Dipartimento di Statistica
"Giuseppe Parenti"

WORKING PAPER 2007/11

Dipartimento di Statistica "G. Parenti" – Viale Morgagni 59 – 50134 Firenze - www.ds.unifi.it

Volatility Spillovers, Interdependence and Comovements: A Markov Switching Approach

Giampiero M. Gallo,
Edoardo Otranto

Università degli Studi
di Firenze

Volatility Spillovers, Interdependence and Comovements: A Markov Switching Approach

Giampiero M. Gallo*

Dipartimento di Statistica "G. Parenti", Università di Firenze

Viale G. B. Morgagni, 59 - 50134 Firenze, Italy (*gallog@ds.unifi.it*)

Edoardo Otranto

Dipartimento di Economia, Impresa e Regolamentazione, Università di Sassari

Via Torre Tonda - 07100 Sassari, Italy (*eotranto@uniss.it*)

This Revision: Sep 7, 2007

Abstract

The transmission mechanisms of volatility between markets can be characterized within a new Markov Switching bivariate model where the state of one variable feeds into the transition probability of the state of the other. A number of model restrictions and hypotheses can be tested to stress the role of one market relative to another (spillover, interdependence, comovement, independence, Granger non causality). The model is estimated on the weekly high–low range of five Asian markets, assuming a central (but not necessarily dominant) role for Hong Kong. The results show plausible market characterizations over the long run with a spillover from Hong Kong to Korea and Thailand, interdependence with Malaysia and comovement with Singapore.

KEY WORDS: Markov Switching, multiple chains, volatility, spillover effect, comovements.

JEL classification: C32 C52 C53

*Corresponding author: Dipartimento di Statistica "G. Parenti", Università di Firenze, Viale G. B. Morgagni, 59 50134 Firenze - Italy, Tel.: +39 055 4237273, Fax: +39 055 4223560

1 Introduction

The diffusion of international investments and capital movements across borders has marked the evolution of financial markets and has changed the profile of correlations among assets denominated in different currencies which are exchanged in geographically separated markets. Volatility in one market reacts to innovations in other markets as a result of financial integration. Moreover, the volatility patterns show frequently evidence of nonlinearities (see, for example, Frijns and Schotman, 2006).

In the financial literature, a stream of research has dealt with spillovers of volatility from one market to another (Lee and Kim, 1993), focussing on shocks to volatility in a GARCH framework (Engle *et al.*, 1990). In recent times, several studies have focused on financial crises (notably, Mexico, Russia, East Asia, Argentina) with the intention of analyzing the sources of the crisis: a recurring question is whether the crises originated in one region and spilled over to other regions (spillover effect) or whether they are the result of an interdependent reaction to some common shock.

Another financial transmission mechanism frequently considered in literature and strictly related to the spillover effect is the contagion, which is frequently detected by changes in correlation coefficients. In discussing the presence and the extension of contagion effects, several authors have concentrated on different aspects, and hence different definitions of contagion: the World Bank site on Financial Crises provides a broad definition of cross-country transmission of shocks which may take place during both "good" and "bad" times, whereas more restrictive definitions are centered around a specific situation of crisis and the consequent increase in the level of interdependence across countries.

From an empirical point of view, methodologies vary considerably (Pericoli and Sbracia, 2003): one can recognize models where the period of the crisis is known and some explanation for its inception is sought. In a Probit/Logit model the crisis is translated into a binary variable and contagion is tantamount to the statistical significance of a dummy variable flagging an existing crisis in another market; in a Leading Indicators model one examines the predictive value of variables linked to economic fundamentals or to foreign markets; in the line of Forbes and Rigobon (2002) one would detect a correlation breakdown in correspondence to the known dates of the crisis.

A different line of research is characterized by volatility spillovers which characterize the structure of interrelationships across markets: the GARCH models put forth by Engle *et al.* (1990) allow to see whether conditional variances are affected by additional information in the form of squared innovations occurring in other markets. This basic idea gets more involved if one considers that volatility clustering may be characterized by the presence of regimes alternating between low and high levels of unconditional volatility. In this respect, a further category of models which has received considerable attention relates to Markov Switching models (MS), diffused in the applied statistics by Hamilton, 1989, and adapted to switching volatility by Hamilton and Susmel (1994), introducing the SWARCH models (another interesting extension deals with stochastic volatility models; see So et al., 1998, and Carvalho and Lopes, 2007). In the context of financial crises the presence of sudden switches ruled by a Markov chain can be accommodated for the variance equation, as in Edwards and Susmel (2001) and (2003), who suggest a bivariate version of the SWARCH model for weekly international stock returns and interest rates, tracking co-dependence in volatility regimes. In these models, the idea of crisis and contagion translates into a sudden change in the volatility of stock returns or interest rates measured in a pair of countries and of their correlation. The MS model provides a framework in which regimes are associated with the various combinations of low and high volatility in each country. The interesting feature of their approach is that one country is *ex ante* considered the *originator* of the crisis (dominant market) and the correlation coefficient is made dependent on the state of such originator country. Contagion is had when the correlation coefficients significantly change value across states. Baele (2005) studies the effect of globalization on market interdependence, using a MS model where switching occurs in the spillover parameters. This paper has some analogies to the approach pursued here since it makes use of three different structures of the transition probability matrices to characterize comovements and independence. He defines a test for contagion along the lines of what is suggested by Bekaert *et al.* (2005).

In our approach we pursue the idea that transmission mechanisms operate in the presence of volatility regimes. To this end, we choose to focus on the conditional expectation of an observable volatility proxy measured on different markets, namely the weekly range. We adopt a new version of the Markov Switching model called the Multi Chain MS model (MCMS, Otranto, 2005), where asymmetries are inserted by making the transition probability of each market dependent on the state of the other markets.

The definition of spillover we propose may be consistent with some definition of contagion (for example, the one of Edwards and Susmel, 2001 and 2003); since it does not refer directly to changes in correlations, but to more general changes in regime, we prefer to use the term spillover to not generate confusion.

In this context, we study market characterizations relying on the following definitions. *Spillover* is seen as a situation in which a switch in regime of a dominating market leads to a change in regime in the dominated market (with a lag). *Interdependence* is seen as a situation in which a switch in regime of one of the markets leads a change in regime of the other markets. Finally, *comovement* is represented by contemporaneous change in regimes. As detailed in what follows, the various hypotheses corresponding to the different market features can be tested within the context of MCMS models which belong to the VAR-MS family (Krolzig, 1997). Modelling weekly volatilities in different markets as linear autoregressions is a common practice in the statistical and econometric literature (see, for example, Diebold and Yilmaz, 2007).

In Section 2 the multivariate models used and their interpretation are introduced; Section 3 contains a discussion of the choice of the proxy of the volatility used with some stylized facts about the markets of interest, whereas in section 4 the methodology exposed will be applied to analyze the characteristics of the Asian markets in the period 1993-2004, including the East Asian crisis of 1997. Concluding remarks follow.

2 The Multi–Chain Markov Switching Model

The presence of multiple regimes can be acknowledged using a popular multivariate model introduced by Hamilton (1990) where parameters are made dependent on a hidden state process ruled by a Markov chain: such a model, the multivariate Markov Switching Model (MS), considers an n-dimensional vector $\boldsymbol{y}_t \equiv (y_{1t}, \ldots, y_{nt})'$, which is assumed to follow a VAR(p) with time-varying parameters:

$$\begin{aligned} \boldsymbol{y}_t &= \boldsymbol{\mu}(s_t) + \sum_{i=1}^{p} \boldsymbol{\Phi}_i(s_t)\boldsymbol{y}_{t-i} + \boldsymbol{\epsilon}_t \\ &\text{and} \\ \boldsymbol{\epsilon}_t &\sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Sigma}(s_t)\right), \end{aligned} \tag{1}$$

where the parameters for the conditional expectation $\boldsymbol{\mu}(s_t)$ and $\boldsymbol{\Phi}_i(s_t)$, $i = 1, \ldots, p$, as well as the variances and covariances of the error terms $\boldsymbol{\epsilon}_t$ in the matrix $\boldsymbol{\Sigma}(s_t)$ all depend upon the state variable s_t which can assume a number q of values (corresponding to different regimes). The transition probability matrix $\boldsymbol{P}$ contains the probabilities of being in a generic state j at time t given that the state at time $t-1$

was i, namely, for a generic element

$$p_{ij} = Pr(s_t = j | s_{t-1} = i), \ i, j = 1, \ldots, q.$$

The properties of this model are well known by now and need not be discussed here: we refer to Hamilton (1994) for the estimation, filtering and smoothing procedures for this model. For this model it is crucial to keep in mind that all variables in the process $\boldsymbol{y}$ depend on the same state variable s_t, and as such they are subject to a common switching.

Such a model is of limited use in deciding whether there is spillover or interdependence, in that it can only signal the common switch of all the variables analyzed from one state to another. In this respect, this model is a good candidate to represent common contemporaneous changes across markets, which we have defined as *comovement*. For the same reasons, it is going to be misleading in cases in which variables are ruled by different states which may be temporally dependent on one another (mutually or in one direction only) or even independent.

The idea behind a Multi–Chain Markov Switching model (MCMS), as suggested by Otranto (2005), is to consider a multivariate process in which the switching mechanism across regimes makes the state for one variable be dependent on the lagged states of all variables. This case could be considered as representative of the situation of *interdependence*, because the change in the state of each variable can be transmitted to all the others with a certain probability. As a special case, one can consider a process in which one variable is assumed to be dominant on the others and the switching dynamics intrinsically asymmetric: a particular state for one variable alters the probability of other variables to change states, but not vice versa. This feature is suitable to describe transmission mechanisms occurring in financial crises, but also to any relationship where a leading variable is present (in Otranto, 2005, new orders are assumed to be leading the turnover at the aggregate level) thus representing the case of *spillover*. Finally, the reciprocal dependence on the state of the other variables could turn out to be not significant, representing the case in which markets are ruled by *independent* state variables.

To fix ideas, let us consider a bivariate case with two latent states for each variable: the dynamics of the two variables are thus subject to state dependence. The transition from one (multi–) state to another is ruled by a Markov chain obtained by letting the transition probabilities for one variable be a function of the (lagged) state of both variables.

In formal terms, as before, $\boldsymbol{y}_t$ is assumed to follow a VAR(p) process (note that $\boldsymbol{s}_t$ is now a vector):

$$\begin{aligned}\boldsymbol{y}_t &= \boldsymbol{\mu}(\boldsymbol{s}_t) + \sum_{i=1}^{p} \boldsymbol{\Phi}_i(\boldsymbol{s}_t)\boldsymbol{y}_{t-i} + \boldsymbol{\epsilon}_t \\ \text{and} & \\ \boldsymbol{\epsilon}_t &\sim \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Sigma}(\boldsymbol{s}_t)\right),\end{aligned} \tag{2}$$

where the parameters for the conditional expectation $\boldsymbol{\mu}(\boldsymbol{s}_t)$ and $\boldsymbol{\Phi}_i(\boldsymbol{s}_t)$, $i = 1, \ldots, p$, as well as the variances and covariances of the error terms $\boldsymbol{\epsilon}_t$ in the matrix $\boldsymbol{\Sigma}(\boldsymbol{s}_t)$ all depend upon the state vector $\boldsymbol{s}_t \equiv (s_{1t}, \ldots, s_{nt})'$ with s_{jt} representing the state associated with variable y_{jt}. Each state can assume a number q of regimes (in principle these could be different across states). The difference with respect to the classical multivariate MS models is that $y_{1,t}$ and $y_{2,t}$ depend on separate but potentially related state variables.

To illustrate how the asymmetric behavior of the variables can be embedded in the model, let us consider the transition probability matrix $\boldsymbol{P}$ with generic element representing

$$\boldsymbol{P} = \left\{\Pr\left[\boldsymbol{s}_t | \boldsymbol{s}_{t-1}\right]\right\}.$$

If we consider, for simplicity, the case $n = q = 2$, the state vector $\boldsymbol{s}_t$ can assume four different values $\{(0,0), (0,1), (1,0), (1,1)\}$ and the matrix $\boldsymbol{P}$ is a 4×4 matrix. Let us suppose that, conditional on (s_{1t-1}, s_{2t-1}), the states s_{1t} and s_{2t} are independent, so that:

$$\Pr\left[s_{1t}, s_{2t} | s_{1t-1}, s_{2t-1}\right] = \Pr[s_{1t} | s_{1t-1}, s_{2t-1}] \Pr[s_{2t} | s_{1t-1}, s_{2t-1}]. \tag{3}$$

The right hand side of equation (3) can be parameterized with logistic functions where the functional dependence on past states is made explicit as follows:

$$\begin{aligned}\Pr\left(s_{1t} = h | s_{1t-1} = h, s_{2t-1}\right) &= \frac{\exp[\alpha_1(h, .) + \beta_1(h, 1)s_{2t-1}]}{1 + \exp[\alpha_1(h, .) + \beta_1(h, 1)s_{2t-1}]} \\ &\text{and} \\ \Pr\left(s_{2t} = h | s_{1t-1}, s_{2t-1} = h\right) &= \frac{\exp[\alpha_2(., h) + \beta_2(1, h)s_{1t-1}]}{1 + \exp[\alpha_2(., h) + \beta_2(1, h)s_{1t-1}]},\end{aligned} \tag{4}$$

for $h = 0, 1$. From (4), it is apparent that the state of the variable i at time $t-1$ influences the probability of variable j to stay in the same regime, and vice versa. Obviously,

$$\Pr\left(s_{jt} = k | s_{jt-1} = h, s_{it-1}\right) = 1 - \Pr\left(s_{jt} = h | s_{jt-1} = h, s_{it-1}\right)$$

for $h, k = 0, 1$, $h \neq k$, and $i, j = 1, 2$, $i \neq j$. Hypothesis testing can be performed on the estimated model (2)–(4) in order to assess the relevance of the dependence structure assumed for the states and whether the presence of asymmetric effects in the dynamics of regimes is supported by the data. Statistical significance of all parameters in (4) will provide evidence in favor of the case of *interdependence*. If the coefficient $\beta_j(h, k) = 0$, the state of the variable i at time $t-1$ influences the probability of variable j to stay in the same regime, but not vice versa, this is evidence in favor of the dominant status of variable i or *spillover*. This property gives meaning to our envisaging spillover as a stable asymmetric relationship between markets and not necessarily related to the effects of single shocks. Finally, the non significance of all the coefficients $\beta_j(h, k)$ and $\beta_i(h, k)$ would show evidence for *independence* between markets.

In this way, the estimated probabilities in (4) will show the impact of the regime of variable i on the transition probabilities for variable j; moreover, we would expect the signs of coefficients $\beta_1(0, 1)$ and $\beta_2(1, 0)$ to be negative and those of coefficients $\beta_1(1, 1)$ and $\beta_2(1, 1)$ to be positive.

Disposing the estimated transition probabilities (3) in a matrix, with rows representing the multiple state at time $t-1$ and columns the multiple state at time t, it is possible to evaluate the most probable scenario (a particular combination of s_{1t} and s_{2t}) at time t, given a certain state at time $t-1$.

The properties of the model from a theoretical point of view coincide with those of a standard Markov switching model: estimation filtering and smoothing can be performed according to the procedures described by Hamilton (1990) and Kim (1994).

3 The Choice of the Volatility Proxy

The Asian markets are a classical example for which there is a large debate to establish the nature of the relationship among markets subject to sudden changes in volatility. For example, Forbes and Rigobon (2002) note that the shock originating from Hong Kong in October 1997 has not implied a significant increase in the correlation coefficients of the other main Asian markets: the conclusion reached is that the series analyzed cannot be considered as subject to a form of spillover from Hong Kong, but rather the markets considered exhibit interdependence. Let us now see what our analysis allows us to say, in view of a more articulate definition of spillover, interdependence, independence and comovements.

We analyze the stock market indices of 5 Asian countries starting from daily data spanning a period between November 29, 1993 and April 26, 2004; the indices are the Hang Seng index (Hong Kong-HSI hereafter), the KOSPI index (South Korea-KS11), the KLSE composite index (Malaysia-KLSE), the Straits Times index (Singapore-STI), the Thailand SET index (Thailand, SETI). The proxy of the volatility is computed as the weekly range of the logarithm of the data (highest recorded minus lowest recorded value, rescaled by the factor $(1/4ln(2))^{1/2}$; see Parkinson, 1980) and results in 544 observations. The choice of a long sample period is motivated by the desire to capture interactions over several years characterized by an increasing degree of financial and real integration and where crises have shaped the interdependence among these markets.

We have tried to construct alternative proxies of the volatility among those suggested by Parkinson (1980) and Garman and Klass (1980). Most of them show a similar behavior, which is a logical consequence of their asymptotical equivalence. Moreover, the range of the logarithms seems to evidence a clearer presence of regimes with respect to a diffuse proxy of the volatility as the log weekly range. Finally, it seems that there is more information in considering a volatility proxy which is not derived by weekly returns measured from mid-week closing prices. The turmoil occurring in the markets is better captured by a range-related variable than it is by squared returns. The latter have been established to be a noisier measure of price variability in the realized volatility literature (Andersen and Bollerslev, 1998, Brunetti et al., 2003, Brunetti and Lildholdt, 2006); moreover, the MS models seem to not represent adequately the squared returns because they do not reproduce correctly their autocorrelation function (Bulla and Bulla, 2006). Therefore our choice seems supported by the data and the related literature.

The choice of the frequency of analysis is always crucial in detecting the direction of a temporal relationship. The use of weekly data is a natural consequence of the definition of spillover and comovement we made. We define spillovers as stable dynamic relationship, that is, say, as the increase in the range on a market leading systematically to the increase in the range on another market with a time lag. On the other side, a situation of comovement would arise if such a lag were not to be present in the data. Increasing the frequency of observation could cause more noise in the data and more uncertainty in the length of the lag by which these spillovers occur. We chose a weekly frequency as a good compromise between seeing everything as interdependent and having a picture clouded by data which may condition the results in the multivariate context of interest here (see, for example, Diebold and Yilmaz, 2007).

The proxy used delivers the HSI series shown in Figure 1; the East Asian crisis shows its most evident effect in the third week of October 1997, in which the

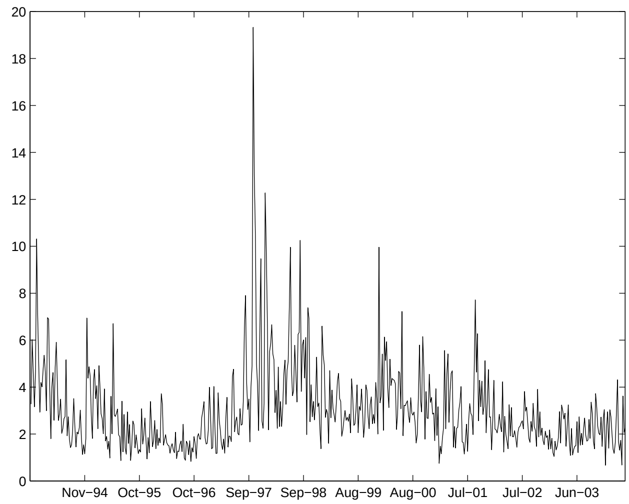

Figure 1: Hang Seng volatility

volatility increased by almost 300 percent. The dramatically high volatility of this period is a common feature of all the series analyzed (Figure 2), with a different degree of persistence and depth, but a similar general behavior. In particular the Korean and Thail markets seem to suffer dramatically from the October crisis, with a surge in volatility which does not get completely reabsorbed in successive periods. Other series seem to absorb the shock, albeit gradually.

The abrupt changes in volatility could be considered as outliers when compared with the rest of the univariate series; by the same token, bursts of market activity do occur and it would be undesirable to manipulate relevant information about sudden changes in volatility, especially when we want to relate these to similar episodes in other markets in a Markov Switching context. As noted by Bollerslev et al. (2007), consider that jumps in the index can be recorded only when all stocks in the index have a jump so that the problem of sparse cases of true outliers (say, high recorded index values due to isolated mispricing) would not really be likely in this context.

The existence of several regimes is clear observing these graphs, as is the presence of some sort of common feature, with possible lags. In general, in a MS framework, it is numerically cumbersome to work with more than four regimes. For this reason we will estimate MMS and MCMS models with 4×4 transition probability matrices.

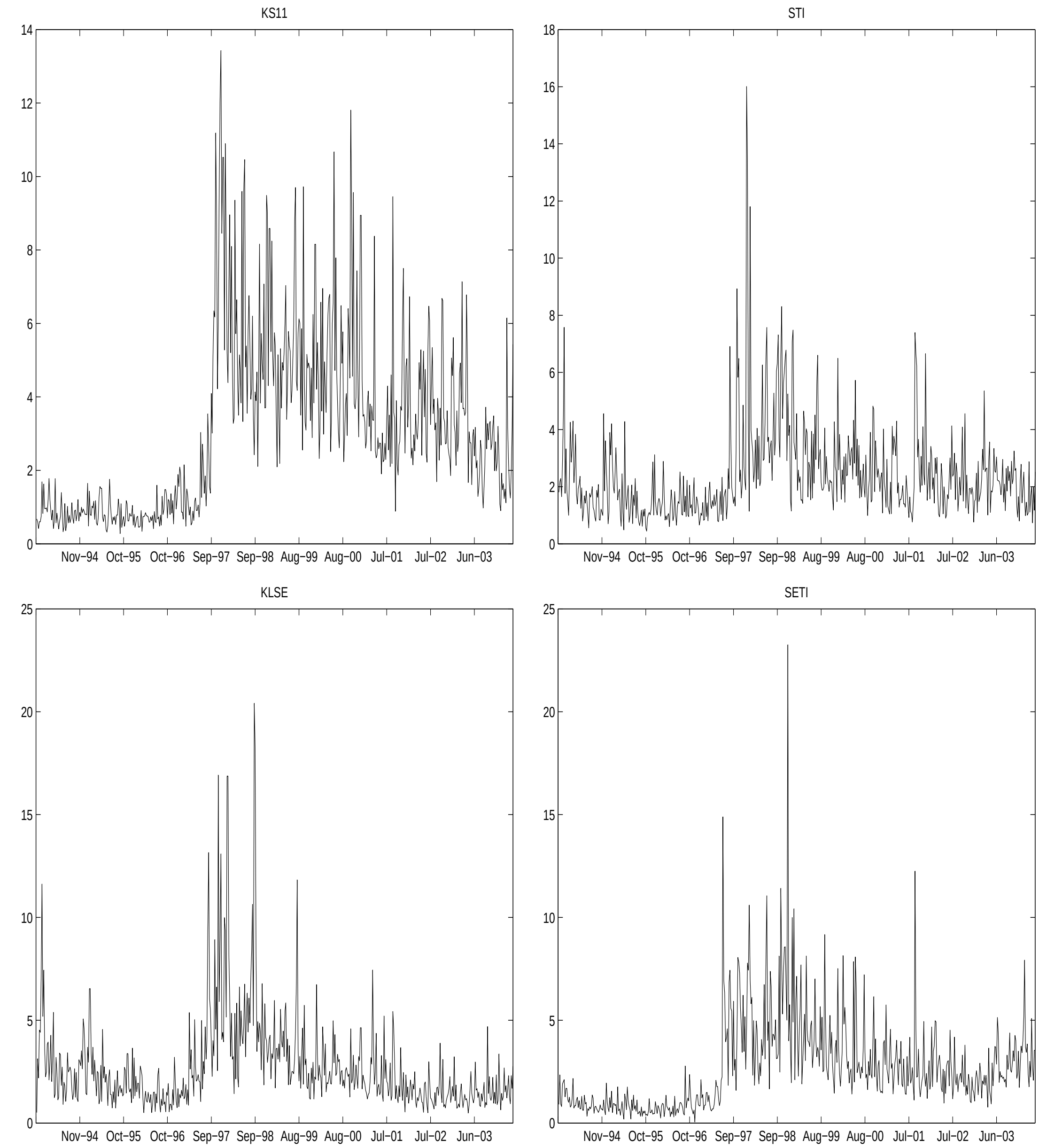

Figure 2: Volatility of Asian markets

4 The Empirical Results

The first step of our approach is to estimate a MCMS model without constraints on the parameters in the transition probability specifications (4) and then to test some restrictions. We shall consider bivariate models keeping HSI as the second variable in all models and letting the data suggest which type of relationship it holds with other markets, given the great influence exerted by Hong Kong on the other Asian economies, a role that it has retained even after the reunification with mainland China. Ideally, one should consider a full model with all markets at once (and not only the ones we have chosen), but the numerical difficulties involved are prohibitive (see, for example, Edwards and Susmel, 2001 and 2003). The assumption that markets should be measured in their relationships to Hong Kong is a reasonable one, we believe: this does not entail that Hong Kong be the originator of the crises nor a dominant market. In making the transition probabilities be dependent on the state of another market we achieve the goal of being able to detect whether such a link exists or not, or whether other characterizations are likely (interdependence, independence, comovement).

Following this hypothesis, we estimate 4 separate MCMS models with 2×2 states; the value 0 represents the ordinary regime, the value 1 occurs in the turbulent regime. We let the intercept of model (2) to vary with both the regimes of the two markets; in other terms, we will have four possible intercepts for each variable. In addition, we consider the autoregressive parameters in (2) not to be state–dependent and the order p equal to 2; for these coefficients the usual stationarity constraints hold. Finally, we suppose a structure of the covariance matrix as:

$$\mathbf{\Sigma}(s_{1,t}, s_{2,t}) = \begin{bmatrix} \sigma_1^2(s_{1t}, .) & \rho(s_{1t}, s_{2t})\sigma_1(s_{1t}, .)\sigma_2(., s_{2t}) \\ \rho(s_{1t}, s_{2t})\sigma_1(s_{1t}, .)\sigma_2(., s_{2t}) & \sigma_2^2(., s_{2t}) \end{bmatrix}$$

In other terms, the variances of each variable (related to fourth moments of returns, which we assume exist) depend only on the variable's own state, whereas the effect of the multi–state affects the correlation coefficient, that varies in [-1,1]. The previous specification implies that the volatility is transmitted from a market to another, causing also some change in the covariance structure, whereas the volatility of volatility depends just on the own state.

After estimating the models, we test some propositions to evaluate the nature of the dependence on the state of the other variable:

State Dependence in the Volatility

1. No dependence of the intercept of y_1 on the state of y_2:
   H_0: $\mu_1(0,0)$=$\mu_1(0,1)$ and $\mu_1(1,0)$=$\mu_1(1,1)$;
2. No dependence of the intercept of y_2 on the state of y_1:
   H_0:$\mu_2(0,0)$=$\mu_2(1,0)$ and $\mu_2(0,1)$=$\mu_2(1,1)$

Dynamic Dependence in the Volatility

3. y_2 does not **linearly** Granger cause y_1:
   H_0:ϕ_{12}^1=ϕ_{12}^2=0
4. y_1 does not **linearly** Granger cause y_2:
   H_0:ϕ_{21}^1=ϕ_{21}^2=0

State Dependence in the Correlations

5. No dependence of the correlation on the state of y_2:
   H_0:$\rho(0,0)$=$\rho(0,1)$ and $\rho(1,0)$=$\rho(1,1)$
6. No dependence of the correlation on the state of y_1:
   H_0:$\rho(0,0)$=$\rho(1,0)$ and $\rho(0,1)$=$\rho(1,1)$

Characterization of Market Dependence

7. No spillover effect from y_2 to y_1:
   H_0:$\beta_1(0,1)$=$\beta_1(1,1)$=0
8. No spillover effect from y_1 to y_2: H_0:$\beta_2(1,0)$=$\beta_2(1,1)$=0
9. No interdependence (no reciprocal spillover):
   H_0:$\beta_1(0,1)$=$\beta_1(1,1)$=$\beta_2(1,0)$=$\beta_2(1,1)$=0
10. Comovement between y_1 and y_2

$$H_0: \begin{array}{l} \alpha_1(0,.) = \alpha_2(.,0), \\ \alpha_1(0,.) + \beta_1(0,1) + \alpha_2(.,1) = 0, \\ \alpha_1(.,1) + \alpha_2(.,0) + \beta_2(1,0) = 0, \\ \text{and} \\ \alpha_1(1,.) + \beta_1(1,1) = \alpha_2(.,1) + \beta_2(1,1) \end{array}$$

Global Causality

11. y_2 does not cause y_1:

    Hypothesis 3 and Hypothesis 7

12. y_1 does not cause y_2:

    Hypothesis 4 and Hypothesis 8

The hypothesis 10 is not intuitive because the MMS model, where $s_{1t} = s_{2t}$ for each t, is not nested into the MCMS model. Thus, the rows and the columns of the transition probability matrix of the independent MCMS model where $s_{1t} \neq s_{2t}$ cannot be constrained so as to obtain the smaller size transition probability matrix of the MMS model. However, one can impose that the profile of the estimated state variable for market 1 be the same as the corresponding state variable for market 2. The analytical derivation of these constraints is developed in the appendix at the end of the paper. The hypotheses 3. and 4. are relative to the classical definition of Granger causality in a linear model. To consider both the linear and nonlinear possible dependencies we test also the hypotheses 11. and 12., which we call global causality; it is analogous to the definition of causality in MS models proposed by Anas et al. (2006). All these hypotheses can be tested by means of classical Wald statistics.

The hypotheses labeled 7. to 10. are the ones which characterize the relationships between markets. The various situations can be summarized as follows:

- **Spillover**: it occurs when hypothesis 7. cannot be rejected and hypothesis 8. is rejected or the other way around.
- **Interdependence** or reciprocal spillover: hypotheses 7., 8., and 9. are rejected.
- **Independence**: it occurs when hypothesis 9. cannot be rejected and hypothesis 10. is rejected.
- **Comovement** or common state variable: it occurs when 10. cannot be rejected, as discussed above.

Table 1: Market Characterization Based on MCMS Models. The '*' and '**' symbols represent **rejection** of the hypothesis at 5%, respectively, 1% significance level, on the basis of a corresponding Wald-type tests on estimated MCMS models.

	Market 1			
Hypotheses	KS11	STI	KLSE	SETI
State Dependence in the Volatility				
1. No dependence of the intercept of Market 1 on the state of HSI	**	**	**	**
2. No dependence of the intercept of HSI on the state of Market 1		**	**	**
Dynamic Dependence in the Volatility				
3. HSI does not linearly Granger cause Market 1	**			**
4. Market 1 does not linearly Granger cause HSI	**	**	*	**
State Dependence in the Correlations				
5. No dependence of the correlation on the state of HSI		**		*
6. No dependence of the correlation on the state of Market 1		*	*	
Characterization of Market Dependence				
7. No spillover from HSI to Market 1	**		**	**
8. No spillover from Market 1 to HSI			**	
9. No interdependence	*		**	**
10. Comovement between Market 1 and HSI	**		**	**
Global Causality				
11. HSI does not cause Market 1	**		**	**
12. Market 1 does not cause HSI	**	**	**	**
Plausible Market Characterization				
Spillover from Hong Kong	×			×
Interdependence			×	
Comovement		×		
Independence				

In Table 1 we summarize the hypothesis testing results of the Wald test statistics for the twelve hypotheses above; the estimated models (2)–(4) show some form of dependence between the couples of series according to the various categories detailed above. In particular, the hypotheses that the intercepts of one market do not depend on the state of the other market, and the linear Granger non causality hypotheses are often rejected (we cannot reject the hypothesis of no dependence of the intercept of Hong Kong on the state of Korea and the hypothesis of linear Granger causality from Hong Kong to Singapore and Malaysia). The correlations seem to be dependent on both the states only in the case of STI/HSI markets, whereas in general there is not evidence of state dependence. Except for STI/HSI, all the bivariate models show a form of reciprocal causality (linear and/or nonlinear) between Hong Kong and the other markets.

As per market dependence as described above, we can say that the Hong Kong market has a spillover effect on both the Korean and Thai markets (hypothesis 7. rejected and hypothesis 8. not rejected). For the Malaysia/Hong Kong markets the evidence favors interdependence (rejection of both hypotheses 7. and 8.). The Singapore case is the only one which shows evidence of comovement with Hong Kong (hypotheses 7, 8, 9, 10 not rejectable).

We show the estimation of the selected models for each pair of markets in Tables 2 to 5. The standard errors are calculated considering the sandwich covariance matrix of the estimators (Bollerslev and Woolridge, 1992). In the last rows we report the p-values relative to the Jarque-Bera test (JB), the Ljung-Box test (LB(4)) and the Ljung-Box test on squared residuals (LBS(4)), both calculated with 4 lags.

Table 2: Estimated parameters of the MCMS Model for Korea/Hong Kong (robust standard errors in parentheses)

Switching coefficients - Constant Term							
Korea Equation				Hong Kong Equation			
$\mu_1(0,0)$	$\mu_1(0,1)$	$\mu_1(1,0)$	$\mu_1(1,1)$	$\mu_2(0,0)$	$\mu_2(1,0)$	$\mu_2(0,1)$	$\mu_2(1,1)$
0.429	0.785	2.070	4.938	1.057	1.403	3.159	3.665
(0.061)	(0.121)	(0.160)	(0.314)	(0.128)	(0.292)	(0.473)	(0.512)
Autoregressive Terms							
Korea Equation				Hong Kong Equation			
ϕ^1_{11}	ϕ^1_{12}	ϕ^2_{11}	ϕ^2_{12}	ϕ^1_{21}	ϕ^1_{22}	ϕ^2_{21}	ϕ^2_{22}
0.284	-0.020	0.197	-0.000	0.028	0.158	-0.000	0.178
(0.029)	(0.021)	(0.027)	(0.000)	(0.051)	(0.045)	(0.001)	(0.031)
Switching coefficients - Standard deviations				Switching coefficients			
Korea Equation		Hong Kong Equation		Correlation Terms			
$\sigma_1(0,.)$	$\sigma_1(1,.)$	$\sigma_2(.,0)$	$\sigma_2(.,1)$	$\rho(0,0)$	$\rho(0,1)$	$\rho(1,0)$	$\rho(1,1)$
0.247	0.949	0.406	1.469	0.000	0.000	0.037	0.039
(0.024)	(0.066)	(0.035)	(0.237)	(0.097)	(0.082)	(0.209)	(0.098)
Probability parameters							
Korea Equation				Hong Kong Equation			
$\alpha_1(0,.)$	$\beta_1(0,1)$	$\alpha_1(1,.)$	$\beta_1(1,1)$	$\alpha_2(.,0)$		$\alpha_2(.,1)$	
1.614	-1.203	1.053	0.000	1.199		0.012	
(0.268)	(0.377)	(0.240)	(0.137)	(0.213)		(0.112)	
p-values of test statistics							
	Korea				Hong Kong		
JB	LB(4)	LBS(4)		JB	LB(4)	LBS(4)	
0.241	0.394	0.009		0.000	0.139	0.230	

Table 3: Estimated parameters of the MS–4 states Model for Singapore/Hong Kong (robust standard errors in parentheses).

Switching coefficients - Constant Term							
Singapore Equation				Hong Kong Equation			
$\mu_1(1)$	$\mu_1(2)$	$\mu_1(3)$	$\mu_1(4)$	$\mu_2(1)$	$\mu_2(2)$	$\mu_2(3)$	$\mu_2(4)$
0.770	0.770	1.528	3.394	0.257	1.019	2.072	3.267
(0.112)	(0.115)	(0.197)	(0.352)	(0.134)	(0.215)	(0.170)	(0.417)
Autoregressive Terms							
Singapore Equation				Hong Kong Equation			
ϕ_{11}^1	ϕ_{12}^1	ϕ_{11}^2	ϕ_{12}^2	ϕ_{21}^1	ϕ_{22}^1	ϕ_{21}^2	ϕ_{22}^2
0.235	-0.007	0.159	0.012	0.171	0.179	0.035	0.139
(0.033)	(0.042)	(0.057)	(0.039)	(0.032)	(0.042)	(0.079)	(0.046)
Switching coefficients - Standard deviations							
Singapore Equation				Hong Kong Equation			
$\sigma_1(1)$	$\sigma_1(2)$	$\sigma_1(3)$	$\sigma_1(4)$	$\sigma_2(1)$	$\sigma_2(2)$	$\sigma_2(3)$	$\sigma_2(4)$
0.457	0.298	0.500	1.744	0.143	0.223	0.503	2.019
(0.072)	(0.031)	(0.051)	(0.277)	(0.052)	(0.060)	(0.061)	(0.316)
Switching coefficients - Correlations							
		$\rho(1)$	$\rho(2)$	$\rho(3)$	$\rho(4)$		
		0.330	0.0989	-0.424	0.536		
		(0.283)	(0.201)	(0.202)	(0.086)		
Transition Probabilities							
	p_{11}	p_{12}	p_{13}	p_{21}	p_{22}	p_{23}	
	0.124	0.420	0.357	0.234	0.364	0.324	
	(0.077)	(0.167)	(0.167)	(0.090)	(0.113)	(0.052)	
	p_{31}	p_{32}	p_{33}	p_{41}	p_{42}	p_{43}	
	0.216	0.320	0.338	0.211	0.122	0.182	
(0.105)	(0.108)	(0.083)	(0.082)	(0.077)	(0.045)		
p-values of test statistics							
Singapore				Hong Kong			
JB	LB(4)	LBS(4)		JB	LB(4)	LBS(4)	
0.180	0.001	0.101		0.000	0.942	0.649	

Table 4: Estimated parameters of the MCMS Model for Malaysia/Hong Kong (robust standard errors in parentheses)

Switching coefficients - Constant Term							
Malaysia Equation				Hong Kong Equation			
$\mu_1(0,0)$	$\mu_1(0,1)$	$\mu_1(1,0)$	$\mu_1(1,1)$	$\mu_2(0,0)$	$\mu_2(1,0)$	$\mu_2(0,1)$	$\mu_2(1,1)$
0.757	2.116	4.855	4.855	1.264	1.327	2.059	5.215
(0.083)	(0.125)	(0.594)	(0.550)	(0.114)	(0.275)	(0.218)	(0.663)
Autoregressive Terms							
Malaysia Equation				Hong Kong Equation			
ϕ_{11}^1	ϕ_{12}^1	ϕ_{11}^2	ϕ_{12}^2	ϕ_{21}^1	ϕ_{22}^1	ϕ_{21}^2	ϕ_{22}^2
0.183	0.025	0.126	0.000	0.066	0.195	0.009	0.135
(0.024)	(0.049)	(0.024)	(0.003)	(0.036)	(0.046)	(0.054)	(0.043)
Switching coefficients - Standard deviations				Switching coefficients			
Malaysia Equation		Hong Kong Equation		Correlation Terms			
$\sigma_1(0,.)$	$\sigma_1(1,.)$	$\sigma_2(.,0)$	$\sigma_2(.,1)$	$\rho(0,0)$	$\rho(0,1)$	$\rho(1,0)$	$\rho(1,1)$
0.314	2.373	0.528	1.207	0.139	0.000	0.581	0.085
(0.022)	(0.354)	(0.037)	(0.199)	(0.086)	(0.184)	(0.163)	(0.141)
Probability parameters							
Malaysia Equation				Hong Kong Equation			
$\alpha_1(0,.)$	$\beta_1(0,1)$	$\alpha_1(1,.)$	$\beta_1(1,1)$	$\alpha_2(.,0)$	$\beta_2(1,0)$	$\alpha_2(.,1)$	$\beta_2(1,1)$
2.507	-1.149	-1.077	0.950	0.963	-1.219	-0.238	0.801
(0.294)	(0.376)	(0.532)	(0.659)	(0.191)	(0.405)	(0.274)	(0.578)
p-values of test statistics							
Malaysia				Hong Kong			
JB	LB(4)	LBS(4)		JB	LB(4)	LBS(4)	
0.004	0.936	0.326		0.000	0.568	0.303	

The residuals (calculated as a weighted sum of the residuals in the four states, with weights given by the filtered probabilities) exhibit non normality in all cases (except Singapore), but this is not surprising given that we are modelling the conditional expectation of a positive valued process and the distribution of residuals is often asymmetric and affected by exceptionally high values (cf. also Figures 1 and 2); furthermore this kind of models implies densities which are mixtures of Normal distributions.

We can note that the signs of the parameters of the logistic functions are consistent with our expectations. In the KS11/HSI case the state of HSI has an impact just on the probabilities in state 0. For the STI/HSI case, the volatility increases from state 1 to state 4 and the intercept of STI is the same in state 1 and 2. All

Table 5: Estimated parameters of the MCMS Model for Thailand/Hong Kong (robust standard errors in parentheses)

Switching coefficients - Constant Term							
Thailand Equation				Hong Kong Equation			
$\mu_1(0,0)$	$\mu_1(0,1)$	$\mu_1(1,0)$	$\mu_1(1,1)$	$\mu_2(0,0)$	$\mu_2(1,0)$	$\mu_2(0,1)$	$\mu_2(1,1)$
0.294	1.018	2.162	3.550	1.238	1.238	1.893	4.324
(0.045)	(0.066)	(0.191)	(0.764)	(0.109)	(0.126)	(0.211)	(0.692)
Autoregressive Terms							
Thailand Equation				Hong Kong Equation			
ϕ_{11}^1	ϕ_{12}^1	ϕ_{11}^2	ϕ_{12}^2	ϕ_{21}^1	ϕ_{22}^1	ϕ_{21}^2	ϕ_{22}^2
0.357	-0.010	0.148	-0.000	0.038	0.230	-0.000	0.146
(0.020)	(0.023)	(0.010)	(0.000)	(0.039)	(0.049)	(0.001)	(0.049)
Switching coefficients - Standard deviations				Switching coefficients - Correlation Terms			
Thailand Equation		Hong Kong Equation					
$\sigma_1(0,.)$	$\sigma_1(1,.)$	$\sigma_2(.,0)$	$\sigma_2(.,1)$	$\rho(0,0)$	$\rho(0,1)$	$\rho(1,0)$	$\rho(1,1)$
0.157	1.740	0.560	1.294	0.000	0.000	0.546	0.044
(0.013)	(0.239)	(0.074)	(0.229)	(0.058)	(0.088)	(0.239)	(0.084)
Probability parameters							
Thailand Equation				Hong Kong Equation			
$\alpha_1(0,.)$	$\beta_1(0,1)$	$\alpha_1(1,.)$	$\beta_1(1,1)$	$\alpha_2(.,0)$		$\alpha_2(.,1)$	
1.544	-1.261	0.199	0.468	0.889		-0.166	
(0.217)	(0.306)	(0.238)	(0.489)	(0.179)		(0.349)	
p-values of test statistics							
Thailand				Hong Kong			
JB	LB(4)	LBS(4)		JB	LB(4)	LBS(4)	
0.000	0.087	0.082		0.000	0.536	0.552	

the intercepts of the MCMS models exhibit a gradual change from the (0,0) to the (1,1) state (note that in the Malaysia case – Table 4 – the intercept does not change between (1,0) and (1,1); in the Hong Kong the same happens between state (0,0) and (1,0), when the other market is Thailand – Table 5).

Finally, we compare the ability of each selected model to describe the volatility behavior with respect to an alternative model, which is the MMS–4 states model for KS11/HSI, KLSE/HSI, SET/HSI, and the MCMS model for STI/HSI. We will follow Hamilton and Susmel (1994) in carrying out a comparison based on in-sample goodness of fit performance using the Mean Square Error (MSE) and Mean Absolute Error (MAE) or their equivalents for the variables expressed in logs ($[LE]^2$ and $|LE|$ respectively, following Hamilton and Susmel's notation).

Table 6: 1-step ahead forecasting performance of MMS and MCMS models in terms of loss functions.

	KS11/HSI		STI/HSI	
	$MMS(4)$	$MCMS$	$MMS(4)$	$MCMS$
MSE	5.371	5.330	4.529	4.664
MAE	2.342	2.292	1.981	2.045
$[LE]^2$	0.554	0.503	0.438	0.465
$\|LE\|$	0.839	0.803	0.742	0.771
	KLSE/HSI		SETI/HSI	
	$MMS(4)$	$MCMS$	$MMS(4)$	$MCMS$
MSE	6.549	6.205	6.000	5.960
MAE	2.423	2.226	2.314	2.214
$[LE]^2$	0.582	0.498	0.612	0.568
$\|LE\|$	0.867	0.795	0.877	0.833

The results are shown in Table 6 and evidence as the selected models have a better performance with respect to the alternatives.

We note that many correlation coefficients between estimated innovations are equal to zero: this suggests that the consideration of the regimes captures the main features of the strong relationship seemingly exhibited by the variables. To reinforce our confidence in the modelling strategy adopted here, a bivariate VAR model on the four pairs of variables was estimated as well (no switching whatsoever): the residuals in each case are strongly correlated.

Table 7: Means and standard deviation of correlation parameters in VAR(2) models estimated on 500 series generated by models described in tables 2-5, with correlation parameters fixed equal to zero.

DGP	mean	st. dev.
$KS11/HSI$	0.229	0.033
STI/HSI	0.607	0.047
$KLSE/HSI$	0.460	0.047
$SETI/HSI$	0.444	0.037

To support our claim that undetected regimes induce spurious correlations in the residuals, we ran a few Monte Carlo experiments. More in detail, we have generated 500 series of length 544 from each data generating processes with parameters obtained by the four estimated models illustrated in tables 2 to 5, but fixing the correlation coefficients equal to zero; successively we have estimated a linear VAR(2) model. The outcome is that when MS and MCMS models with uncorrelated disturbances are simulated and then estimated by a VAR the residuals are cross-correlated. The means and standard deviations of the 500 correlation coefficients are shown in Table 7; the presence of spurious correlation is evident, especially when the data are generated by the MMS model relative to STI/HSI.

5 Concluding Remarks

In this paper we propose a new model, based on correlated Markov chains, to represent the case of interdependence among financial markets, with the case of spillover and independent markets as particular cases. The fact that the two last cases are nested in the more general model provides the possibility to test statistically the various scenarios. The case of comovement among variables, though, which is characterized by a classical Markov Switching model is not nested in the MCMS model: we resorted to a separate test for common dynamics of the two state variables.

The applications show the relevant role of Hong Kong as a dominant market over the period considered: it turns out that a plausible market characterization from the estimated models and the hypothesis testing performed is that Hong Kong has a leading role relative to Korea and to Thailand. Malaysia shows some form of interdependence while for the case of Singapore the estimated model points rather to a situation of comovement between the two markets.

The estimation of a bivariate model is forced by the difficulty of increasing the number of variables in the model without stumbling into the usual numerical problems encountered in Markov Switching models with higher number of regimes. A n-variate model with k states per variable would have a transition matrix of order k^n, which is rapidly intractable (flat likelihood function) for even moderate numbers of n or k above 2. There is therefore a trade-off between the depth of the economic interpretation which one would have available if more than two markets were to be compared and the numerical difficulties which accompany such an effort.

The definitions of spillover, interdependence, comovement and independence are consistent with large part of the literature, but we should stress that their practical characterization is different in terms of the statistical instruments utilized. For example, Forbes and Rigobon (2002) base their analysis only on the behavior of the correlation coefficients, and on a significant increase changing from a state of low to another of high volatility (with the periods of low and high volatility established a priori). In our approach, the analysis is not limited to specific episodes of crisis, the periods of high and low volatility are selected by the model itself. An important result is that the presence of correlation between the residuals disappears if one takes into proper consideration the existence of regimes and the peculiar structure of the dynamics behind them.

A Comovement in the MCMS Model

In this Appendix we demonstrate that testing the null of comovement against the hypothesis of MCMS model is equivalent to verifying a set of linear restrictions on the MCMS model.

The case of comovement corresponds to the case in which the state of y_{1t} and y_{2t} is the same for each t; this situation can justify the adoption of a classical MS model. The MS model is not nested into the MCMS model given the different number of states: hence the classical tests based on the likelihood function cannot be applied.

In view of Hamilton (1994), a Markov chain can be represented as an AR(1) process:

$$\boldsymbol{\xi}_{t+1} = \boldsymbol{P}'\boldsymbol{\xi}_t + \boldsymbol{v}_{t+1},$$

where $\boldsymbol{\xi}_t$ is a vector containing 1 in correspondence of the state at time t, $\boldsymbol{P}$ is the transition probability matrix and $\boldsymbol{v}_t$ is a vector innovation with zero mean. In our case, the multiple states are (0,0), (0,1), (1,0), (1,1); correspondingly, for example, $\boldsymbol{\xi}_t = [0, 0, 0, 1]'$ points to a value of the multiple state at time t as $(1, 1)$.

The conditional expectation of $\boldsymbol{\xi}_{t+1}$ is:

$$E(\boldsymbol{\xi}_{t+1}|\boldsymbol{\xi}_t) = \boldsymbol{P}'\boldsymbol{\xi}_t.$$

If we are interested in the behavior of the single regimes s_{1t} and s_{2t}, let us note that they can be represented as the vectors $\boldsymbol{\xi}_t^*$, respectively, $\boldsymbol{\xi}_t^{**}$. Correspondingly, their expected values are given by the 2×1 vectors:

$$E(\boldsymbol{\xi}_{t+1}^*|\boldsymbol{\xi}_t) = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \boldsymbol{P}'\boldsymbol{\xi}_t$$

and

$$E(\boldsymbol{\xi}_{t+1}^{**}|\boldsymbol{\xi}_t) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \boldsymbol{P}'\boldsymbol{\xi}_t.$$

To investigate the presence of comovement, as defined in the main body of the paper, let us test the equality of the two previous vectors, that is,

$$\left[\begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right] \boldsymbol{P}' = \left[\begin{array}{cccc} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right] \boldsymbol{P}'. \quad (5)$$

It is easy to verify that the two rows provide equal constraints: once it is verified that the first element of $\boldsymbol{\xi}_t^*$ is equal to the first element of $\boldsymbol{\xi}_t^{**}$ for each t, automatically the second elements of the two vectors will be equal, as each of them are the complements to 1 of the previous corresponding elements. Let us denote the probability $\Pr[s_{1t} = i, s_{2t} = j | s_{1t-1} = w, s_{2t-1} = z]$ by $p(ij|wz)$; as a consequence, the $\boldsymbol{P}$ matrix is:

$$\left[\begin{array}{cccc} p(00|00) & p(01|00) & p(10|00) & p(11|00) \\ p(00|01) & p(01|01) & p(10|01) & p(11|01) \\ p(00|10) & p(01|10) & p(10|10) & p(11|10) \\ p(00|11) & p(01|11) & p(10|11) & p(11|11) \end{array}\right]$$

Developing the first (or the second) equation of (5), the four constraints to be verified are:

$$\begin{array}{rcl} \Pr\left[s_{1t}=0, s_{2t}=1 | s_{1t-1}=0, s_{2t-1}=0\right] & = & \Pr\left[s_{1t}=1, s_{2t}=0 | s_{1t-1}=0, s_{2t-1}=0\right], \\ \Pr\left[s_{1t}=0, s_{2t}=1 | s_{1t-1}=0, s_{2t-1}=1\right] & = & \Pr\left[s_{1t}=1, s_{2t}=0 | s_{1t-1}=0, s_{2t-1}=1\right], \\ \Pr\left[s_{1t}=0, s_{2t}=1 | s_{1t-1}=1, s_{2t-1}=0\right] & = & \Pr\left[s_{1t}=1, s_{2t}=0 | s_{1t-1}=1, s_{2t-1}=0\right] \\ & \text{and} & \\ \Pr\left[s_{1t}=0, s_{2t}=1 | s_{1t-1}=1, s_{2t-1}=1\right] & = & \Pr\left[s_{1t}=1, s_{2t}=0 | s_{1t-1}=1, s_{2t-1}=1\right] \end{array} \quad (6)$$

Recalling the hypothesis of conditional independence (3) and the parameterization (4), we obtain that (6) corresponds to the four nonlinear constraints:

$$\frac{\exp[\alpha_1(0,.)]}{1+\exp[\alpha_1(0,.)]}\frac{1}{1+\exp[\alpha_2(.,0)]} = \frac{1}{1+\exp[\alpha_1(0,.)]}\frac{\exp[\alpha_2(.,0)]}{1+\exp[\alpha_2(.,0)]},$$

$$\frac{\exp[\alpha_1(0,.)+\beta_1(0,1)]}{1+\exp[\alpha_1(0,.)+\beta_1(0,1)]}\frac{\exp[\alpha_2(.,1)]}{1+\exp[\alpha_2(.,1)]} = \frac{1}{1+\exp[\alpha_1(0,.)+\beta_1(0,1)]}\frac{1}{1+\exp[\alpha_2(.,0)]},$$

$$\frac{1}{1+\exp[\alpha_1(.,1)]}\frac{1}{1+\exp[\alpha_2(.,0)+\beta_2(1,0)]} = \frac{\exp[\alpha_1(1,.)]}{1+\exp[\alpha_1(1,.)]}\frac{\exp[\alpha_2(.,0)+\beta_2(1,0)]}{1+\exp[\alpha_2(.,0)+\beta_2(1,0)]},$$

and

$$\frac{1}{1+\exp[\alpha_1(1,.)+\beta_1(1,1)]}\frac{\exp[\alpha_2(.,1)+\beta_2(1,1)]}{1+\exp[\alpha_2(.,1)+\beta_2(1,1)]} = \frac{\exp[\alpha_1(1,.)+\beta_1(1,1)]}{1+\exp[\alpha_1(1,.)+\beta_1(1,1)]}\frac{1}{1+\exp[\alpha_2(.,1)+\beta_2(1,1)]}$$

After simple algebraic manipulations, the previous nonlinear relationships among the probabilities parameters are equivalent to the following linear restrictions:

$$\begin{aligned}\alpha_1(0,.) &= \alpha_2(.,0), \\ \alpha_1(0,.)+\beta_1(0,1)+\alpha_2(.,1) &= 0, \\ \alpha_1(.,1)+\alpha_2(.,0)+\beta_2(1,0) &= 0 \\ \text{and} & \\ \alpha_1(1,.)+\beta_1(1,1) &= \alpha_2(.,1)+\beta_2(1,1)\end{aligned}$$

In the simultaneous presence of these four constraints, we can think of common dynamics for the state variables and therefore of comovement.

Acknowledgements

We would like to thank the anonymous referees for their detailed reading of the paper and many suggestions which lead to the clarification of the focus of the paper. Herman van Dijk and Alessandra Amendola provided support and encouragement to this line of research. All remaining errors are ours. Financial support from Italian MIUR under grants 2002134775_004, 2004139322_007 and 2006137221_001 is gratefully acknowledged.

References

[1] Anas, J., Billio, M., Ferrara, L. and Lo Duca, M., 2006. Euro-zone business cycle analysis with multivariate Markov switching models. In: G.L. Mazzi and G. Savio (Eds.), Growth and Cycle in the Euro-zone, Palgrave-MacMillan, New-York.

[2] Andersen, T.G. and Bollerslev, T., 1998. Answering the skeptics: yes, standard volatility models do provide accurate forecasts. International Economic Review, 39 885-905.

[3] Baele, L., 2005. Volatility spillover effects in European equity markets. Journal of Financial and Quantitative Analysis, 40.

[4] Bekaert, G., Harvey, C.R. and Ng, A., 2005. Market integration and contagion. Journal of Business, 78.

[5] Bollerslev, T., Law, T.H. and Tauchen, G. Risk, jumps, and diversification. Mimeo (Duke University, 2007).

[6] Bollerslev, T. and Wooldridge, J.M., 1992. Quasi-maximum likelihood estimation and inference in dynamic models with time varying covariances. Econometric Reviews, 11 143-172.

[7] Brunetti, C. and Lildholdt, P.M., 2006. Relative efficiency of return- and range-based volatility estimators. Mimeo (Penn Institute for Economic Research).

[8] Brunetti, C., Mariano, R. S., Scotti, C. and Tan, A.H.H., 2003. Markov switching GARCH models of currency crises in Southeast Asia. PIER Working Paper No. 03-008.

[9] Bulla, J. and Bulla, I., 2006. Stylized facts of financial time series and hidden semi-Markov models. Computational Statistics and Data Analysis, 51 2192-2209.

[10] Carvalho, C. M. and Lopes, H.F., 2007. Simulation-based sequential analysis of Markov switching stochastic volatility models, Computational Statistics and Data Analysis, 51 4526-4542.

[11] Diebold, F.X. and Yilmaz, K., 2007. Measuring financial asset return and volatility spillovers, with application to global equity markets. PIER Working Paper N0. 07-002.

[12] Edwards, S. and Susmel, R., 2001. Volatility dependence and contagion in emerging equity markets. Journal of Development Economics, 66 505-532.

[13] Edwards, S. and Susmel, R., 2003. Interest-rate volatility in emerging markets. The Review of Economics and Statistics, 85 328-348.

[14] Engle, R.F., Ito, T. and Lin, W., 1990. Meteor showers or heat waves? Heteroskedastic intra-daily volatility in the foreign exchange market. Econometrica, 58 525-542.

[15] Forbes, K.J. and Rigobon, R., 2002. No contagion, only interdependence: measuring stock market comovements. The Journal of Finance, 62 2223-2261.

[16] Frijns, B. and Schotman, P.C., 2006. Nonlinear dynamics in Nasdaq dealer quotes. Computational Statistics and Data Analysis, 51 2246-2266.

[17] Garman, M.B. and Klass, M.J., 1980. On the estimation of security price volatilities from historical data. The Journal of Business, 53 67-78.

[18] Hamilton, J.D., 1989. A new approach to the economic analysis of nonstationary time series and the business cycle. Econometrica, 57 357-384.

[19] Hamilton, J.D., 1990. Analysis of time series subject to changes in regime. Journal of Econometrics, 45 39-70.

[20] Hamilton, J.D., 1994. State–Space models. In: R.F. Engle and D.L. McFadden (Eds.), Handbook of Econometrics, vol. IV, North-Holland, Amsterdam, 3039-3080.

[21] Hamilton, J.D. and Susmel, R., 1994. Autoregressive conditional heteroskedasticity and changes in regime. Journal of Econometrics, 64 307-333.

[22] Kim, C.J., 1994. Dynamic linear models with Markov-switching. Journal of Econometrics, 60 1-22.

[23] Krolzig, H.M., 1997. Markov switching vector autoregressions: modelling, statistical inference and application to business cycle analysis. Springer, Berlin.

[24] Lee, S. and Kim, K.J., 1993. Does the October 1987 crash strengthen the comovements among national stock markets?. Review of Financial Studies, 3 89-102.

[25] Otranto, E., 2005. The multi-chain Markov switching model. Journal of Forecasting, 24 523-537.

[26] Parkinson, M., 1980. The extreme value method for estimating the variance of the rate of return. Journal of Business, 53 61-65.

[27] Pericoli, M. and Sbracia, M., 2003. A primer on financial contagion. Journal of Economic Surveys, 17 571-608.

[28] So, M.K.P., Lam, K. and Li, W.K., 1998. A stochastic volatility model with Markov switching. Journal of Business and Economic Statistics, 16 244253.