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The Effect of Seasonal
Adjustment on the Properties
of Business Cycle Regimes

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ABSTRACT

We study the impact of seasonal adjustment on the properties of business cycle expansion and recession regimes using analytical, simulation and empirical methods. Analytically, we show that the X-11 adjustment filter both reduces the magnitude of change at turning points and reduces the depth of recessions, with specific effects depending on the length of the recession. A simulation analysis using Markov switching models confirms these properties, with particularly undesirable effects in delaying the recognition of the end of a recession. However, seasonal adjustment can have desirable properties in clarifying the true regime when this is well underway. The empirical findings, based on four coincident US business cycle indicators, reinforce the analytical and simulation results by showing that seasonal adjustment leads to the identification of longer and shallower recessions than obtained using unadjusted data.

1. Introduction

The analysis of business cycle regimes has been a focus of interest for economists for many decades, dating back at least to the seminal work of Burns and Mitchell (1946). The essential implication of this stream of research is that expansions and recessions have distinct characteristics, which was interpreted by Hamilton (1989) in terms of a Markov-switching model with regime-dependent mean growth rates. A huge literature has subsequently developed, studying the nonlinear aspects of business cycle fluctuations in key indicators such as gross domestic product (GDP) and industrial production. Almost without exception, however, empirical studies of the business cycle employ data after seasonal adjustment.

Nevertheless, links between seasonality and the business cycle have not been ignored, with a number of studies showing that the seasonal patterns in important variables change with the stage of the business cycle (Canova and Ghysels, 1994, Cecchetti and Kashyap, 1996, Krane and Wascher, 1999, Matas-Mir and Osborn, 2004a, Osborn and Matas-Mir, 2003). One implication is that the use of seasonally adjusted data discards information relevant to the business cycle. More seriously, however, it also implies that the use of seasonally adjusted data may distort business cycle inferences. Perhaps surprisingly, despite a large literature on many aspects of seasonal adjustment, its impact on the properties of business cycle expansion and recession regimes appears to have been almost entirely overlooked.

Although there is now substantial evidence that nonlinearity is needed to capture the (nonseasonal) features of the business cycle (see, for example, Galvão, 2002 or Morley and Piger, 2005), it is unclear whether conventional seasonal adjustment aids or detracts from such analyses. This is an important question, because most macroeconomic policymakers and commentators in the US and other countries rely on official seasonally adjusted data, so that their views of business cycle expansions and recessions might be distorted, or perhaps

clarified, by the routine use of seasonal adjustment¹. We are aware of only two papers relevant to this issue. In a simulation and empirical study in a Markov-switching framework, Franses and Paap (1999) find that seasonal adjustment reduces the estimated probabilities of regime switches, while the empirical analysis of US GDP conducted by Luginbuhl and de Vos (2003) finds seasonal adjustment to result in longer and shallower estimated recessions. However, the generality of these results remains an open issue.

This paper examines the impact of seasonal adjustment on the estimated characteristics of business cycle regimes, focusing particularly on recessions. We complement and extend previous analyses by studying the impact of adjustment on business cycle phases from an analytical perspective, which produces the novel finding that the effect of adjustment depends on the length of the regime. In the simulation analysis that follows, we consider whether adjustment distorts the detection of business cycle regimes and the extent to which the regime-dependent mean growth rate is affected. To assess the empirical validity of these findings, we analyse the properties of estimated regimes in quarterly GDP, industrial production, employment and sales data for the US.

We primarily focus on the widely-used linear version of the X-11 seasonal adjustment program of the US Bureau of the Census, which remains the core of the Census X-12-ARIMA program (Findley *et al.*, 1998). We make no assumptions about seasonality changing over the business cycle, but focus on the implications of adjustment for the detection and properties of regimes. Quarterly data are assumed throughout, since this is the frequency typically employed when modelling business cycle regimes.

The plan of the paper is as follows. Section 2 examines the effect of adjustment on recession characteristics in the context of a simple regime-dependent model with known

¹ Christiano and Todd (2002) claim that a simple model of seasonality independent of the business cycle can capture key characteristics of the short-run dynamic relationships between observed US macroeconomic time series. However, their conclusions are based on a linear model and may not extend to a nonlinear regime-dependent business cycle model.

turning point dates. Subsequently nonlinear Markov switching models are applied to simulated data (Section 3), with both known and stochastic regimes, and considering a range of plausible values for the regime means. The empirical analysis in Section 4 verifies earlier findings through a comparison of the implications of models estimated using unadjusted and official seasonally adjusted data. Conclusions (Section 5) complete the paper.

2. Seasonal Adjustment Filters and Regimes

After some general points about the X-11 filter, Subsection 2.2 contains our analysis of the effect of the filter in a simple regime-dependent model.

2.1 The X-11 Filter

Many statistical agencies across the world base seasonal adjustment on procedures developed within the US Bureau of the Census, specifically the X-11 program. The X-11 filters are also incorporated into statistical software programs, allowing other users ready access to them. Although now further developed as X-12-ARIMA, the adjustment filters of X-11 remain the essence of this new program (see Findley *et al.*, 1998, or Ghysels and Osborn, 2001, Chapter 5). Due to their widespread use, we concentrate our analysis on the X-11 filters, whose properties and implications have been studied by many authors, including Bell and Hillmer (1984), Burridge and Wallis (1984), Franses and Paap (1999), Ghysels and Perron (1993, 1996), Sims (1974) and Wallis (1974).

As discussed in a number of studies, including Ghysels and Osborn (2001, Chapter 4), seasonal adjustment involves the application of a sequence of moving average filters. Therefore, denoting the original observed time series as y_t , the seasonally adjusted (or filtered) series y_t^{SA} is obtained as

$$y_t^{SA} = v(L) y_t \quad (1)$$

where, when applied to historical data, X-11 can be well approximated² by a symmetric two-sided linear moving average filter

$$v(L) = \sum_{i=-m}^m v_i L^i . \quad (2)$$

The coefficients v_i sum to unity over leads/lags $i = -m, \dots, 0, \dots, m$, with specific values for quarterly series given in Laroque (1977) and Ghysels and Perron (1993), with the latter also presenting those for the monthly case. Although often referred to as “weights”, it should be noted that some filter coefficients in (2), especially at seasonal lags, are negative. Nevertheless, the smoothing involved in this filter at nonseasonal lags is not trivial; for example, the coefficients for the quarterly case imply positive weights of 5.5 and 3.4 percent, respectively, for observations five and six quarters away from observation t in both directions.

The two-sided filter (2) cannot be used to seasonally adjust the most recent observations, since it then requires unknown future values³. This problem is solved in X-12-ARIMA, as well as in X-11-ARIMA used by Statistics Canada (Dagum, 1980), by employing an ARIMA (autoregressive integrated moving average) model to generate forecasts of the required future values, with the two-sided filter then applied; see Findley *et al.* (1998). Therefore, the two-sided filter of (2) plays a fundamental role in X-11 seasonal adjustment for both concurrent and historical data.

² Various options are available in both X-11 and X-12-ARIMA to deal with outliers, additive versus multiplicative adjustment, etc, which make the filter nonlinear; see Ghysels, Granger and Siklos (1996). Although nonlinearities introduced by seasonal adjustment are potentially important, our focus is on the impact of the linear filtering which is the core of seasonal adjustment.

³ The Monte Carlo analysis in an early version of this paper, Matas-Mir and Osborn (2004b), also analysed the implications of the one-sided X-11 filter.

2.2 Seasonal Adjustment of Regimes

A simple regime-dependent business cycle model is given by

$$y_t = \mu_0 + \mu_d s_t + x_t \quad (3)$$

where s_t is a binary regime indicator, μ_0 is the mean of y_t in the regime when $s_t = 0$, μ_d is the difference between the means in regimes $s_t = 1$ and $s_t = 0$; the properties of the zero-mean series x_t (which may be seasonal) are, for convenience, assumed to be invariant over regimes.

With the usual convention $\mu_d > 0$, $s_t = 1, 0$ can be interpreted as relating to expansions and recessions, respectively.

To focus on a single recession episode, consider the sequence

$$s_t = \begin{cases} 1 & t = \dots, T_B - 2, T_B - 1 \\ 0 & t = T_B, T_B + 1, \dots, T_C - 1 \\ 1 & t = T_C, T_C + 1, \dots \end{cases} \quad (4)$$

Since recessions are typically of relatively short duration, the filter of (2) applied at the points $t = T_B$ and $t = T_C$ (immediately subsequent to a peak and trough, respectively) may have a substantial overlap. A single switch between business cycle regimes can be regarded as a structural break, for which Ghysels and Perron (1996) examine the effect of seasonal adjustment. In such a context, (4) can be interpreted as a reversed structural break. However, the overlap of the adjustment filters applied $t = T_B$ and $t = T_C$ implies that the effect of adjustment on the regime indicator (4) before, during and subsequent to recessions may differ from the effects implied by two distinct (and distant) regime switches⁴.

Since the weights in (2) sum to unity, seasonal adjustment of (3) yields

$$y_t^{SA} = \mu_0 + \mu_d s_t^{SA} + x_t^{SA} \quad (5)$$

where x_t^{SA} retains its zero mean property. Of principal interest for our analysis, the regime indicator s_t is transformed in (5) to become

$$s_t^{SA} = v(L)s_t \quad (6)$$

which is no longer binary. This immediately suggests that, compared with (3), seasonal adjustment leads to a blurring of the distinction between regimes.

Given the timing of the regime switches as defined in (4), together with the properties of the seasonal adjustment filter (specifically symmetry and unit sum for the weights), it is relatively straightforward to show that (see Appendix 1)

$$s_t^{SA} = \begin{cases} \sum_{i=-m}^{T_B-t-1} v_i & T_B - m \leq t < T_C - m \\ \sum_{i=-m}^{T_B-t-1} v_i + \sum_{i=T_C-t}^m v_i & T_C - m \leq t < T_B \\ 2 \sum_{i=T_C-t}^m v_i + \sum_{i=T_B+1}^{T_C-t-1} v_{2i} & T_B \leq t < T_C \\ \sum_{i=-m}^{t-T_C} v_i + \sum_{i=t-T_B+1}^m v_i & T_C \leq t < T_B + m \\ \sum_{i=-m}^{t-T_C} v_i & T_B + m \leq t < T_C + m \\ 1 & \text{otherwise} \end{cases} \quad (7)$$

when we assume that $T_C < T_B + m$, so that the second regime-switch at T_C occurs within the period where the filtered values are affected by the initial switch at T_B . Since $m = 28$ for the quarterly X-11 filter, this is a realistic assumption in the context of a recession.

Figure 1 plots the values given by (7) for the two-sided linear quarterly X-11 filter for cases where $T_C = T_B + k$ for $k = 2, 3, 4, 5$, corresponding to recession durations between two and five quarters. The vertical scale sets $T_B = 0$, with the interpretation that periods are considered in relation to the start of a recession. As implied by (7), the transformed state variable is symmetrical around the recession.

Except for the case when the recession lasts exactly one year, seasonal adjustment leads to substantial distortion. Considering first this one-year special case, Ghysels and Perron

⁴ The relatively long duration of expansions implies that the filters applied at the trough and subsequent peak

(1993) show that the strong seasonal pattern in the filter weights of X-11 result in an annual cycle of distortions when a single structural break occurs. However, when this structural break is reversed after one year, the two annual cycles of distortions effectively cancel out, so that seasonal adjustment then has little impact on either turning point detection or on the magnitude of the intermediate regime.

A number of features are common to the other cases examined, when the recession regime has a duration of two, three or five quarters. Firstly, seasonal adjustment reduces the magnitude of the shift in mean between regimes, so that the depth of recessions is reduced. This is most marked when the duration is two quarters, when $s_t^{SA} \approx 0.1$ during the recession, instead of the true value of zero. Interpreted in terms of (3), this implies a reduction of around 10 percent in the mean shift, so that a recession of six months duration appears shallower after seasonal adjustment than actually the case. Interestingly, when the duration is five quarters, the regime shift is enhanced for the central three quarters of this regime, but nevertheless is reduced on average over the five quarters by around 3 percent. Since the filter weights sum to unity, any reduction (on average) of the recession depth implies a decrease in the average value in expansions. However, as expansions are generally of relatively long duration, this effect will generally be negligible.

Secondly, the magnitude of the regime change is reduced at the actual turning points. More specifically, Figure 1 implies a reduction of the step change by approximately 20 percent for all three relevant durations. This effect is a mixture of the reduction in the value of the regime indicator for the last period of an expansion and the “leakage” of expansion indicator values into the first seasonally adjusted recession value. A symmetrical effect also occurs at the end of the recession. While the two constituent parts differ for different regime

will typically not overlap.

durations, the total effect remains effectively constant for the cases considered (except, of course, for the special case of a one year duration, already discussed).

The third noteworthy feature is the distortion of the regime indicator for expansion observations shortly before and after recessions. In particular, this follows a marked seasonal pattern. Indeed, the pattern of increase followed by decrease shown in Figure 1 four and five quarters after the end of a recession (except for the case $k = 4$) gives spurious indication of a business cycle peak and a return to the recession regime.

These effects occur because X-11 implicitly assumes evolving seasonality. When a regime shift (or structural break) occurs, the filter is unable to distinguish fully between changing seasonality and the nonseasonal break, and hence it effectively allocates part of the regime change to a change in the seasonal pattern, as illustrated in Figure 1.

In summary, seasonal adjustment by X-11 reduces the average depth of recessions and marginally decreases average growth during expansions, makes the detection of business cycle turning points more difficult by reducing the magnitude of the regime change when it occurs, and introduces spurious evidence of business cycle turning points one and two years before and after a recession. In combination, these effects may reinforce each other. For example, the combination of the reduction in the magnitude of change at a trough and the spurious evidence of a turning point one year later may, in practice, lead to the trough being dated later than it actually occurred. This will further conflate recession and expansion observations, thereby reinforcing the tendency for seasonal adjustment to reduce the depth of a recession.

3. Seasonal Adjustment and Markov Switching Models: A Monte Carlo Study

The analysis of the previous section treats the turning point dates as known, whereas the true regimes are in fact unobserved. In common with much business cycle regime literature, we now estimate (3) through the use of a two-regime Markov switching model. The results are discussed after the methodology of our Monte Carlo study is outlined⁵.

3.1 Method

The simple model (3) is used as the data generating process (DGP), with $x_t = \varepsilon_t \sim NID(0, 1)$ and $\mu_d > 0$. Seasonally adjusted values are obtained by applying the quarterly linear X-11 seasonal adjustment filter for observations $t = 30, \dots, T - 29$, where initial and final values are lost due to the two-sided nature of the filter. To ensure comparability, the initial and final 29 observations are also discarded when analysing the unadjusted data. All experiments employ 10,000 replications.

Estimation employs a first-order Markov switching assumption for s_t with constant transition probabilities, so that:

$$\begin{aligned} P(s_t = 1 | s_{t-1} = 1) &= p & P(s_t = 0 | s_{t-1} = 1) &= 1 - p \\ P(s_t = 0 | s_{t-1} = 0) &= q & P(s_t = 1 | s_{t-1} = 0) &= 1 - q \end{aligned} \quad (8)$$

Although our DGP has no dynamics, seasonal adjustment induces serial correlation. To control for this, we adopt autoregressive augmentation with both adjusted and unadjusted data. Thus, our Monte Carlo experiments are based on estimating the model

$$\phi(L)y_t = \delta_0 + \delta_d s_t + e_t, \quad t = 34, \dots, T - 29 \quad (9)$$

⁵ Estimation is conducted in GAUSS, based on the procedures of van Norden and Vigfusson (1996). Further details can be found in the earlier version of this paper, Matas-Mir and Osborn (2004b).

with $\phi(L)$ being a fourth-order polynomial in the lag operator⁶ and e_t is a disturbance term which the researcher assumes to be an *iid* normal variate. This specification implies that the switching process changes the intercept⁷ and has been considered by, among many others, Hamilton (1990). For both filtered and unfiltered data, an additional four observations are required to create the autoregressive lags for the estimation of (9).

To further investigate the analytical implication of Section 2 that seasonal adjustment makes regime identification more difficult, our first set of simulations of (3) examines its impact on the detection of given business cycle regimes. For this purpose, the regimes are based on the National Bureau of Economic Research (NBER) business cycle chronology for the US⁸. We assume the use of quarterly data over 1951:I to 1996:IV, yielding $T = 183$ values for a growth rate y_t . There are five NBER recessions in this period, with durations from two to five quarters. We set $\mu_0 = -0.5$ and $\mu_d = 1.2$, implying expected negative annual growth of approximately 2 percent in recessions, with positive annual growth of approximately 2.8 percent in expansions.

A second experiment employs a Markov switching DGP. While this analysis also considers regime identification before and after filtering, the focus here is on the implications for regime means and transition probabilities. It is intuitively obvious that if seasonal adjustment adversely affects separation of recessions and expansions (and consequently affects their estimated characteristics), the effects may be accentuated when the underlying regimes are relatively close. Therefore, we investigate values of μ_d from 1.2 to 2.5 (with $\sigma = 1$ for both regimes). Here, in line with the typical sample size in empirical studies with

⁶ Although the lag length of four is frequently used in empirical analysis of quarterly data, whether seasonally adjusted or not, it only approximately removes the autocorrelation induced by seasonal adjustment. However, this fixed lag length is adopted for reasons of practicality in our Monte Carlo analysis.

⁷ The intercept-switching model is more practical than a mean-switching one in the context of a large Monte Carlo study, because it avoids the dependence of the conditional distribution of y_t on lags of the state process.

⁸ NBER turning point dates are converted to regimes by assuming that the new regime starts at the beginning of the quarter after the month of the turning point.

quarterly data, we generate $T = 160$ observations per replication using transition probabilities $p = 0.9$ and $q = 0.65$, which are also representative values for quarterly data.

Regimes are identified for the Markov switching model using the full-sample smoother probabilities $P(s_t = j | \mathbf{Y}_T; \hat{\theta})$, which deliver the optimal probabilistic inference that state j applied at time t , based on complete sample information on y_t to time T (Hamilton, 1989). These probabilities are a by-product of parameter estimation, with $\hat{\theta}$ being the vector of maximum likelihood parameter estimates for the Markov switching model of (8)/(9).

When regimes are stochastic, the performances of filtered and unfiltered data cannot be compared over replications for specific regime episodes. In this case, we summarise regime identification performance using the quadratic probability score (QPS), given by

$$\text{QPS} = \frac{2}{T_U - T_L + 1} \sum_{t=T_L}^{T_U} [P(s_t = 1 | \mathbf{Y}_T; \hat{\theta}) - s_t]^2 \quad (10)$$

with T_L and T_U being (respectively) the lower and upper sample observations for which regime inferences are obtained. The series and parameter estimates in (10) relate to filtered or unfiltered data, as appropriate.

For the estimated intercept-switching models, the implied regime-dependent means for $s_t = 0, 1$ are recovered using the weighted means

$$\hat{\mu}_j = \frac{\sum_{t=1}^T y_t P(s_t = j | \mathbf{Y}_T; \hat{\theta})}{\sum_{t=1}^T P(s_t = j | \mathbf{Y}_T; \hat{\theta})}, \quad j = 0, 1. \quad (11)$$

3.2 Results

This subsection first discusses the results relating to regime detection and then examines the impact of adjustment on the estimation of regime means and transition probabilities.

3.2.1. Regime detection

Regime identification results obtained from the simulations using the NBER chronology as the true regimes are shown in Figure 2, which summarises the empirical distributions for the full-sample probabilities for the filtered and unfiltered data sets corresponding to the recession (lower) regimes, with vertical lines denoting actual recession observations. The distributions of estimates are summarized by graphing the third quartile, median and first quartile values corresponding to each quarter.

The third quartile, shown in panel (a), indicates that seasonally adjusted data tend to point to the onset of recession too early and delay the recognition of recovery. Although there is a tendency for these features to occur also with unfiltered data, the pattern is more marked after seasonal adjustment. In other words, the filtered data obscures the dates of the regime changes, as predicted by the analysis of Section 2. However, although that analysis indicates symmetry for the distortion prior to the beginning of the regime and subsequent to its completion, the estimated regimes for the Markov switching models in Figure 2(a) show that filtered data capture the start of recession regimes more adequately than their end. This asymmetry presumably results from an interaction of the effects of seasonal adjustment with regime inference within the Markov-switching model.

At the same time, however, it is also clear that once an expansion regime is well under way, the third quartile lower regime probabilities after adjustment are generally (and correctly) closer to zero than their unfiltered counterparts. Therefore, the smoothing inherent in filtering reduces the chances that low-valued observations generated within an expansion are mistakenly attributed to a recession regime.

Turning to the median and first quartile of probabilities, panels (b) and (c) respectively of Figure 2, seasonal adjustment has the desirable effect of signalling recession periods more strongly than the unfiltered data. This effect is most noticeable when the lower regime lasts a

year or longer (see those after $t = 74$, $t = 90$ and $t = 122$ in the graph). Interestingly, the median and first quartile probabilities during the short recession of two quarters (commencing after $t = 114$) are very similar whether filtered or unfiltered data are used.

Although not associated with seasonal adjustment, it is also notable that the Markov switching model may completely miss genuine recessions. More specifically, when the recession duration is a year or less, the first quartile values in Figure 2(c) never rise above 0.5 during the recession whether adjusted or unadjusted data are employed, and they do not reach 0.5 using unadjusted data even when the recession duration is five quarters. Indeed, within the short recession of two quarters, the median probability barely reaches 0.5 for either type of data.

For the Markov switching DGP, the overall QPS of (10) is computed for each replication, with separate QPS values also computed for observations in the upper and lower regimes, classified according to the actual s_t . Table 1 summarises the results, using the median QPS. It is not surprising that, in all cases, regime detection as measured by QPS deteriorates as the difference between regime means is reduced; that is, QPS increases as μ_d decreases.

Table 1 also shows that the effects of filtering are not negligible. The median overall QPS is up to 21 percent larger with adjusted data, with the unadjusted data performing relatively better in terms of overall QPS when there is a larger distance between the regimes. In expansions, QPS can be increased by up to 50 percent by filtering, which is at least partly due to the later recognition of business cycle troughs. However, when the regime means are relatively close, with $\mu_d = 1.2, 1.4, 1.6$, filtered data performs better than unfiltered data in detecting the occurrence of recessions, which is compatible with the performance in Figure 2 using the NBER dates. Nevertheless, in such cases both perform poorly. For example, a QPS

of 0.5 (approximately the median value for both adjusted and unadjusted data when $\mu_d = 1.6$) would result from a recession probability being 0.5 throughout the regime.

In summary, seasonal adjustment acts against timely detection of regime switches and Figure 2 indicates that this cost is particularly evident in the belated recognition of the beginning of recovery from recession. Similarly, it unambiguously worsens performance according to QPS both overall and within expansions. However, it can improve identification of a recession when expansion and recession regimes are relatively close in terms of their underlying mean growth rates, although identification of the true regime is then poor irrespective of whether seasonal adjustment is applied or not.

3.3.2 Regime means and transition probabilities

Table 2 summarises the empirical distributions of estimation errors (estimated less actual) for the regime means and transition probabilities, in terms of median, mean, skewness and kurtosis. Due to the large skewness exhibited in many cases, the median and mean values often differ substantially. Kurtosis is also a feature for estimation of the regime means. Due to this substantial non-normality, the discussion focuses on the median estimation error as providing a more representative summary measure than the mean.

The median estimate of the recession regime mean, μ_0 , in panel (a) always underestimates the true (negative) value when seasonally adjusted data are used. The magnitude of this underestimation ranges from 0.22 to 0.12, implying estimated mean declines (at an annual rate) of between approximately 1.1 to 1.5 percent rather than the true 2 percent. The extent of underestimation is greatest when μ_d is largest, with this being the balance of two opposing effects. That is, the better identification of the recession regime that occurs in this case is outweighed by the greater impact of contamination from the expansion regime that occurs when the regime means differ substantially.

It is worth noting that the (sample) mean error is always less than the median error for estimation of the lower regime mean when filtered data are used. For example, when $\mu_d = 1.4$, the median error is 0.14, compared with a mean error of 0.06. However, the practical importance of the latter is questionable, since the median error implies a 50-50 chance of underestimating the depth of the recession by 0.55 percent in annual growth terms. There seems little comfort in the average error being substantially lower due to the possible occurrence of very large errors in the opposite direction.

In contrast to the filtered case, the recession regime mean is always estimated relatively accurately by the median value when unfiltered data are employed.

Not surprisingly, seasonal adjustment plays much less role for estimation of the upper regime mean, $\mu_1 = \mu_0 + \mu_d$, in panel (b). Nevertheless, whether the data are filtered or not, there is an increasing tendency for the median value to overestimate the true mean as μ_d declines. Further, in terms of the absolute value of the median error, filtering leads to more accurate estimates in all cases except the relatively extreme one where $\mu_d = 2.5$.

Further light is shed on these issues by the estimation errors in the transition probabilities, shown in panels (c) and (d) of Table 2. From panel (c) it is clear that filtering leads to overestimation of the lower regime transition probability q , with (not surprisingly) this being most acute when the regimes are relatively close and hence regime identification of individual observations is most difficult. This effect is not negligible: when $\mu_d = 1.2$, the median error of 0.115 for this transition probability implies a mean recession duration of 4.25 quarters rather than the true 2.8 quarters. In turn, the overestimation of this probability under unfiltered data is much less pronounced, with the estimated duration at the median for $\mu_d = 1.2$ being 3.3 quarters. The pattern of overestimation of the recession transition probability q is, of course, inherently linked with the delayed recognition of troughs evident in Figure 2, which (as already noted) is particularly marked after seasonal adjustment.

The median error in estimating the expansion regime transition probability p is relatively small, compared both with the corresponding recession probability and in relation to the true DGP value $p = 0.95$. Nevertheless, as with the expansion regime mean, the smoothing property of seasonal adjustment acts to improve the accuracy of estimation of the corresponding transition probability when $\mu_d = 1.2, 1.4$.

Therefore, these simulation results confirm the implication of Section 2 that seasonal adjustment leads to underestimation of the depth of a recession. This appears to be due partly to the Markov switching model belatedly recognising the occurrence of a business cycle trough (and hence mis-classifying initial expansion observations as being part of the recession) and to the smoothing in seasonal adjustment leading to “leakage” of a portion of expansion regime observations into ones from the lower regime, as analysed in Section 2.

Our simulation results substantially extend those of Franses and Paap (1999) relating to the estimation of transition probabilities. From a business cycle analysis perspective, their simulations consider only an extreme case, which correspond to $\mu_d = 4$ when $\sigma = 1$. Indeed, such a set-up is not representative of any of the key US business cycle indicators analysed in Section 4. As evident from Table 2, the largest (median) errors in estimating the transition probabilities, and hence regime duration, occur when regimes are in relatively close proximity, so that the empirical relevance of results based on large values of μ_d is questionable.

4. Seasonal Adjustment and the US Business Cycle

In order to investigate the empirical impact of seasonal adjustment on the estimated characteristics of business cycle regimes, we apply Markov switching models to quarterly seasonally adjusted and unadjusted values of four variables that are key to the decisions of the

NBER Business Cycle Dating Committee, namely real GDP, industrial production (IP), employment and sales⁹, in order to investigate the impact of adjustment on regime inference. Data issues are discussed in Appendix 2, but it is important to note that published seasonally adjusted values are used and hence the results provide a check on the analysis of earlier sections based on the linear X-11 filter. All series are analysed as quarterly growth rates, namely after taking the first differences of the logarithms.

Although some individual indicators, particularly GDP, may mimic the NBER dates better than others, NBER turning point dates are nevertheless estimates obtained using seasonally adjusted values for a range of series. Therefore, it is appropriate to consider separately the four variables above when examining business cycle effects.

Business cycle regimes are here derived from a Markov-switching model where the mean switches with the regime¹⁰, so that

$$\phi(L)(y_t - [\mu_1^+(1 - s_t) + \mu_1^- s_t]) = \sigma z_t, \quad (12)$$

where $z_t \sim NID(0, 1)$ and s_t again represents the unobserved Markov process with transition probabilities defined in (8). For reasons of parsimony in estimation, deterministic seasonality in the unadjusted series is removed by a prior regression. As discussed in Appendix 2, this prior regression is also used to investigate, and where appropriate take account of, changes in the deterministic seasonal pattern. For both adjusted and unadjusted data, the autoregressive lag order k in (12) is determined by minimizing¹¹ BIC over $k = 1, \dots, 5$, with estimation by maximum likelihood.

However, the assumption of constant disturbance variance over time in (12) may be invalid, since there is now a large body of evidence documenting a break in the variance of

⁹ Although personal income is also used by the NBER, seasonally unadjusted data for this series could not be obtained.

¹⁰ Estimation of this model is feasible in this context, since we do not face the same constraints as in the Monte Carlo analysis of Section 3.

US real output growth around the mid-1980s (see, among others, McConnell and Perez-Quiros, 2000, Sensier and van Dijk, 2004). To capture this, our models for IP and GDP follow McConnell and Perez-Quiros (2000) by allowing an endogenous break in the variance through a second unobservable Markov process, so that the models for these variables have the form

$$\phi(L)(y_t - \{(1-r_t)[\mu_1^+(1-s_t) + \mu_1^-s_t] + r_t[\mu_2^+(1-s_t) + \mu_2^-s_t]\}) = [(1-r_t)\sigma_1 + r_t\sigma_2]z_t \quad (13)$$

where s_t and r_t are independent Markov switching processes determining the state of the economy (expansion or recession) and the state of the variance (high or low)¹², respectively. Following McConnell and Perez-Quirós (2000), (13) also allows the strength of expansions and recessions to change in the low-variance state compared to the high-variance one.

Table 3 presents the estimated models for both seasonally adjusted (SA) and seasonally unadjusted (NSA) data. For all four series, and in accord with the simulation results in Section 3, the use of seasonally adjusted data leads to recessions that are more persistent compared with unadjusted data, with estimated recession transition probability q larger after seasonal adjustment. This is reflected in the estimated expected recession regime duration, also shown in Table 3. Note, in particular, that GDP recessions are estimated to last less than three quarters on average using seasonally adjusted data, but over a year using adjusted data.

Although the estimated expansion regime transition probabilities are generally similar irrespective of seasonal adjustment, they result in substantially different estimated average expansion regime durations, due to the proximity of these values to unity. Indeed, for IP and

¹¹ For GDP, both seasonally adjusted and unadjusted, this procedure led to high-order AR models whose smoothed probabilities did not reflect satisfactorily US expansions and recessions. Therefore, the reported results derive from models using the second-best BIC in each case.

¹² Our results for the variance process are very similar to those of McConnell and Perez-Quirós (2000), with the high-variance state dominating up to 1984:I and a low-variance period in force for the remainder of the sample, with this being the case irrespective of whether seasonally adjusted or unadjusted data are used.

sales, the use of filtered data leads to expansions that are estimated to last 6 and 9 quarters more, respectively, compared with NSA data¹³.

Figure 3 presents the estimated smoothed probabilities for the recession regime, with the NBER recessions indicated by shading. It is unsurprising that the use of adjusted data leads to recession regimes more closely corresponding to those dated by the NBER, due to their use of such data when constructing the NBER turning point chronology. However, it is also evident that the use of unadjusted data would sometimes result in a shorter recession; see, for example, the regime probabilities for GDP during the recession around 1970. Again for GDP, it is also clear that seasonal adjustment conflates what would be identified with unadjusted data as a “double dip” (or perhaps “triple dip”) recession during 2000 to 2002, with this pattern also evident for IP. It is also interesting to note that the recession dates for 1990 are more closely matched using unadjusted than seasonally adjusted GDP.

This empirical evidence also corroborates the simulation results that the regime probabilities obtained using seasonally adjusted data tend to delay the recognition of recovery, compared with unadjusted data. This is particularly evident for GDP in the recessions of the early 1970s, 1990s and 2000s, for IP in the 2000 recession and for employment in the 1990 and 2000 recessions¹⁴. Adjustment also sometimes leads to the onset of recession being dated earlier, which is particularly notable for GDP.

In almost all cases, and as anticipated from the analysis of earlier sections, the use of unadjusted data leads to deeper estimated recessions in Table 3. The single exception is for IP in the high variance state. Indeed, the feature of filtering implying less severe recessions is especially notable for GDP in the low-variance state (after 1984), where the seasonally

¹³ The relatively long estimated average expansion regime duration for these two series in Table 3 is partly an artefact of the fact that the starting point of these series omits the three NBER recessions of the 1950s and 1960s.

¹⁴ It is interesting to note that the idea, frequently cited by commentators, that the US recovery after the 2000 recession seems to be a “jobless recovery” is borne out by the smoother probabilities when using seasonally adjusted data, but substantially less so using seasonally unadjusted data.

adjusted estimate of the mean implies an average annual positive growth of 0.8 percent in the low growth (or recession) regime, whereas unadjusted data yield an estimated 0.6 percent decline. The estimated average decline in employment during recessions (0.36 percent in annual terms) is also mild compared to the 0.6 percent implied by seasonally unadjusted data.

5. Concluding Remarks

This paper has considered the effects of seasonal adjustment on the characteristics of business cycle regimes, both analytically and when these regimes are identified through the use of a Markov switching model. We show that seasonal adjustment distorts information about the extent and timing of turning points that underlie regime identification, while leading to apparently less deep recessions.

Nevertheless, the picture is not entirely straightforward, because in some instances seasonal adjustment can have the effect of clarifying the regime. Indeed, through the smoothing inherent in seasonal adjustment, the filtered data tend to produce less false turning point signals, albeit they will detect the occurrence of actual turning points with more difficulty. As measured by summary statistics of regime tracking like the Quadratic Probability Score, however, the filtering deteriorates the fit of regimes in the Markov switching model overall, with this result being mainly dominated by a belated signal of the occurrence of a business cycle trough. Our analytical results shed light on why this occurs, as (except when a recession lasts exactly a year), seasonal adjustment reduces the magnitude of business cycle turning points by the order of 20 percent even when the turning point dates are known.

Empirical analysis based on US business cycle indicators confirms that these results carry over to observed data series. It is particularly notable for US GDP that the use of

unadjusted data would, in general, imply shorter and sharper recessions than when adjusted data are employed, which confirms the empirical result of Luginbuhl and de Vos (2003) based on a different model. Interestingly, unadjusted GDP and industrial production point to a “double dip” recession in the US during 2000 to 2002, rather than a single recession regime.

Although the simulation and empirical analyses employ Markov switching models as a convenient way to estimate business cycle regimes, the analytical results of Section 2 imply that seasonal adjustment will have qualitatively similar effects on regime characteristics irrespective of the method used to identify business cycle turning points.

APPENDIX 1

Seasonally Adjusted Regime Indicator

In order to obtain (7), consider first the application of seasonal adjustment to the structural break indicator series

$$b_t^B = \begin{cases} 0 & t = \dots, T_B - 2, T_B - 1 \\ 1 & t = T_B, T_B + 1, \dots \end{cases} \quad (\text{A.1})$$

where the break occurs at $t = T_B$. When the two-sided filter of (2) is applied in the context of the structural break (A.1), the filtered series is given by

$$b_t^{B,SA} = \begin{cases} 0 & t < T_B - m \\ \sum_{i=T_B-t}^m v_i & T_B - m \leq t < T_B \\ 1 - \sum_{i=t-T_B+1}^m v_i & T_B \leq t < T_B + m \\ 1 & t \geq T_B + m \end{cases} \quad (\text{A.2})$$

where we use the symmetry of the filter and the fact that the weights sum to unity. As indicated by a comparison of (A.1) and (A.2), all filtered values for $T_B - m \leq t < T_B + m$ are influenced by the structural break. Although they do not present an expression such as (A.2), Ghysels and Perron (1996) graphically present the distortion implied by (A.2) compared with (A.1).

Defining the break series b_t^C , where the break occurs at $t = T_C$, in an analogous way to (A.1), then the regime indicator of (4) can be obtained as

$$s_t = 1 - b_t^B + b_t^C \quad (\text{A.3})$$

and hence, with linear seasonal adjustment,

$$s_t^{SA} = 1 - b_t^{B,SA} + b_t^{C,SA}. \quad (\text{A.4})$$

Substitution from appropriate expressions from (A.2) into (A.4) then yield (7).

APPENDIX 2

Data Issues

For the empirical analysis of Section 4, real GDP are obtained from the Bureau of Economic Analysis (BEA) over 1953:I to 2003:IV. Although the BEA does not publish seasonally unadjusted real GDP, we derive such a series by multiplying seasonally adjusted real values by the ratio of the unadjusted to adjusted nominal values, as in Miron and Zeldes (1988). As little seasonality is anticipated in prices relative to seasonality in real GDP, this should provide a good approximation (see Barsky and Miron, 1989).

Wholesale and retail sales data cover 1967:I to 2004:II and are constructed employing the same procedure as for GDP. Seasonally adjusted real series are from BEA, while nominal seasonally unadjusted and adjusted values are obtained from the OECD Main Economic Indicators (MEI) database¹⁵. Wholesale and retail sales are summed to yield total sales. Industrial production (IP) and employment are from the OECD MEI database, over 1962:I to 2004:II and 1960:I to 2004:II respectively, in both cases seasonally adjusted and unadjusted.

The sample period used is the longest available in each case, with the exception of industrial production, where data are available from 1960:I. However, Polzehl *et al.* (2004) find evidence of a break in the variance of this series around the summer of 1961. To avoid a potential break early in the sample, our analysis of IP starts in 1962:I.

¹⁵ To check the compatibility of these two sources, we computed the correlation of the log-differences between the nominal seasonally adjusted BEA and OECD series; the correlation was 0.995. All models using the BEA sales indicator include a dummy variable that is unity in 1997Q1 and zero elsewhere to link the real series based in 1992 dollars to that based in 1997 dollars, which are published separately by the BEA.

Two obvious approaches for allowing for deterministic seasonal effects in a Markov switching model for seasonally unadjusted data are to augment the model with dummy variables to account for deterministic seasonality, with all parameters jointly estimated, or to remove deterministic seasonal effects prior to estimating the Markov switching model. For practical reasons we adopt this second approach in the empirical application of Section 4¹⁶.

The preliminary regression starts from a simple AR model with seasonal dummies, where the AR lag length minimizes BIC over lags $k = 1, \dots, 5$, provided that the selected model yields non-rejection of serial correlation using an LM test to order 5 at significance level 0.01. However, visual examination of some series (especially GDP) points to the possibility that the nature of seasonality may have changed over our sample period. Therefore, using the selected lag length, we test for a break in the deterministic seasonal pattern at unknown break date t_B using the asymptotic critical p -values of Hansen (1997). More specifically, we apply the test within the model

$$y_t = \mu + \sum_{i=1}^3 \delta_i \tilde{D}_{i,t} + I_t \sum_{i=1}^3 \delta_i^* \tilde{D}_{i,t} + \sum_{j=1}^k \phi_j y_{t-j} + \varepsilon_t, \quad (\text{A.5})$$

where the seasonal dummies are constructed as $\tilde{D}_{i,t} = D_{i,t} - D_{4,t}$ ($D_{q,t}$ being the conventional binary seasonal dummy variable for quarter q) and I_t is an indicator variable such that $I_t = 0$, $t < t_B$ and $I_t = 1$, $t \geq t_B$; the null hypothesis under scrutiny is $H_0 : \delta_i^* = 0$. The results are shown in Table A.1. For employment, sales and IP, the null hypothesis cannot be rejected at the 5 percent significance level, so we adopt the two-stage procedure outlined above.

For GDP, however, there is strong evidence in Table A.1 of a break in the seasonal pattern. Further, there is marked visual evidence of a seasonal deterministic trend in the early part of the sample, which then apparently disappears. Based on this evidence we also apply the Hansen test in the context of the model

$$y_t = \mu + \sum_{i=1}^3 (\delta_i + \tau_i t) \tilde{D}_{i,t} + I_t \sum_{i=1}^3 (\delta_i^* + \tau_i^* t) \tilde{D}_{i,t} + \sum_{j=1}^k \phi_j y_{t-j} + \varepsilon_t, \quad (\text{A.6})$$

with the null hypothesis $H_0 : \delta_i^* = \tau_i^* = 0$. This hypothesis is rejected at the 1 percent significance level, so that the first-step regression in this case is based on (A.6) including the break terms with $t_B = 1977\text{Q2}$.

¹⁶ In practice, we have found these methods usually deliver very similar results. However, on occasion we found that joint estimation leads to convergence problems, presumably due to the relatively large number of parameters in the non-linear optimisation. Therefore, we adopt the two-step approach.

Having uncovered evidence of a break in deterministic seasonality for GDP, (A.6) is applied with no autoregressive dynamics ($k = 0$) for the purpose of purging the series of deterministic seasonality. This ensures that overall seasonal mean effects are estimated, with dynamics then captured within the Markov switching model.

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Table 1. Quadratic Probability Score for Business Cycle Regimes Identified in Markov Switching Data Generating Processes.

μ_d	<i>Filtered data</i>			<i>Unfiltered data</i>			<i>Ratio</i>		
	<i>Overall</i>	<i>Upper</i>	<i>Lower</i>	<i>Overall</i>	<i>Upper</i>	<i>Lower</i>	<i>Overall</i>	<i>Upper</i>	<i>Lower</i>
2.5	0.128	0.065	0.268	0.106	0.051	0.228	1.211	1.271	1.176
2	0.195	0.096	0.389	0.169	0.078	0.361	1.150	1.228	1.077
1.8	0.228	0.114	0.442	0.204	0.090	0.440	1.116	1.275	1.003
1.6	0.264	0.138	0.500	0.242	0.102	0.526	1.093	1.357	0.950
1.4	0.307	0.170	0.556	0.285	0.118	0.622	1.078	1.444	0.893
1.2	0.354	0.218	0.599	0.326	0.140	0.722	1.085	1.563	0.830

Notes: Results refer to the Monte Carlo analysis based on (3) and (9) as the data generating process, with $\mu_0 = -0.5$, $p = 0.9$ and $q = 0.65$, for a sample of $T = 160$ observations. The estimated model is given by (9), with 10,000 replications employed.

Table 2. Properties of Estimation Errors in Markov Switching Data Generating Processes.

<i>Lower regime mean μ_0, filtered data</i>					<i>Lower regime mean μ_0, unfiltered data</i>			
μ_d	<i>Median</i>	<i>Mean</i>	<i>Skewness</i>	<i>Kurtosis</i>	<i>Median</i>	<i>Mean</i>	<i>Skewness</i>	<i>Kurtosis</i>
2.5	0.224	0.200	-1.149	5.598	0.003	-0.045	-1.739	12.591
2	0.193	0.149	-1.366	5.429	-0.006	-0.091	-1.529	5.537
1.8	0.181	0.120	-1.489	5.301	-0.007	-0.098	-1.466	5.441
1.6	0.168	0.093	-1.477	4.652	-0.007	-0.114	-1.666	6.267
1.4	0.138	0.056	-1.580	5.126	-0.009	-0.129	-1.888	7.943
1.2	0.123	0.017	-1.702	5.218	-0.028	-0.160	-1.907	7.989
<i>Upper regime mean μ_1, filtered data</i>					<i>Upper regime mean μ_1, unfiltered data</i>			
μ_d	<i>Median</i>	<i>Mean</i>	<i>Skewness</i>	<i>Kurtosis</i>	<i>Median</i>	<i>Mean</i>	<i>Skewness</i>	<i>Kurtosis</i>
2.5	-0.047	-0.041	0.494	2.376	0.013	0.025	2.038	21.811
2	-0.019	0.006	2.830	22.748	0.028	0.063	4.041	38.101
1.8	-0.001	0.041	3.722	31.273	0.042	0.094	4.592	48.694
1.6	0.025	0.073	3.249	24.61	0.065	0.139	4.272	35.777
1.4	0.052	0.117	3.520	24.275	0.095	0.196	3.903	26.48
1.2	0.087	0.164	3.437	22.035	0.130	0.249	3.364	18.21
<i>Lower regime transition probability q, filtered data</i>					<i>Lower regime transition probability q, unfiltered data</i>			
μ_d	<i>Median</i>	<i>Mean</i>	<i>Skewness</i>	<i>Kurtosis</i>	<i>Median</i>	<i>Mean</i>	<i>Skewness</i>	<i>Kurtosis</i>
2.5	0.031	0.004	-1.118	1.833	-0.004	-0.031	-1.030	1.445
2	0.048	0.009	-1.122	1.340	0.006	-0.036	-0.999	0.861
1.8	0.057	0.012	-1.117	1.072	0.016	-0.034	-0.975	0.593
1.6	0.072	0.022	-1.127	0.953	0.027	-0.029	-0.959	0.393
1.4	0.094	0.033	-1.139	0.882	0.038	-0.018	-0.957	0.297
1.2	0.115	0.046	-1.176	0.888	0.05	-0.011	-0.967	0.184
<i>Upper regime transition probability p, filtered data</i>					<i>Upper regime transition probability p, unfiltered data</i>			
μ_d	<i>Median</i>	<i>Mean</i>	<i>Skewness</i>	<i>Kurtosis</i>	<i>Median</i>	<i>Mean</i>	<i>Skewness</i>	<i>Kurtosis</i>
2.5	0.007	-0.003	-1.595	4.760	-0.001	-0.012	-3.228	26.811
2	0.008	-0.015	-3.208	18.877	-0.002	-0.025	-2.999	15.979
1.8	0.007	-0.024	-3.149	15.331	-0.003	-0.036	-3.007	13.486
1.6	0.005	-0.031	-2.637	10.52	-0.007	-0.051	-2.614	9.137
1.4	0.004	-0.042	-2.614	9.465	-0.012	-0.070	-2.377	6.706
1.2	-0.001	-0.055	-2.445	7.730	-0.018	-0.087	-2.104	4.766

Notes: See Table 1. The estimation error is calculated as the estimated value less the actual value of the corresponding parameter.

Table 3. Estimation Results for Markov Switching Models for US Business Cycle Indicators.

(a) Parameter estimates.

<i>Parameter</i>	<i>GDP</i>		<i>IP</i>		<i>Employment</i>		<i>Sales</i>	
	<i>SA</i>	<i>NSA</i>	<i>SA</i>	<i>NSA</i>	<i>SA</i>	<i>NSA</i>	<i>SA</i>	<i>NSA</i>
μ_1^+	1.2589 (0.1540)	1.2655 (0.1749)	1.0454 (0.2304)	1.0653 (0.2245)	0.5747 (0.0597)	0.5539 (0.0769)	1.2358 (0.1067)	1.3010 (0.1189)
μ_1^-	-0.2993 (0.2634)	-0.4246 (0.3064)	-3.5091 (0.7845)	-3.5086 (1.1314)	-0.0908 (0.1410)	-0.1469 (0.1929)	-1.3007 (0.3825)	-1.6236 (0.4281)
μ_2^+	0.9236 (0.0740)	0.9268 (0.0668)	0.9350 (0.1115)	0.9928 (0.1167)	-	-	-	-
μ_2^-	0.2094 (0.2144)	-0.1511 (0.2964)	-0.9813 (0.3620)	-1.1058 (0.3866)	-	-	-	-
σ_1^2	0.7880 (0.1306)	1.1473 (0.2123)	1.6030 (0.2601)	2.2366 (0.3853)	0.1217 (0.0164)	0.1852 (0.0261)	1.1423 (0.1516)	1.9284 (0.2809)
σ_2^2	0.1751 (0.0337)	0.2125 (0.0477)	0.2777 (0.0520)	0.4960 (0.1242)	-	-	-	-
ϕ_1	0.0693 (0.0984)	-0.0874 (0.1036)	0.4792 (0.0881)	0.2879 (0.0850)	0.3427 (0.0917)	0.2858 (0.1151)	0.0289 (0.1104)	-0.2830 (0.0901)
ϕ_2	-	-	0.0137 (0.0973)	0.1210 (0.0746)	-	-0.1917 (0.0688)	-	-
ϕ_3	-	-	0.1599 (0.0963)	-0.0857 (0.0766)	-	-0.0095 (0.0745)	-	-
ϕ_4	-	-	-0.0872 (0.0852)	0.4033 (0.0810)	-	0.5563 (0.0709)	-	-
ϕ_5	-	-	-0.1850 (0.0766)	-0.5381 (0.0777)	-	-0.3018 (0.1012)	-	-
p	0.9182 (0.0343)	0.9019 (0.0395)	0.9633 (0.0172)	0.9527 (0.0288)	0.9431 (0.0277)	0.9572 (0.0304)	0.9582 (0.0217)	0.9333 (0.0336)
q	0.7646 (0.0914)	0.6491 (0.1299)	0.6100 (0.1341)	0.5341 (0.1683)	0.8270 (0.0998)	0.7748 (0.1236)	0.6534 (0.1513)	0.5278 (0.1712)
<i>Log-L</i>	-240.90	-276.92	-235.45	-250.81	73.62	40.87	-104.92	-149.11

Notes: *SA* refers to seasonally adjusted data and *NSA* to non-seasonally adjusted data. Values in parentheses are standard errors; μ_i^+ , μ_i^- refer to the estimated means in the upper and lower regimes, respectively; μ_2^+ , μ_2^- , and σ_2 refers to the lower variance state, which is not applicable for employment and sales.

(b) Estimated average durations of expansions and recessions.

	<i>GDP</i>		<i>IP</i>		<i>Employment</i>		<i>Sales</i>	
	<i>SA</i>	<i>NSA</i>	<i>SA</i>	<i>NSA</i>	<i>SA</i>	<i>NSA</i>	<i>SA</i>	<i>NSA</i>
<i>Expansions</i>	12.22	10.19	27.24	21.24	17.57	23.36	23.92	14.99
<i>Recessions</i>	4.24	2.84	2.56	2.14	5.78	4.44	2.89	2.12

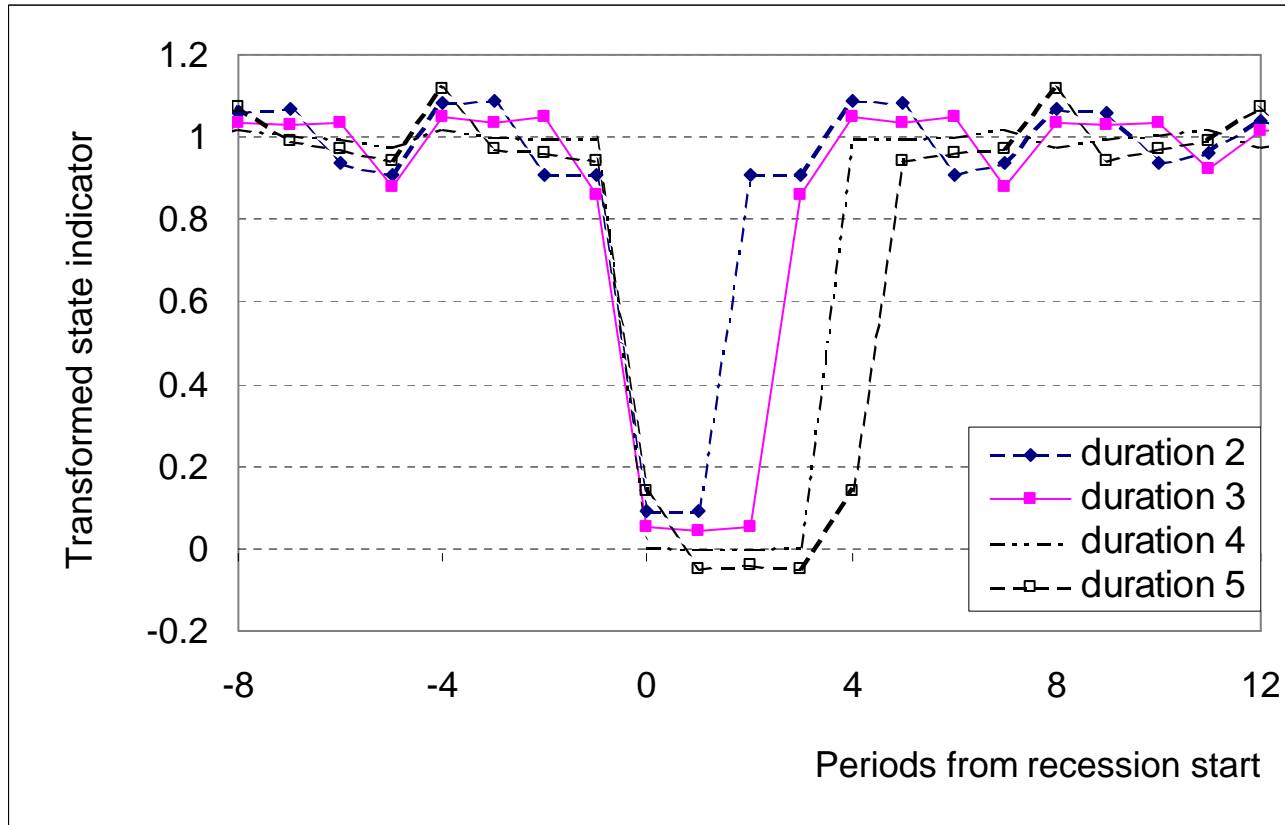
Notes: Estimated average durations are expressed in quarters and are obtained using the estimated p and q in panel (a)

Table A.1. Results of Tests for Break in Deterministic Seasonality for US Business Cycle Indicators.

<i>Series</i>	<i>Break date</i>	<i>P-value</i>	<i>AR order</i>
GDP	1977Q2	0.00	5
IP	1981Q3	0.06	5
Employment	1983Q2	0.86	5
Sales	1974Q3	0.07	1

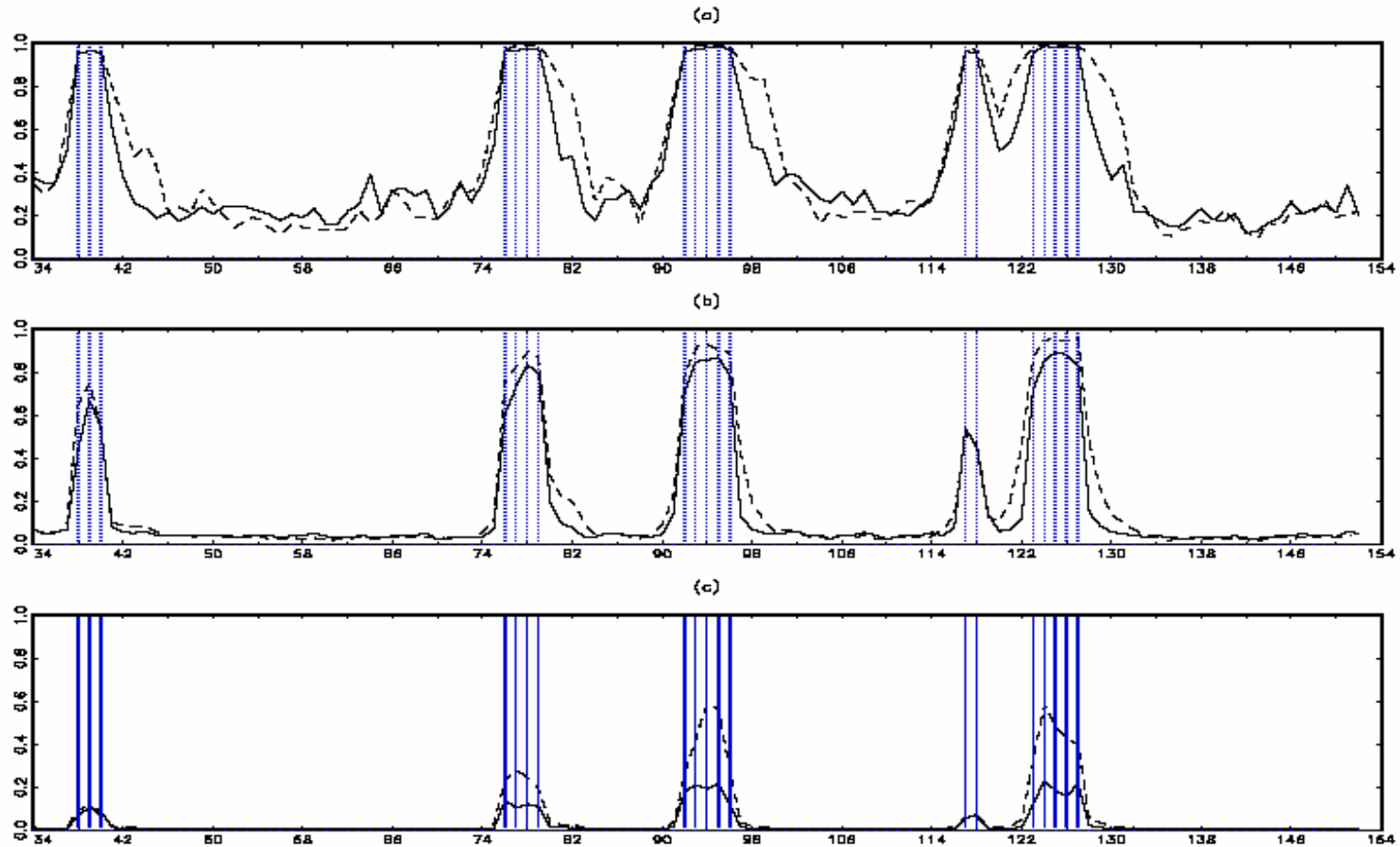
Notes: Seasonal components are seasonal means for all the series except GDP, for which a seasonal trend is also allowed prior to the break date. The AR order reports the number of autoregressive lags used in the test.

Figure 1. The Business Cycle Regime Indicator after Seasonal Adjustment using the Linear X-11 Filter.



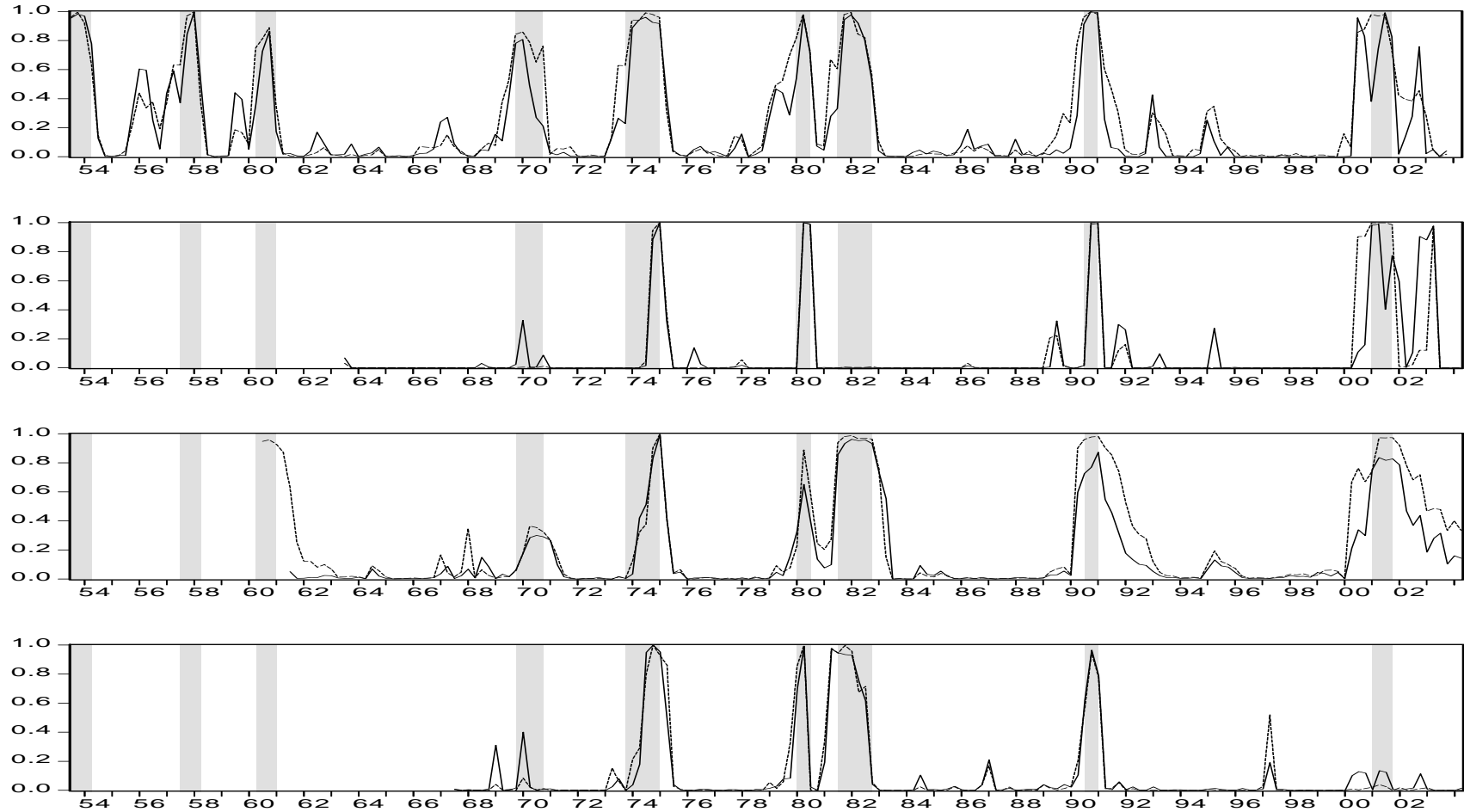
Notes: The graph shows the results of applying the linear X-11 quarterly seasonal adjustment filter to the business cycle regime indicator of equation (4) for recessions of duration two to five quarters.

Figure 2. Markov Switching Recession Probabilities from Monte Carlo Analysis using NBER Business Cycle Dates. Panel (a) refers to the third quartile, panel (b) to the median and panel (c) to the first quartile.



Notes: Results refer to the Monte Carlo analysis based on (3) as the data generating process, with $\mu_0 = -0.5$, $\mu_d = 1.2$, for a sample of $T = 183$ observations, with true regimes given by the business cycle phases dated by the NBER over 1951:I to 1996:IV. The estimated model is (9), with 10,000 replications employed. Horizontal lines show estimated recession probabilities for seasonally adjusted (dashed line) and non-seasonally adjusted (solid line) data, where seasonal adjustment is by the linear X-11 filter; vertical lines indicate NBER recession observations.

Figure 3. Estimated Recession Probabilities for US Business Cycle Indicators.



Notes: Lines show estimated recession probabilities for seasonally adjusted (dashed line) and non-seasonally adjusted (solid line) data for GDP, IP, employment, sales (respectively, from top to bottom). Shading indicates NBER recession observations.

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