2t}|s_{1t-1}, s_{2t-1}] \tag{3}$$

The right hand side of equation (3) can be parameterized with logistic functions where the functional dependence on past states is made explicit as follows:

$$\begin{aligned} \Pr(s_{1t} = h|s_{1t-1} = h, s_{2t-1}) &= \frac{\exp[\alpha_1(h,.) + \beta_1(h,1)s_{2t-1}]}{1 + \exp[\alpha_1(h,.) + \beta_1(h,1)s_{2t-1}]} \\ \Pr(s_{2t} = h|s_{1t-1}, s_{2t-1} = h) &= \frac{\exp[\alpha_2(.,h) + \beta_2(1,h)s_{1t-1}]}{1 + \exp[\alpha_2(.,h) + \beta_2(1,h)s_{1t-1}]}, \end{aligned} \tag{4}$$

for $h = 0, 1$. From (4), it is apparent that the state of the variable i at time $t-1$ influences the probability of variable j to stay in the same regime, and vice versa. Obviously,

$$\Pr(s_{jt} = k|s_{jt-1} = h, s_{it-1}) = 1 - \Pr(s_{jt} = h|s_{jt-1} = h, s_{it-1})$$

for $h, k = 0, 1$, $h \neq k$, and $i, j = 1, 2$, $i \neq j$. Hypothesis testing can be performed on the estimated model (2)–(4) in order to assess the relevance of the dependence structure assumed for the states and whether the presence of asymmetric effects in the dynamics of regimes is supported by the data. Statistical significance of all parameters in (4) will provide evidence in favor of the case of *interdependence*. If the coefficient $\beta_j(h,k) = 0$, the state of the variable i at time $t-1$ influences the probability of variable j to stay in the same regime, but not vice versa, this is evidence in favor of the dominant status of variable i or *contagion*. This property gives meaning to our envisaging contagion as a stable asymmetric relationship between markets and not necessarily related to the effects of single shocks. Finally, the non significance of all the coefficients $\beta_j(h,k)$ and $\beta_i(h,k)$ would show evidence for *independence* between markets.

In this way, the estimated probabilities in (4) will show the impact of the regime of variable i on the transition probabilities for variable j; moreover, we would expect the signs of coefficients $\beta_1(0,1)$ and $\beta_2(1,0)$ to be negative and those of coefficients $\beta_1(1,1)$ and $\beta_2(1,1)$ to be positive.

Disposing the estimated transition probabilities (3) in a matrix, with rows representing the multiple state at time $t-1$ and columns the multiple state at time t, it is possible to evaluate the most probable scenario (a particular combination of s_{1t} and s_{2t}) at time t, given a certain state at time $t-1$.

The properties of the model from a theoretical point of view coincide with those of a standard Markov switching model: estimation filtering and smoothing can be performed according to the procedures described by Hamilton (1990) and Kim (1994). It should be clear that in practice some restrictions will have to be imposed on the general model (2) in order to make it tractable from a computational point of view, also to retain interpretability of the results according to the specific application at hand.

4 Hong Kong's Role in Asian Markets

The Asian markets are a classical example for which there is a large debate to establish the nature of the relationship among markets subject to sudden changes in volatility. For example, Forbes and Rigobon (2002) note that the shock originating from Hong Kong in October 1997 has not implied a significant increase in the correlation coefficients of the other main Asian markets: the conclusion reached is that the series analyzed cannot be considered as subject to a form of contagion from Hong Kong, but rather the markets considered exhibit interdependence. Let us now see what our analysis allows us to say, in view of a more articulate definition of contagion, interdependence, independence and comovements.

4.1 The Data

We analyze the stock market indices of 5 Asian countries starting from daily data spanning a period between November 29, 1993 and April 26, 2004; the indices are the Hang Seng index (Hong Kong-HSI hereafter), the KOSPI index (South Korea-KS11), the KLSE composite index (Malaysia-KLSE), the Straits Times index (Singapore-STI), the Thailand SET index (Thailand, SETI). The proxy of the volatility is computed as the weekly range of the logarithm of the data (highest recorded minus lowest recorded value) and results in 544 observations.

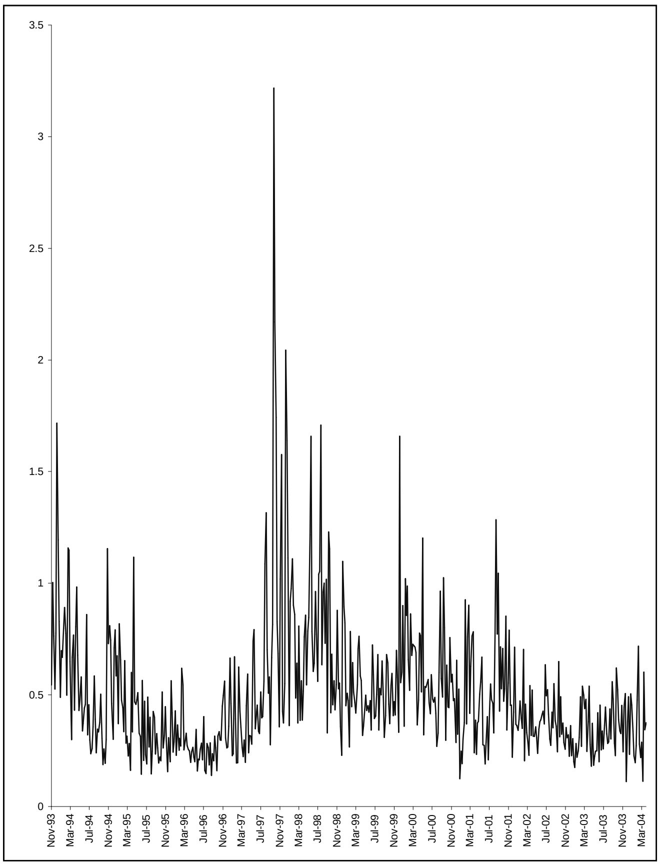

Figure 1: Hang Seng volatility

The proxy used delivers the HSI series shown in Figure 1; the East Asian crisis shows its most evident effect in the third week of October 1997, in which the

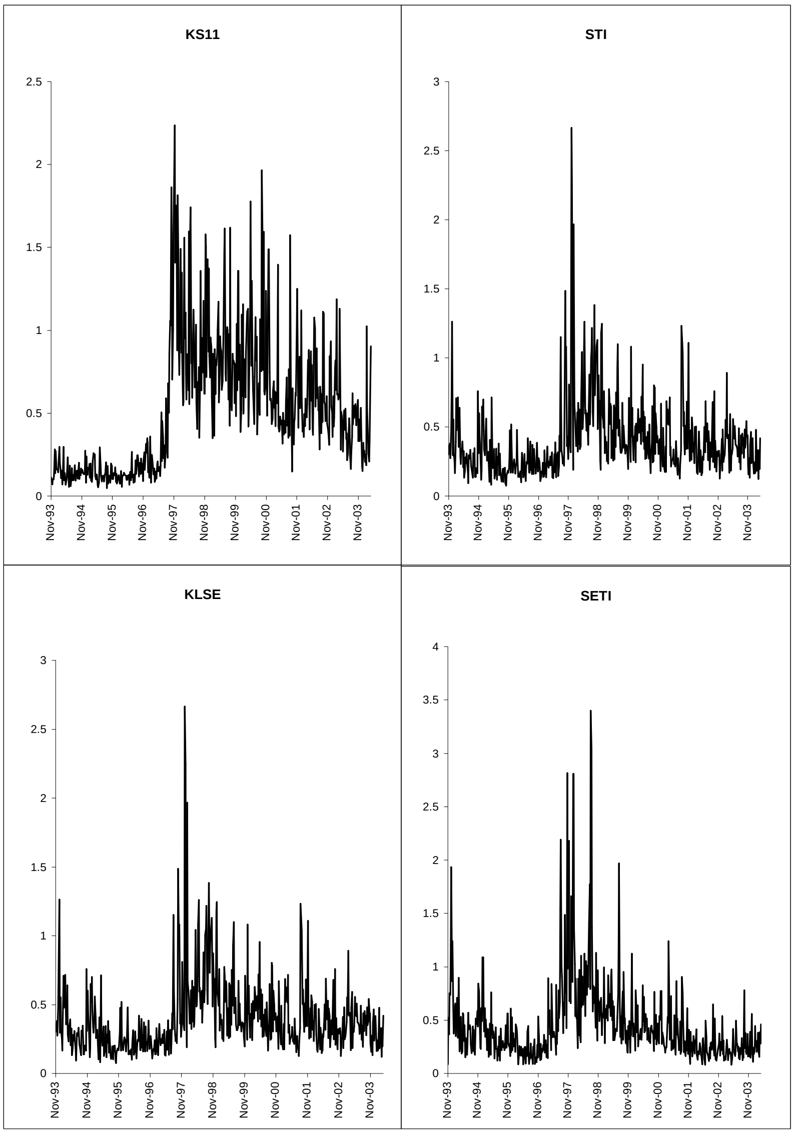

Figure 2: Volatility of Asian markets

volatility increased by almost 300 percent. The dramatically high volatility of this period is a common feature of all the series analyzed (Figure 2), with a different degree of persistence and depth, but a similar general behavior. In particular the Korean market seems to suffer dramatically from the October crisis, increasing its volatility by 81 percent and never really reverting to lower volatility levels in successive periods. Other series seem to absorb the shock, albeit gradually.

4.2 The Presence of Regimes

The existence of at least two regimes is clear observing these graphs, as is the presence of some sort of common feature, with possible lags. The presence of regimes can be detected by means of the nonparametric Bayesian procedure of Otranto and Gallo (2002). With this approach, using diffuse priors for the distributions of the number of regimes, we obtain the posterior distributions shown in Table 1.[4] The series HSI, KS11 and KLSE show a strong evidence in favor of three regimes, whereas STI seems to exhibit only two regimes. The case of SETI is not clear, since very similar probabilities are present for 3 and 4 states. At any rate, for each series the situation of no regimes (1 state) is ruled out; these results are therefore consistent with the proposed approach.

Table 1 approximately here

4.3 The Empirical Results

The first step of our approach is to estimate a MCMS model without constraints on the parameters in the transition probability specifications (4) and then to test some restrictions. We shall consider bivariate models keeping HSI as the second variable in all models and letting the data suggest which type of relationship it holds with other markets, given the great influence exerted by Hong Kong on the other Asian economies.

Following this hypothesis, we estimate 4 separate MCMS models with 2×2 states; the value 0 represents the ordinary regime, the value 1 occurs in the turbulent regime. In addition, we consider the autoregressive parameters in (2) not to be state–dependent and the order p equal to 2; for these coefficients the usual stationarity constraints hold. Finally, we suppose a structure of the covariance matrix as:

[4]We use the same priors of Otranto and Gallo (2002), with the hyperparameter A, regulating the prior probabilities of the number of regimes, equal to 0.15.

$$\mathbf{\Sigma}(s_{1,t}, s_{2,t}) = \begin{bmatrix} \sigma_1^2(s_{1t}, .) & \rho(s_{1t}, s_{2t})\sigma_1(s_{1t}, .)\sigma_2(., s_{2t}) \\ \rho(s_{1t}, s_{2t})\sigma_1(s_{1t}, .)\sigma_2(., s_{2t}) & \sigma_2^2(., s_{2t}) \end{bmatrix}$$

In other terms, the variances of each variable (related to fourth moments of returns) depend only on the variable's own state, whereas the effect of the multi–state affects the correlation coefficient, that varies in [-1,1].

We allow the intercept of model (2) to vary with both the regimes of the two markets; in other terms, we will have four possible intercepts for each variable.

After the estimation of the models, we would like to test some proposition to evaluate the presence of dependence on the state of the other variable, in particular the case of contagion or independence. The testable propositions are:

State Dependence in the Mean Equation

1. No dependence of the intercept of y_1 on the state of y_2:
   H_0: $\mu_1(0,0)$=$\mu_1(0,1)$ and $\mu_1(1,0)$=$\mu_1(1,1)$;
2. No dependence of the intercept of y_2 on the state of y_1:
   H_0:$\mu_2(0,0)$=$\mu_2(1,0)$ and $\mu_2(0,1)$=$\mu_2(1,1)$

Dynamic Dependence in the Mean Equation

3. y_2 does not Granger cause y_1:
   H_0:ϕ_{12}^1=ϕ_{12}^2=0
4. y_1 does not Granger cause y_2:
   H_0:ϕ_{21}^1=ϕ_{21}^2=0

State Dependence in the Correlations

5. No dependence of the correlation on the state of y_2:
   H_0:$\rho(0,0)$=$\rho(0,1)$ and $\rho(1,0)$=$\rho(1,1)$
6. No dependence of the correlation on the state of y_1:
   H_0:$\rho(0,0)$=$\rho(1,0)$ and $\rho(0,1)$=$\rho(1,1)$

Characterization of Market Dependence

7. No contagion from y_2 to y_1:
   H_0:$\beta_1(0,1)$=$\beta_1(1,1)$=0
8. No contagion from y_1 to y_2: H_0:$\beta_2(1,0)$=$\beta_2(1,1)$=0
9. No interdependence (no reciprocal contagion):
   H_0:$\beta_1(0,1)$=$\beta_1(1,1)$=$\beta_2(1,0)$=$\beta_2(1,1)$=0
10. Comovement between y_1 and y_2

$$H_0 : \begin{array}{l} \alpha_1(0,.) = \alpha_2(.,0) \\ \alpha_1(0,.) + \beta_1(0,1) + \alpha_2(.,1) = 0 \\ \alpha_1(.,1) + \alpha_2(.,0) + \beta_2(1,0) = 0 \\ \alpha_1(1,.) + \beta_1(1,1) = \alpha_2(.,1) + \beta_2(1,1) \end{array}$$

The last hypothesis is not intuitive because the MMS model, where $s_{1t} = s_{2t}$ for each t, is not nested into the MCMS model. Thus, the rows and the columns of the transition probability matrix of the independent MCMS model where $s_{1t} \neq s_{2t}$ cannot be constrained so as to obtain the smaller size transition probability matrix of the MMS model. However, one can impose that the profile of the estimated state variable for market 1 be the same as the corresponding state variable for market 2. The analytical derivation of these constraints is developed in the appendix at the end of the paper.

All these hypotheses can be tested by means of classical Wald statistics and are consistent with the idea of Granger causality for MS VAR models proposed by Warne (2000).

The hypotheses labeled 7. to 10. are the ones which characterize the relationships between markets. The various situations can be summarized as follows:

- **Contagion**: it occurs when hypothesis 7. cannot be rejected and hypothesis 8. is rejected or the other way around.
- **Interdependence** or reciprocal contagion: hypotheses 7., 8., and 9. are rejected.
- **Independence**: it occurs when hypothesis 9. cannot be rejected and hypothesis 10. is rejected.

- **Comovement** or common state variable: it occurs when 10. cannot be rejected, as discussed above.

Table 2 approximately here

In Table 2 we summarize the hypothesis testing results of the Wald test statistics for the ten hypotheses above; the estimated models (2)–(4) show some form of dependence between the couples of series according to the various categories detailed above. In particular, the hypotheses that the intercepts of one market do not depend on the state of the other market, and the Granger non causality hypotheses are rejected (strongly in all cases, except for the Granger noncausality test in the direction Thailand to Hong Kong which is rejected at the 5% significance level). The correlations seem to be dependent on the states in all cases except for the Korean/Hong Kong markets.

As per market dependence as described above, we can say that the Hong Kong market has a contagion effect on both the Korean and Thailand markets (hypothesis 7. rejected and hypothesis 8. not rejected). For the Malaysia/Hong Kong markets the evidence favors interdependence (rejection of both hypotheses 7. and 8.). The Singapore case is a puzzling one: if one follows the testing procedure above, one finds a p-value associated with the test statistic for hypothesis 9. equal to 0.066 and a p-value for the corresponding statistic for hypothesis 10. equal to 0.025, which would characterize the case as borderline between independence and (more so) comovement. Moreover, there is also mild support for the case of contagion from HSI (the no contagion hypothesis 7. is also rejected only at 5% significance level, but not at 1%). We can direct the analysis to gather some more evidence between the two cases of comovement (represented by a bivariate MS model) or independence (represented by a bivariate MCMS model with independent s_{1t} and s_{2t}). This allows us also to shed some light on the different characteristics of the various models.

In view of the results in Table 1, where we assessed that STI seems to possess two regimes and HSI three, we will estimate two MS models, one with two regimes and one with three. We will follow Hamilton and Susmel (1994) in carrying out a comparison based on in-sample goodness of fit performance using the Mean Square Error (MSE) and Mean Absolute Error (MAE) or their equivalents for the variables expressed in logs ($[LE]^2$ and $|LE|$ respectively, following Hamilton and Susmel's notation). The results are shown in Table 3 (the boldface figures indicate the best performance).

Table 3 approximately here

The MS model with 3 states (labels henceforth are to states 1, 2, and 3) clearly performs better than the others, with the MS model with 2 states coming in as second: we interpret this to be evidence in favor of a case of comovement between STI and HSI.

We show the estimation of the selected models for each pair of markets in Tables 4, 5, 6, 7, reporting in the last rows the p-values relative to the Jarque-Bera test (JB), the Ljung-Box test (LB(10)) and the Ljung-Box test on squared residuals (LBS(10)), both calculated with 10 lags.

Table 4 approximately here

Table 5 approximately here

Table 6 approximately here

Table 7 approximately here

The residuals of HSI exhibit non normality in all cases, but this is not surprising given that we are modelling the conditional expectation of a positive valued process and the distribution of residuals is often asymmetric and affected by exceptionally high values (cf. also Figures 1 and 2).

It is interesting to note that many correlation coefficients between estimated innovations are equal to zero which may suggest that the consideration of the regimes captures the main features of the strong relationship seemingly exhibited by the variables. To investigate this issue, we have estimated a bivariate VAR model on the four pairs of variables as well: the residuals in each case are strongly correlated. To support our claim that undetected regimes induce spurious correlations in the residuals, we ran a few Monte Carlo experiments. The outcome is that when MS and MCMS models with uncorrelated disturbances are simulated and then estimated by a VAR the residuals are cross-correlated (the results are not reported here for the sake of space, but they are available upon request).

We can note that the signs of the parameters of the logistic functions are consistent with our expectations. In the KS11/HSI case the state of HSI has an impact just on the probabilities in state 0. For the STI/HSI case, the switch to state 3 from state 1 or 2 is not likely, whereas the change from state 3 to state 1 is more likely than the change to state 2; we can interpret the state 1 as the case of low volatility and the state 2 as the high volatility one, whereas the state 3, given its infrequent occurrence accompanied by low persistence, can be considered as an extremely turbulent state. All the intercepts of the MCMS models exhibit a gradual change from the (0,0) to the (1,1) state (note that in the Malaysia case - Table 6 - the intercept does not change between (1,0) and (1,1)).

5 Concluding Remarks

In this paper we propose a new model, based on correlated Markov chains, to represent the case of interdependence among financial markets, with the case of contagion and independent markets as particular cases. The fact that the two last cases are nested in the more general model provides the possibility to test statistically the various scenarios. The case of comovement among variables, though, which is characterized by a classical Markov Switching model is not nested in the MCMS model: we resorted to a separate test for common dynamics of the two state variables.

The applications show the relevant role of Hong Kong as a dominant market over the period considered: it turns out that a plausible market characterization from the estimated models and the hypothesis testing performed is that Hong Kong has a leading role relative to Korea and to Thailand. Malaysia shows some form of interdependence while for the case of Singapore the estimated models and the evaluation of several loss functions in one-step-ahead prediction points rather to a situation of comovement between the two markets.

The estimation of a bivariate model is forced by the difficulty of increasing the number of variables in the model without stumbling into the usual numerical problems encountered in Markov Switching models with higher number of regimes. A n-variate model with k states per variable would have a transition matrix of order k^n, which is rapidly intractable (flat likelihood function) for even moderate numbers of n or k above 2. There is therefore a trade-off between the depth of the economic interpretation which one would have available if more than two markets were to be compared and the numerical difficulties which accompany such an effort.

The definitions of contagion, interdependence, comovement and independence are consistent with large part of the literature, but we should stress that their practical characterization is different in terms of the statistical instruments utilized. For example, Forbes and Rigobon (2002) base their analysis only on the behavior of the correlation coefficients, and on a significant increase changing from a state of low to another of high volatility (with the periods of low and high volatility established a priori). In our approach, the analysis is not limited to specific episodes of crisis, the periods of high and low volatility are selected by the model itself. An important result is that the presence of correlation between the residuals disappears if one takes into proper consideration the existence of regimes and the peculiar structure of the dynamics behind them.

Appendix

In this Appendix we demonstrate that testing the null of comovement against the hypothesis of MCMS model is equivalent to verifying a set of linear restrictions on the MCMS model.

The case of comovement corresponds to the case in which the state of y_{1t} and y_{2t} is the same for each t; this situation can justify the adoption of a classical MS model. The MS model is not nested into the MCMS model given the different number of states: hence the classical tests based on the likelihood function cannot be applied.

In view of Hamilton (1994), a Markov chain can be represented as an AR(1) process:

$$\boldsymbol{\xi}_{t+1} = \boldsymbol{P}'\boldsymbol{\xi}_t + \boldsymbol{v}_{t+1},$$

where $\boldsymbol{\xi}_t$ is a vector containing 1 in correspondence of the state at time t, $\boldsymbol{P}$ is the transition probability matrix and $\boldsymbol{v}_t$ is a vector innovation with zero mean. In our case, the multiple states are (0,0), (0,1), (1,0), (1,1); correspondingly, for example, $\boldsymbol{\xi}_t = [0,0,0,1]'$ points to a value of the multiple state at time t as $(1,1)$.

The conditional expectation of $\boldsymbol{\xi}_{t+1}$ is:

$$E(\boldsymbol{\xi}_{t+1}|\boldsymbol{\xi}_t) = \boldsymbol{P}'\boldsymbol{\xi}_t.$$

If we are interested in the behavior of the single regimes s_{1t} and s_{2t}, let us note that they can be represented as the vectors $\boldsymbol{\xi}_t^*$, respectively, $\boldsymbol{\xi}_t^{**}$. Correspondingly, their expected values are given by the 2×1 vectors:

$$\begin{aligned} E(\boldsymbol{\xi}^*_{t+1}|\boldsymbol{\xi}^*_t) &= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \boldsymbol{P}'\boldsymbol{\xi}_t \\ E(\boldsymbol{\xi}^{**}_{t+1}|\boldsymbol{\xi}^{**}_t) &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \boldsymbol{P}'\boldsymbol{\xi}_t. \end{aligned}$$

To investigate the presence of comovement, as defined in the main body of the paper, let us test the equality of the two previous vectors, that is,

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \boldsymbol{P}' = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \boldsymbol{P}'. \tag{5}$$

It is easy to verify that the two rows provide equal constraints: once it is verified that the first element of $\boldsymbol{\xi}_t^*$ is equal to the first element of $\boldsymbol{\xi}_t^{**}$ for each t, automatically the second elements of the two vectors will be equal, as each of them are the complements to 1 of the previous corresponding elements. Let us denote the probability $\Pr[s_{1t} = i, s_{2t} = j | s_{1t-1} = w, s_{2t-1} = z]$ by $p(ij|wz)$; as a consequence, the $\boldsymbol{P}$ matrix is:

$$\begin{bmatrix} p(00|00) & p(01|00) & p(10|00) & p(11|00) \\ p(00|01) & p(01|01) & p(10|01) & p(11|01) \\ p(00|10) & p(01|10) & p(10|10) & p(11|10) \\ p(00|11) & p(01|11) & p(10|11) & p(11|11) \end{bmatrix}$$

Developing the first (or the second) equation of (5), the four constraints to be verified are:

$$\begin{array}{rcl} \Pr\left[s_{1t} = 0, s_{2t} = 1 | s_{1t-1} = 0, s_{2t-1} = 0\right] & = & \Pr\left[s_{1t} = 1, s_{2t} = 0 | s_{1t-1} = 0, s_{2t-1} = 0\right] \\ \Pr\left[s_{1t} = 0, s_{2t} = 1 | s_{1t-1} = 0, s_{2t-1} = 1\right] & = & \Pr\left[s_{1t} = 1, s_{2t} = 0 | s_{1t-1} = 0, s_{2t-1} = 1\right] \\ \Pr\left[s_{1t} = 0, s_{2t} = 1 | s_{1t-1} = 1, s_{2t-1} = 0\right] & = & \Pr\left[s_{1t} = 1, s_{2t} = 0 | s_{1t-1} = 1, s_{2t-1} = 0\right] \\ \Pr\left[s_{1t} = 0, s_{2t} = 1 | s_{1t-1} = 1, s_{2t-1} = 1\right] & = & \Pr\left[s_{1t} = 1, s_{2t} = 0 | s_{1t-1} = 1, s_{2t-1} = 1\right] \end{array} \quad (6)$$

Recalling the hypothesis of conditional independence (3) and the parameterization (4), we obtain that (6) corresponds to the four nonlinear constraints:

$$\begin{array}{rcl} \frac{\exp[\alpha_1(0,.)]}{1+\exp[\alpha_1(0,.)]}\frac{1}{1+\exp[\alpha_2(.,0)]} & = & \frac{1}{1+\exp[\alpha_1(0,.)]}\frac{\exp[\alpha_2(.,0)]}{1+\exp[\alpha_2(.,0)]} \\ \frac{\exp[\alpha_1(0,.)+\beta_1(0,1)]}{1+\exp[\alpha_1(0,.)+\beta_1(0,1)]}\frac{\exp[\alpha_2(.,1)]}{1+\exp[\alpha_2(.,1)]} & = & \frac{1}{1+\exp[\alpha_1(0,.)+\beta_1(0,1)]}\frac{1}{1+\exp[\alpha_2(.,0)]} \\ \frac{1}{1+\exp[\alpha_1(.,1)]}\frac{1}{1+\exp[\alpha_2(.,0)+\beta_2(1,0)]} & = & \frac{\exp[\alpha_1(1,.)]}{1+\exp[\alpha_1(1,.)]}\frac{\exp[\alpha_2(.,0)+\beta_2(1,0)]}{1+\exp[\alpha_2(.,0)+\beta_2(1,0)]} \\ \frac{1}{1+\exp[\alpha_1(1,.)+\beta_1(1,1)]}\frac{\exp[\alpha_2(.,1)+\beta_2(1,1)]}{1+\exp[\alpha_2(.,1)+\beta_2(1,1)]} & = & \frac{\exp[\alpha_1(1,.)+\beta_1(1,1)]}{1+\exp[\alpha_1(1,.)+\beta_1(1,1)]}\frac{1}{1+\exp[\alpha_2(.,1)+\beta_2(1,1)]} \end{array}$$

After simple algebraic manipulations, the previous nonlinear relationships among the probabilities parameters are equivalent to the following linear restrictions:

$$\begin{array}{rcl} \alpha_1(0,.) & = & \alpha_2(.,0) \\ \alpha_1(0,.) + \beta_1(0,1) + \alpha_2(.,1) & = & 0 \\ \alpha_1(.,1) + \alpha_2(.,0) + \beta_2(1,0) & = & 0 \\ \alpha_1(1,.) + \beta_1(1,1) & = & \alpha_2(.,1) + \beta_2(1,1) \end{array}$$

In the simultaneous presence of these four constraints, we can think of common dynamics for the state variables and therefore of comovement.

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Tables

Table 1: Empirical Posterior Distribution of the number of regimes

	1	2	3	4	5	6	7	8
HSI	0.000	0.000	0.872	0.118	0.008	0.002	0.000	0.000
KS11	0.000	0.000	0.768	0.200	0.024	0.008	0.000	0.000
STI	0.000	0.806	0.182	0.012	0.000	0.000	0.000	0.000
KLSE	0.000	0.000	0.850	0.124	0.024	0.002	0.000	0.000
SETI	0.000	0.018	0.420	0.494	0.052	0.016	0.000	0.000

Table 2: Market Characterization Based on MCMS Models

Hypotheses	Market 1 KS11	STI	KLSE	SETI
State Dependence in the Mean Equation				
1. No dependence of the intercept of Market 1 on the state of HSI	**	**	**	**
2. No dependence of the intercept of HSI on the state of Market 1	**	**	**	**
Dynamic Dependence in the Mean Equation				
3. HSI does not Granger cause Market 1	**	**	**	**
4. Market 1 does not Granger cause HSI	**	**	**	*
State Dependence in the Correlations				
5. No dependence of the correlation on the state of HSI		**	**	**
6. No dependence of the correlation on the state of Market 1		**	**	**
Characterization of Market Dependence				
7. No contagion from HSI to Market 1	**	*	**	**
8. No contagion from Market 1 to HSI			**	
9. No interdependence	**		**	**
10. Comovement between Market 1 and HSI	**	*	**	**
Plausible Market Characterization				
Contagion from Hong Kong	×			×
Interdependence			×	
Comovement		×		
Independence		×		

Note: The '*' and '**' symbols represent **rejection** of the hypothesis at 5%, respectively, 1% significance level, on the basis of a corresponding Wald-type tests on estimated MCMS models.

Table 3: STI/HSI: Loss functions for MCMS and MS models

	MSE	MAE	$[LE]^2$	$\|LE\|$
MS 2 states	12.776	3.346	0.450	0.756
MS 3 states	**12.734**	**3.337**	**0.447**	**0.751**
MCMS	12.936	3.369	0.458	0.763

Table 4: Estimated parameters of the MCMS Model for Korea/Hong Kong (standard errors in parentheses)

Switching coefficients - Constant Term							
Korea Equation				Hong Kong Equation			
$\mu_1(0,0)$	$\mu_1(0,1)$	$\mu_1(1,0)$	$\mu_1(1,1)$	$\mu_2(0,0)$	$\mu_2(1,0)$	$\mu_2(0,1)$	$\mu_2(1,1)$
0.714	1.307	3.446	8.222	1.760	2.336	5.260	6.103
(0.064)	(0.210)	(0.181)	(0.264)	(0.104)	(0.381)	(0.367)	(0.426)
Autoregressive Terms							
Korea Equation				Hong Kong Equation			
ϕ_{11}^1	ϕ_{12}^1	ϕ_{11}^2	ϕ_{12}^2	ϕ_{21}^1	ϕ_{22}^1	ϕ_{21}^2	ϕ_{22}^2
0.284	-0.020	0.197	-0.000	0.028	0.158	-0.000	0.178
(0.007)	(0.007)	(0.006)	(0.008)	(0.009)	(0.013)	(0.004)	(0.012)
Switching coefficients - Standard deviations				Switching coefficients - Correlation Terms			
Korea Equation		Hong Kong Equation					
$\sigma_1(0,.)$	$\sigma_1(1,.)$	$\sigma_2(.,0)$	$\sigma_2(.,1)$	$\rho(0,0)$	$\rho(0,1)$	$\rho(1,0)$	$\rho(1,1)$
0.412	1.580	0.675	2.446	0.000	0.000	0.037	0.039
(0.012)	(0.041)	(0.016)	(0.050)	(0.050)	(0.079)	(0.064)	(0.051)
Probability parameters							
Korea Equation				Hong Kong Equation			
$\alpha_1(0,.)$	$\beta_1(0,1)$	$\alpha_1(1,.)$	$\beta_1(1,1)$	$\alpha_2(.,0)$		$\alpha_2(.,1)$	
1.614	-1.203	1.053	0.000	1.119		0.012	
(0.246)	(0.362)	(0.221)	(0.357)	(0.166)		(0.188)	
p-values of test statistics							
Korea				Hong Kong			
JB	LB(10)	LBS(10)		JB	LB(10)	LBS(10)	
0.241	0.360	0.040		0.000	0.306	0.311	

Table 5: Estimated parameters of the MS–3 states Model for Singapore/Hong Kong (standard errors in parentheses)

Switching coefficients - Constant Term					
Singapore Equation			Hong Kong Equation		
$\mu_1(1)$	$\mu_1(2)$	$\mu_1(3)$	$\mu_2(1)$	$\mu_2(2)$	$\mu_2(3)$
1.298	2.737	6.801	1.445	3.623	6.847
(0.056)	(0.071)	(0.357)	(0.063)	(0.092)	(0.427)

Autoregressive Terms							
Singapore Equation				Hong Kong Equation			
ϕ_{11}^1	ϕ_{12}^1	ϕ_{11}^2	ϕ_{12}^2	ϕ_{21}^1	ϕ_{22}^1	ϕ_{21}^2	ϕ_{22}^2
0.209	-0.007	0.156	-0.000	0.121	0.145	-0.004	0.163
(0.014)	(0.015)	(0.013)	(0.011)	(0.016)	(0.015)	(0.014)	(0.012)

Switching coefficients - Standard deviations						Switching coefficients Correlation Terms		
Singapore Equation			Hong Kong Equation					
$\sigma_1(1)$	$\sigma_1(2)$	$\sigma_1(3)$	$\sigma_2(1)$	$\sigma_2(2)$	$\sigma_2(3)$	ρ_1	$\rho(2)$	$\rho(3)$
0.528	0.845	2.932	0.539	1.045	3.370	0.108	0.000	0.456
(0.019)	(0.031)	(0.082)	(0.016)	(0.040)	(0.096)	(0.041)	(0.057)	(0.058)

Transition Probabilities					
p_{11}	p_{12}	p_{21}	p_{22}	p_{31}	p_{32}
0.521	0.410	0.459	0.464	0.307	0.195
(0.039)	(0.040)	(0.044)	(0.047)	(0.065)	(0.064)

p-values of test statistics					
Singapore			Hong Kong		
JB	LB(10)	LBS(10)	JB	LB(10)	LBS(10)
0.058	0.013	0.003	0.000	0.129	0.938

The three states are labelled as states 1, 2, and 3.

Table 6: Estimated parameters of the MCMS Model for Malaysia/Hong Kong (standard errors in parentheses)

Switching coefficients - Constant Term							
Malaysia Equation				Hong Kong Equation			
$\mu_1(0,0)$	$\mu_1(0,1)$	$\mu_1(1,0)$	$\mu_1(1,1)$	$\mu_2(0,0)$	$\mu_2(1,0)$	$\mu_2(0,1)$	$\mu_2(1,1)$
1.261	3.523	8.084	8.084	2.105	2.210	3.428	8.683
(0.051)	(1.095)	(1.091)	(1.156)	(0.081)	(0.233)	(0.190)	(0.320)
Autoregressive Terms							
Malaysia Equation				Hong Kong Equation			
ϕ_{11}^1	ϕ_{12}^1	ϕ_{11}^2	ϕ_{12}^2	ϕ_{21}^1	ϕ_{22}^1	ϕ_{21}^2	ϕ_{22}^2
0.183	0.025	0.126	0.000	0.066	0.195	0.009	0.135
(0.009)	(0.010)	(0.007)	(0.010)	(0.011)	(0.016)	(0.013)	(0.017)
Switching coefficients - Standard deviations				Switching coefficients - Correlation Terms			
Malaysia Equation		Hong Kong Equation					
$\sigma_1(0,.)$	$\sigma_1(1,.)$	$\sigma_2(.,0)$	$\sigma_2(.,1)$	$\rho_{0,0}$	$\rho(0,1)$	$\rho(1,0)$	$\rho(1,1)$
0.523	3.951	0.879	2.009	0.139	0.000	0.581	0.085
(0.013)	(0.222)	(0.020)	(0.030)	(0.037)	(0.055)	(0.052)	(0.056)
Probability parameters							
Malaysia Equation				Hong Kong Equation			
$\alpha_1(0,.)$	$\beta_1(0,1)$	$\alpha_1(1,.)$	$\beta_1(1,1)$	$\alpha_2(.,0)$	$\beta_2(1,0)$	$\alpha_2(.,1)$	$\beta_2(1,1)$
2.507	-1.149	-1.077	0.950	0.963	-1.219	-0.238	0.801
(0.270)	(0.365)	(0.480)	(0.616)	(0.147)	(0.403)	(0.184)	(0.458)
p-values of test statistics							
Malaysia				Hong Kong			
JB	LB(10)	LBS(10)		JB	LB(10)	LBS(10)	
0.004	0.725	0.104		0.000	0.708	0.162	

Table 7: Estimated parameters of the MCMS Model for Thailand/Hong Kong (standard errors in parentheses)

Switching coefficients - Constant Term							
Thailand Equation				Hong Kong Equation			
$\mu_1(0,0)$	$\mu_1(0,1)$	$\mu_1(1,0)$	$\mu_1(1,1)$	$\mu_2(0,0)$	$\mu_2(1,0)$	$\mu_2(0,1)$	$\mu_2(1,1)$
0.490	1.695	3.599	5.910	2.061	2.061	3.153	7.200
(0.029)	(0.299)	(0.293)	(0.407)	(0.081)	(0.273)	(0.241)	(0.349)
Autoregressive Terms							
Thailand Equation				Hong Kong Equation			
ϕ_{11}^1	ϕ_{12}^1	ϕ_{11}^2	ϕ_{12}^2	ϕ_{21}^1	ϕ_{22}^1	ϕ_{21}^2	ϕ_{22}^2
0.357	-0.010	0.148	-0.000	0.038	0.230	-0.000	0.146
(0.005)	(0.006)	(0.005)	(0.003)	(0.013)	(0.014)	(0.012)	(0.013)
Switching coefficients - Standard deviations				**Switching coefficients - Correlation Terms**			
Thailand Equation		Hong Kong Equation					
$\sigma_1(0,.)$	$\sigma_1(1,.)$	$\sigma_2(.,0)$	$\sigma_2(.,1)$	$\rho_{0,0}$	$\rho(0,1)$	$\rho(1,0)$	$\rho(1,1)$
0.261	2.898	0.933	2.154	0.000	0.000	0.546	0.044
(0.007)	(0.047)	(0.022)	(0.036)	(0.043)	(0.073)	(0.044)	(0.054)
Probability parameters							
Thailand Equation				Hong Kong Equation			
$\alpha_1(0,.)$	$\beta_1(0,1)$	$\alpha_1(1,.)$	$\beta_1(1,1)$	$\alpha_2(.,0)$		$\alpha_2(.,1)$	
1.544	-1.261	0.199	0.468	0.889		-0.166	
(0.213)	(0.311)	(0.202)	(0.383)	(0.136)		(0.178)	
p-values of test statistics							
Thailand				Hong Kong			
JB	LB(10)	LBS(10)		JB	LB(10)	LBS(10)	
0.000	0.089	0.087		0.000	0.027	0.806	