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# Comparison of Volatility Measures: a Risk Management Perspective

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# Comparison of Volatility Measures: a Risk Management Perspective

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**Abstract**

Volatility measurement has received a boost from the availability of ultra–high frequency data (UHFD) sampled at different frequencies which need to be complemented by appropriate methods to project volatility behavior. In this paper we take a risk management perspective and address the issue of forecasting Value–at–Risk (VaR) using different volatility measures: realized volatility, bipower realized volatility, two scales realized volatility, as well as the daily range. For the sample and assets chosen, volatility clustering occurs around a changing level in average volatility; other features such as persistence and shape appear to change with the UHFD sampling frequency. Building on the existing literature, we propose a novel modeling approach that captures the features of the series called P–Spline Multiplicative Error Model. Such an approach consists of a dynamic model with a flexible trend specification bonded with a penalized maximum likelihood estimation strategy that enhances forecasting ability. Results show that exploiting UHFD volatility measures, VaR predictive ability is improved upon relative to a baseline GARCH approach but the range is not outperformed and that there are relevant gains from modeling volatility trends and the nonnormality of the conditional return distribution.

**Keywords:** Volatility Measures, Value–at–Risk, Forecasting, GARCH, MEM, Spline GARCH, P-Spline MEM, Shrinkage Estimation.

**JEL:** C22, C51, C52, C53

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# 1 Introduction

Measurement and forecasting latent volatility has many important applications in many areas of finance including asset allocation, option pricing and risk management. The two tasks have been successfully accomplished within the same ARCH framework (Engle (1982), Bollerslev, Engle & Nelson (1994)) for the past 25 years. Alternative measurements based on different assumptions and different information sets have been in use for a while, such as historical standard deviations, range, implied volatilities; in recent times the properties of volatility proxies derived from the availability of intra-daily data sampled at high frequency have been the object of a sizeable strand of research (e.g. Andersen & Bollerslev (1998), Andersen, Bollerslev, Diebold & Ebens (2001) Barndorff-Nielsen & Shephard (2002), Andersen, Bollerslev, Diebold & Labys (2003)). Under suitable assumptions they converge (as the sampling frequency of the intra-daily data increases) to the integrated variance, that is the integral of instantaneous (or spot) volatility of an underlying continuous time process over a short period. While it is possible, in theory, to construct ex–post measures of return variability with arbitrary precision, their relationship to the latent underlying process (e.g. with or without jumps) and how to forecast volatility on the basis of existing information is still open to question.

Not knowing what latent process best describes the data generating process, in this work we address the forecasting issue from a pragmatic point of view, trying to establish to which extent different volatility measures improve upon the out–of–sample forecasting ability of standard methods. Several metrics can be used to evaluate the forecasting performance: a Mincer Zarnowitz type regression where each forecast is contrasted against a suitable 'target' (typically one of the measures themselves), implied volatility measures (such as VIX), or, within a risk management framework, the quality of the derived measures of Value at Risk (VaR) or Expected Shortfall (ES) which have emerged as a prominent measure of market risk. A VaR forecasting application is an interesting battleground (Andersen et al. (2003)), so to speak, for comparing different volatility measures. Here it is limited to a single asset, but it could be extended to a multivariate context.

In this work we compare the VaR forecasting ability of three Ultra–High Frequency Data (UHFD) based volatility measures computed each day using data sampled at different frequencies: realized volatility (Andersen & Bollerslev (1998), Andersen et al. (2003)), bipower realized volatility (Barndorff-Nielsen & Shephard (2004)) and two scales realized volatility (Zhang, Mykland & Aït-Sahalia (2005) which vary in their theoretical properties according to the nature of the underlying data generating process. For comparison purposes we also include the range (Parkinson (1980)). The out–of–sample risk management forecasting comparison we build up uses a two-step procedure to forecast VaR. The first step consists of using a dynamic specification for modeling the volatility measures. We perform this task using a Multiplicative Error Model (MEM) (c.f. Engle (2002), Engle & Gallo (2006)) and the novel P–Spline MEM, a modeling approach able to capture the long and short run dynamics of the volatility series which builds up on the recent proposal of Engle & Rangel (2007). Such an approach is based on a flexible specification of the volatility trend based on B-splines bonded with a penalized maximum likelihood estimation strategy which enhances forecasting ability (Eilers & Marx (1996)). The second step consists of modeling returns using a conditional heteroskedastic model based on the volatility predictions from different measures. We then evaluate the VaR performance within a risk management framework assessing the accuracy and adequacy of VaR forecasts against a GARCH benchmark.

The out–of–sample forecasting results on a sample of NYSE blue chips hint that UHFD volatility measures are more accurate than the benchmark ARCH type model but they do not appear to outperform the range. We find that the sampling frequency of the intra-daily data plays a bigger role in forecasting than the choice of the UHFD volatility measures and "high" frequencies (20/30 minutes) perform better than "low" frequencies (30 seconds/1 minute). The sampling frequency appears to have an impact on both the distributional and dynamic features of the UHFD volatility measures. Our findings are consistent with the claim that at very high frequencies microstructure dynamics bias volatility dynamics, so that there is a limit to the benefits of increasing the sampling frequency. However, the ranking between volatility measures is dependent on the choice of the forecasting models, and the differences between the forecasting abilities become smaller when using better model specifications. The P–Spline MEM appears to capture satisfactorily the series dynamics and it systematically improves out–of–sample forecasting ability over simpler specifications.

The closest contributions to our paper is the work by Andersen et al. (2003) and Giot & Laurent (2004) that contain VaR forecasting applications using realized volatility. Initial work on realized volatility includes Andersen & Bollerslev (1998), Andersen et al. (2001), Barndorff-Nielsen & Shephard (2002), Meddahi (2002) and Andersen et al. (2003). Recent extensions and refinement of the early results are found in, *inter alia*, Bandi & Russell (2003), Oomen (2005), Zhang (2006), Barndorff-Nielsen, Hansen, Lunde & Shepard (2006*a*). Stylized facts on pre February 2001 equity data are described in Andersen et al. (2001) and Ebens (1999). MEMs are a generalization of the ARCH and ACD for modeling nonnegative time series proposed in Engle (2002) and have had a significant application in comparing different volatility indicators in Engle & Gallo (2006). Further extensions and applications are presented in Chou (2005), Cipollini, Engle & Gallo (2006) and Lanne (2006). Engle & Rangel (2007) proposed the spline modeling approach for capturing volatility trends. The P-Spline modeling approach was proposed in the context of smoothing in the GLM framework by Eilers & Marx (1996). This type of modelling different frequencies of evolution of volatility is alternative to traditional approaches which take long-range dependence into account in the form of ARFIMA–type of models on the logarithm of realized volatility (e.g. Andersen et al. (2003), Martens & Zein (2004), Koopman, Jungbacker & E. (2005), Deo, Hurvich & Lu (2006), Pong, Shacketon, Taylor & Xu (2004)) and regression models mixing information at different frequencies (e.g. the so called Heterogeneous AR (HAR) model of Corsi (2004) extended by Andersen, Bollerslev & Diebold (2007) and Ghysels, Santa-Clara & Valkanov (2006) in a MIDAS framework). The evaluation of the VaR forecasts is based on the contributions by Christoffersen (1998), Sarma, Thomas & Shah (2003), Engle & Manganelli (2004) and Kuester, Mittnik & Paolella (2006).

The rest of the paper is organized as follows. Section 2 describes the VaR modeling framework based on volatility measures. Section 3 defines the volatility measures used in this work and summarizes the stylized fact of the series. Section 4 discusses the dynamic specifications for the volatility measures. Section 5 discusses a conditionally heteroskedastic model for returns based on the volatility measures. Section 6 presents the VaR forecasting results. Concluding remarks follow in Section 7.

# 2 A Value–at–Risk Framework for the Comparison

There is a wide variety of methods for forecasting VaR in the literature: Historical Simulation, Extreme Value Theory, Conditional Autoregressive Value at Risk (CAViaR) and so forth. Kuester et al. (2006) contains a review and comparison of many proposals.

Our VaR modelling approach builds up on the contribution of Giot & Laurent (2004) for forecasting VaR using realized volatility. Let $r_t$ be the daily (close–to–close) return at time $t$ on a single asset. We assume that

$$r_t = \sqrt{h_t}\, z_t, \qquad z_t \sim F,$$

where $h_t$ is the conditional variance of the daily return at time $t$ and $z_t$ is an i.i.d. unit variance and possibly skewed and leptokurtic random variable from some appropriate cumulative distribution $F$. The 1 day ahead 100(1-p)% VaR is defined as the maximum 1 day ahead loss, that is

$$\mathsf{VaR}^p_{t|t-1} \equiv -F^{-1}(p)\sqrt{h_t},$$

assuming that $h_t$ is known conditional on the information available at time $t-1$. In a GARCH framework one would model the conditional variance of returns, project it one day ahead and use some distributional assumption on $F$ (either parametric or empirical based) to provide the proper quantile of the distribution of the standardized residuals.

If a series for a return variance proxy is directly available, one can depart from this standard procedure. Let such a generic proxy (computed according to definition $m$) be denoted as $rv_{(m,\delta)\,t}$ computed using intra-daily data sampled at frequency $\delta$ on day $t$ and let $rv_{(m,\delta)\,t|t-1}$ denote its expectation conditional on the information available at time $t-1$, using some suitable model specification. Then we assume that the conditional variance of returns is some function of $rv_{(m,\delta)\,t|t-1}$ and a vector of unknown parameters $\psi$:

$$h_t = f(\; rv_{(m,\delta)\,t|t-1} \mid \psi \;).$$

We need to specify (i) a model that captures the dynamics of the volatility measures in order to obtain the conditional expectations of volatility, (ii) a model that connects the conditional variance of returns with the conditional expectation of the volatility measures and (iii) an appropriate distribution for the standardized return distribution.

# 3 Definitions and Stylized Facts

The intuition behind UHFD volatility measures dates at least back to Merton (1980). Authors including Andersen, Bollerslev, Diebold brought back the idea in the mid–90's in correspondence with the availability of large databases containing detailed information of financial transactions in several financial markets.

## 3.1 Volatility Measures

The building blocks of the UHFD volatility measures are intra–daily prices. Let $p_{i,t}$ denote the $i$-th log intra–daily price of day $t$ sampled at frequency $\delta$. The series is constructed by taking the last recorded tick-by–tick price every $\delta$ units of time starting from an initial time of the day (typically the opening) until the closing[1]. Note that overnight information is not included in these series and this has to be taken into account for in the modeling of daily (close–to–close) returns (c.f. Gallo (2001), Martens (2002), Fleming, Kirby & Ostdiek (2003) and Hansen & Lunde (2005)).

The **Realized Volatility** (Andersen et al. (2001)) has become the benchmark UHFD volatility measure, commonly used in applied work. It is defined as

$$rv_{(\mathsf{V},\delta)\,t} \equiv \sum_{i=2}^{n} (p_{i,t} - p_{i-1,t})^2.$$

Under appropriate assumptions including the absence of jumps and microstructure noise, $rv_{(\mathsf{V},\delta),t}$ convergences to the latent volatility as the sampling frequency increases.

The **Bipower Realized Volatility** (Barndorff-Nielsen & Shephard (2004)is one of the first variants of realized volatility proposed in the literature as a robust UHFD volatility measure in the presence of *infrequent jumps*. It is defined as

$$rv_{(\mathsf{B},\delta)\,t} \equiv \frac{\pi}{2} \sum_{i=3}^{n} |p_{i,t} - p_{i-1,t}||p_{i-1,t} - p_{i-2,t}|.$$

Under appropriate assumptions including the absence of microstructure noise, bipower realized volatility converges to the latent volatility while realized volatility converges to the latent volatility plus a component depending on the jumps.

The **Two Scales Realized Volatility** (Zhang et al. (2005)) is the first consistent estimator of latent volatility in the presence of *iid* microstructure noise. The definition of this measure requires some further notation. Let $p^{\mathsf{f}}_{i,t}$ denote the $i$–th log intra–daily price of day $t$ sampled at some "very high" fixed frequency $\delta_{\mathsf{f}}$ and let $p^{g}_{j,t} = p_{g+(\delta/\delta_{\mathsf{f}})(j-1),t}$, with $g = 1, ..., G$ and $G = \delta/\delta_{\mathsf{f}}$, denote the log intra–daily price series obtained by sampling observations from $p_{i,t}$ at frequency $\delta$ starting from $G$ different initial times of day. Define $rv^{g}_{(V,\delta)\,t} \equiv \sum_{j=2}^{n_g} (p^{g}_{j,t} - p^{g}_{j-1,t})^2$ and $rv_{(V,\delta_{\mathsf{f}})\,t} \equiv \sum_{i=1}^{n} (p^{\mathsf{f}}_{i,t} - p^{\mathsf{f}}_{i-1,t})^2$. Then the two scales realized volatility is defined as

$$rv_{(\mathsf{TS},\delta)\,t} \equiv \frac{1}{G} \sum_{g=1}^{G} rv^{g}_{(\mathsf{V},\delta)\,t} - \frac{n_g}{n} rv_{(\mathsf{V},\delta_{\mathsf{f}})\,t}.$$

The expression "two scales" derives from the fact that this estimator combines the information from a slow ($\delta$) and fast ($\delta_{\mathsf{f}}$) time scale. Under appropriate assumption ($\delta \to 0$ and $\delta^2/\delta_{\mathsf{f}} \to 0$), $rv_{(\mathsf{TS},\delta)\,t}$ converges to the latent volatility while realized volatility diverges to infinity.

For comparison purposes we also consider a rather old measure of volatility proposed in the literature: the daily **Range**. The Range volatility estimator can be defined as

$$rv_{(\mathsf{R})\,t} \equiv \frac{1}{4\log(2)} (p_{\mathsf{high},t} - p_{\mathsf{low},t})^2$$

where $p_{\mathsf{high},t}$ and $p_{\mathsf{low},t}$ are respectively the max and min log intra-daily prices of day $t$. Interestingly, a number of authors have recently devoted attention to the range and found

[1]Recently, a number of researchers have claimed that sampling in tick time is more appropriate than sampling in calendar time, see also Renault & Werker (2004).

evidence of good forecasting performance of the models exploiting this indicator (c.f. Alizadeh, Brandt & Diebold (2002), Chou (2005), Brandt & Jones (2006), Engle & Gallo (2006), Ghysels et al. (2006), Christensen & Podolskij (2006)).

## 3.2 Data and Stylized Facts

Our empirical investigation is carried out on of three NYSE stocks: Boeing (BA), General Electric (GE) and Johnson and Johnson (JNJ). The data is extracted from the NYSE-TAQ database. All the series analyzed in this study are constructed using "cleaned" mid quotes from the NYSE between 9:30 and 16:05. For each stock we construct the series of volatility measures defined in the previous section and the series of daily returns. The volatility series are constructed for 12 intra–daily frequencies ranging from 30 seconds to 1 hour[2]. For two scales realized volatility the "high" fixed frequency is taken equal to 15 seconds. The sample period is from February 2001 to December 2006 and contains 1465 daily observations. The analysis of these years is challenging in that this sample contains periods of very high volatility (early 2000s recession following the collapse of the Dot-com bubble, 9/11) followed by a period of very low volatility. Moreover, at the end of January 2001 the NYSE changed its ticksize[3] and this event is bound to have had some impact on the empirical properties of the UHFD volatility measures established in studies on earlier samples. Further discussion of the features of the UHFD series can be found in Brownlees & Gallo (2006).

While the details of the stylized facts are reported in the appendix, it is worthwhile to pinpoint some features of the series which we use as guidance for the subsequent modeling effort:

- Upon visual inspection of the graphs (Figure 2), volatility clustering occurs around a changing level in average volatility (higher in the early part of the sample).
- The persistence and shape of the UHFD volatility measures appears to be frequency dependent. Serial correlation is higher at higher frequencies while skewness and kurtosis decrease.
- Since the mean of realized volatility across sampling frequencies in excess of 30 seconds is substantially constant, it seems that the impact of *iid* microstructure noise for these series is less noticeable than in earlier/other datasets (c.f. Barndorff-Nielsen et al. (2006*a*)). Whether this is common to other series computed from mid-quotes of highly liquid stocks on a period when the minimum tick–size is USD 0.01 is open to further empirical investigation.
- There is evidence of *dependence* in microstructure noise. The increase in the serial correlation and cross correlation between UHFD measures at higher frequencies hints at the presence of *dependent* microstructure noise that appears to bias the UHFD volatility measures (c.f. Barndorff-Nielsen et al. (2006*a*)).

[2]At the end of January 2001 the NYSE changed the minimum price variation of all securities from USD 0.0625 (1/16) to USD 0.01.

[3]The frequencies are: 30s, 1m, 2m, 3m, 4m, 5m, 6m, 10m, 15m, 20m, 30m and 1h.

- Almost all volatility measures systematically underestimate the variance of returns. This is due to the fact that the volatility measures are based only on *intra–daily* information while the daily return is made up of an *intra–daily* and *overnight* component.

- Daily returns standardized by the square root of the volatility measures do not exhibit ARCH effects but do not always appear to be normal.

# 4 Modeling Volatility Measures

The volatility measures series exhibit different features depending on the choice of the measure adopted and, eventually, the sampling frequency of the UHFD. The MEM class is a convenient family of specifications for modeling and forecasting volatility measures that impose reasonable amount of assumptions on the data.

## 4.1 A Family of Dynamic Models for Volatility Measures

Let $rv_{(m,\delta)\,t}$ be the measure of volatility $m$ sampled at frequency $\delta$ and let $\mathcal{F}_{t-1}$ be the information set at $t-1$. The Multiplicative Error Model for the volatility measure $rv_{(m,\delta)\,t}$ is defined as

$$rv_{(m,\delta)\,t} = \sigma^2_{(m,\delta)\,t}\,\varepsilon_{(m,\delta)\,t}, \tag{1}$$

where, conditional on $\mathcal{F}_{t-1}$, $\sigma^2_{(m,\delta)\,t}$ is a nonnegative predictable process function of $\theta$,

$$\sigma^2_{(m,\delta)\,t} = \sigma^2_{(m,\delta)\,t}(\theta);$$

and $\varepsilon_{(m,\delta)\,t}$ unit expected value iid innovation term,

$$\varepsilon_{(m,\delta)\,t}|\mathcal{F}_{t-1} \sim Gamma(\phi, 1/\phi).$$

It then follows from standard properties of the gamma distribution that conditional on time $t$, the volatility measures is distributed as

$$rv_{(m,\delta)\,t+1}|\mathcal{F}_t \sim Gamma\left(\phi, \sigma^2_{(m,\delta)\,t+1}/\phi\right),$$

and the conditional expectation of the volatility measures $rv_{(m,\delta)\,t}$ is

$$rv_{(m,\delta)\,t+1|t} \equiv \mathsf{E}(rv_{(m,\delta)\,t+1}|\mathcal{F}_t) = \sigma^2_{(m,\delta)\,t+1}.$$

Discussions and extensions on the properties of this model class can be found in Engle (2002), Engle & Gallo (2006), Cipollini et al. (2006).

The are a number of reasons why we argue that MEMs are a suitable specification for modeling volatility measures. The MEM is a nonnegative time series model and hence it always produces nonnegative predictions. It provides unbiased predictions of the volatility measures without having to transform predictions and imposing extra assumptions on the data.The Gamma innovation term assumption is a rather flexible distributional assumption that is able to capture the different shapes exhibited by different volatility proxies. Lastly, if $\sigma^2_{(m,\delta)\,t} = E(rv_{(m,\delta)\,t}|\mathcal{F}_{t-1})$, the expected value of the score of $\theta$ evaluated at the true parameters is zero irrespective of the Gamma assumption on $\epsilon_{(m,\delta)\,t}|\mathcal{F}_{t-1}$, the ML estimator is also a QML estimator (White (1994)).

## 4.2 Base MEM

The challenge for successful forecasting lies in choosing an appropriate specification for the volatility measures dynamics $\sigma^2_{(m,\delta)\,t}$ in Equation 1. The Base MEM specification for $\sigma^2_{(m,\delta)\,t}$ adopted in this work is

$$\sigma^2_{(m,\delta)\;t} = \omega + \alpha rv_{(m,\delta)\;t-1} + \beta \sigma^2_{(m,\delta)\;t-1} + \alpha^- rv^-_{(m,\delta)\;t-1} \tag{2}$$

with $rv^-_{(m,\delta)\;t-1} \equiv rv_{(m,\delta)\;t-1} 1_{\{r_{t-1}<0\}}$. The base specification of Equation 2 represents the analog of the GARCH(1,1) model with leverage effects (Glosten, Jagannanthan & Runkle (1993)).

Tables 1, 2 and 3 about here.

The Base MEM specification of Equation 2 is estimated over the whole sample via maximum likelihood. The left hand side of Tables 1, 2 and 3 report the parameter estimates and residual diagnostics of the Base model. The model does not always appear to be able to capture the dynamics of the series as the Ljung–Box test statistic is sometimes significant at standard significance levels. The GE residuals appear to be quite dirty while the BA and JNJ residuals are much better behaved. Interestingly, evidence of autocorrelation in the GE residuals decreases as the sampling frequency decreases. The estimation results exhibit IGARCH type effects: the estimated persistence of shocks varies between 0.97 to 1.00, suggesting the presence of a unit root in the variance. The shape of the innovation distribution appears to change with the sampling frequency: the higher the frequency, the more mound-shaped it is.

## 4.3 P-Spline MEM

The Base MEM estimation results suggest the presence of a unit root in the volatility measures which is consistent with the presence of long range dependence in the series. A modeling approach that captures long range dependence and is consistent with the stylized facts of Section 3 consists of specifying a trend component in the volatility dynamics. Early theoretical and empirical economic justification of such an approach can be found in the work by Olsen & Associates research institute (e.g. Mullër, Dacorogna, Davé, Olsen, Pictet & von Weizsäcker (1997)). The Olsen researchers find that volatility dynamics are well described by a model with a short and a long term component as a results of the interactions of different agents with different time–horizons in the financial markets: the long-term component is determined by "fundamentals" while the short-term component generates volatility clusters around the long-term component.

Following Engle & Rangel (2007), a flexible MEM specification for $\sigma^2_{(m,\delta)\,t}$ capable of capture long and short run dynamics is given by

$$\sigma^2_{(m,\delta)\;t} = \tau_{(m,\delta)\;t}\; g_{(m,\delta)\;t}, \tag{3}$$

where

$$\tau_{(m,\delta)\;t} \equiv \exp\left\{\sum \gamma_i B_i(t)\right\}, \tag{4}$$

## BA

| | Freq. | Base Pers. | Base $\hat{\phi}$ | Base $Q_{10}(\hat{\epsilon}_t)$ | P-Spline Pers. | P-Spline $\hat{\phi}$ | P-Spline $\hat{\lambda}$ | P-Spline $Q_{10}(\hat{\epsilon}_t)$ |
|---|---|---|---|---|---|---|---|---|
| $V$ | 30s | 0.99 | 6.2 | 0.082 | 0.82 | 7.88 | 9 | 0.601 |
| | 1m | 0.98 | 6.32 | 0.075 | 0.82 | 7.35 | 4 | 0.477 |
| | 2m | 0.98 | 5.29 | 0.065 | 0.82 | 6.34 | 1 | 0.330 |
| | 3m | 0.98 | 4.75 | 0.269 | 0.82 | 5.47 | 6 | 0.666 |
| | 4m | 0.98 | 4.34 | 0.098 | 0.82 | 5.04 | 3 | 0.291 |
| | 5m | 0.98 | 4.14 | 0.462 | 0.83 | 4.7 | 13 | 0.965 |
| | 6m | 0.98 | 3.83 | 0.076 | 0.84 | 4.17 | 12 | 0.260 |
| | 10m | 0.98 | 3.19 | 0.472 | 0.79 | 3.47 | 5 | 0.963 |
| | 15m | 0.99 | 2.58 | 0.391 | 0.78 | 2.71 | 6 | 0.845 |
| | 20m | 0.99 | 2.26 | 0.176 | 0.70 | 2.4 | 1 | 0.775 |
| | 30m | 0.99 | 1.79 | 0.990 | 0.77 | 1.89 | 5 | 0.998 |
| | 1h | 0.99 | 1.29 | 0.686 | 0.70 | 1.36 | 1 | 0.760 |
| $B$ | 30s | 0.98 | 5.76 | 0.059 | 0.81 | 7.02 | 5 | 0.558 |
| | 1m | 0.98 | 5.74 | 0.078 | 0.82 | 6.58 | 4 | 0.444 |
| | 2m | 0.98 | 4.89 | 0.030 | 0.82 | 5.74 | 4 | 0.239 |
| | 3m | 0.98 | 4.1 | 0.246 | 0.83 | 4.99 | 11 | 0.652 |
| | 4m | 0.97 | 4.27 | 0.042 | 0.82 | 4.74 | 9 | 0.110 |
| | 5m | 0.98 | 3.87 | 0.305 | 0.84 | 4.39 | 17 | 0.881 |
| | 6m | 0.99 | 3.53 | 0.102 | 0.85 | 3.92 | 3 | 0.341 |
| | 10m | 0.98 | 2.9 | 0.366 | 0.79 | 3.18 | 4 | 0.823 |
| | 15m | 0.98 | 2.27 | 0.069 | 0.74 | 2.57 | 4 | 0.515 |
| | 20m | 0.99 | 2.03 | 0.128 | 0.67 | 2.29 | 3 | 0.726 |
| | 30m | 0.99 | 1.67 | 0.966 | 0.80 | 1.77 | 5 | 0.995 |
| | 1h | 0.99 | 1.15 | 0.657 | 0.61 | 1.25 | 1 | 0.557 |
| $TS$ | 30s | 0.98 | 6.31 | 0.055 | 0.83 | 7.57 | 11 | 0.436 |
| | 1m | 0.98 | 5.61 | 0.034 | 0.82 | 7.04 | 3 | 0.288 |
| | 2m | 0.98 | 5.07 | 0.088 | 0.84 | 6.22 | 14 | 0.454 |
| | 3m | 0.98 | 4.65 | 0.193 | 0.83 | 5.57 | 4 | 0.626 |
| | 4m | 0.98 | 4.45 | 0.152 | 0.83 | 5.11 | 4 | 0.626 |
| | 5m | 0.98 | 4.13 | 0.158 | 0.82 | 4.76 | 3 | 0.704 |
| | 6m | 0.98 | 4.23 | 0.164 | 0.82 | 4.49 | 12 | 0.730 |
| | 10m | 0.98 | 3.31 | 0.149 | 0.79 | 3.68 | 7 | 0.928 |
| | 15m | 0.98 | 2.81 | 0.106 | 0.74 | 3.1 | 2 | 0.848 |
| | 20m | 0.98 | 2.5 | 0.083 | 0.71 | 2.77 | 5 | 0.729 |
| | 30m | 0.98 | 2.04 | 0.219 | 0.67 | 2.29 | 4 | 0.719 |
| | 1h | 0.98 | 1.44 | 0.787 | 0.61 | 1.65 | 3 | 0.789 |
| $R$ | | 0.97 | 1.84 | 0.514 | 0.69 | 1.96 | 5 | 0.927 |

Table 1: Estimation results for the volatility models. For each volatility measures, sampling frequency (when applicable) and volatility model (Base or P-Spline) the table reports the estimated persistence ($\hat{\alpha} + \hat{\beta} + \hat{\alpha}^{-}/2$), shape parameter $\hat{\phi}$ and the p–value of the Ljung–Box test of the residuals $rv_{(m,\delta)\,t}/\hat{\sigma}^2_{(m,\delta)\,t}$. Moreover the table reports the selected shrinkage coefficients $\hat{\lambda}$ for the P-Spline MEM.

## GE

| | Freq. | Base Pers. | $\hat{\phi}$ | $Q_{10}(\hat{\epsilon}_t)$ | P-Spline Pers. | $\hat{\phi}$ | $\hat{\lambda}$ | $Q_{10}(\hat{\epsilon}_t)$ |
|---|---|---|---|---|---|---|---|---|
| $V$ | 30s | 1.00 | 6.02 | 0.022 | 0.90 | 8.15 | 2 | 0.187 |
| | 1m | 1.00 | 4.91 | 0.156 | 0.89 | 7.15 | 3 | 0.258 |
| | 2m | 0.99 | 4.54 | 0.010 | 0.86 | 5.99 | 2 | 0.171 |
| | 3m | 0.99 | 4.57 | 0.032 | 0.84 | 5.35 | 2 | 0.299 |
| | 4m | 0.99 | 4.17 | 0.028 | 0.85 | 4.8 | 3 | 0.282 |
| | 5m | 1.00 | 3.51 | 0.083 | 0.85 | 4.47 | 3 | 0.383 |
| | 6m | 1.00 | 3.25 | 0.051 | 0.85 | 4.18 | 3 | 0.237 |
| | 10m | 1.00 | 2.63 | 0.059 | 0.87 | 3.64 | 3 | 0.313 |
| | 15m | 1.00 | 2.47 | 0.139 | 0.86 | 2.96 | 4 | 0.468 |
| | 20m | 1.00 | 2.64 | 0.074 | 0.86 | 2.69 | 4 | 0.536 |
| | 30m | 0.99 | 1.94 | 0.119 | 0.86 | 2.15 | 2 | 0.190 |
| | 1h | 1.00 | 1.42 | 0.916 | 0.80 | 1.47 | 2 | 0.886 |
| $B$ | 30s | 0.99 | 4.73 | 0.023 | 0.90 | 7.06 | 3 | 0.149 |
| | 1m | 0.99 | 4.88 | 0.023 | 0.88 | 6.34 | 2 | 0.196 |
| | 2m | 0.99 | 4.33 | 0.004 | 0.85 | 5.55 | 2 | 0.159 |
| | 3m | 0.99 | 4.23 | 0.020 | 0.84 | 4.97 | 2 | 0.351 |
| | 4m | 1.00 | 3.38 | 0.058 | 0.85 | 4.47 | 2 | 0.531 |
| | 5m | 0.99 | 3.23 | 0.047 | 0.85 | 4.15 | 3 | 0.339 |
| | 6m | 1.00 | 2.94 | 0.008 | 0.84 | 3.93 | 2 | 0.219 |
| | 10m | 0.99 | 2.84 | 0.030 | 0.87 | 3.41 | 3 | 0.321 |
| | 15m | 0.99 | 2.33 | 0.232 | 0.85 | 2.71 | 5 | 0.447 |
| | 20m | 1.00 | 2.05 | 0.210 | 0.86 | 2.46 | 5 | 0.636 |
| | 30m | 0.99 | 1.66 | 0.243 | 0.85 | 1.94 | 3 | 0.365 |
| | 1h | 0.99 | 1.26 | 0.743 | 0.81 | 1.33 | 3 | 0.957 |
| $TS$ | 30s | 0.99 | 5.93 | 0.024 | 0.88 | 7.69 | 1 | 0.171 |
| | 1m | 1.00 | 4.85 | 0.027 | 0.87 | 6.68 | 1 | 0.197 |
| | 2m | 1.00 | 4.64 | 0.015 | 0.86 | 5.89 | 2 | 0.152 |
| | 3m | 0.99 | 4.11 | 0.035 | 0.85 | 5.38 | 1 | 0.252 |
| | 4m | 0.99 | 4.2 | 0.039 | 0.85 | 5.02 | 3 | 0.304 |
| | 5m | 0.99 | 4.18 | 0.035 | 0.84 | 4.76 | 2 | 0.309 |
| | 6m | 0.99 | 3.66 | 0.043 | 0.84 | 4.57 | 3 | 0.288 |
| | 10m | 1.00 | 3.28 | 0.017 | 0.84 | 3.99 | 3 | 0.166 |
| | 15m | 1.00 | 2.79 | 0.010 | 0.83 | 3.57 | 3 | 0.084 |
| | 20m | 1.00 | 2.95 | 0.006 | 0.82 | 3.28 | 4 | 0.065 |
| | 30m | 0.99 | 2.32 | 0.022 | 0.80 | 2.8 | 4 | 0.125 |
| | 1h | 0.99 | 1.81 | 0.608 | 0.74 | 2.03 | 3 | 0.570 |
| $R$ | | 1.00 | 1.76 | 0.681 | 0.75 | 1.9 | 3 | 0.939 |

Table 2: Estimation results for the volatility models. For each volatility measures, sampling frequency (when applicable) and volatility model (Base or P-Spline) the table reports the estimated persistence ($\hat{\alpha} + \hat{\beta} + \hat{\alpha}^{-}/2$), shape parameter $\hat{\phi}$ and the p–value of the Ljung–Box test of the residuals $rv_{(m,\delta)\,t}/\hat{\sigma}^2_{(m,\delta)\,t}$. Moreover the table reports the selected shrinkage coefficients $\hat{\lambda}$ for the P-Spline MEM.

## JNJ

| | | Base | | | P-Spline | | | |
|---|---|---|---|---|---|---|---|---|
| | Freq. | Pers. | $\hat{\phi}$ | $Q_{10}(\hat{\epsilon}_t)$ | Pers. | $\hat{\phi}$ | $\hat{\lambda}$ | $Q_{10}(\hat{\epsilon}_t)$ |
| $V$ | 30s | 0.99 | 5.31 | 0.261 | 0.84 | 7.54 | 5 | 0.861 |
| | 1m | 0.99 | 4.25 | 0.441 | 0.85 | 6.43 | 6 | 0.868 |
| | 2m | 1.00 | 3.58 | 0.103 | 0.85 | 5.33 | 10 | 0.581 |
| | 3m | 0.99 | 3.25 | 0.840 | 0.85 | 4.63 | 19 | 0.978 |
| | 4m | 1.00 | 3.66 | 0.473 | 0.84 | 4.18 | 3 | 0.983 |
| | 5m | 0.99 | 2.92 | 0.237 | 0.82 | 3.64 | 6 | 0.367 |
| | 6m | 1.00 | 3.09 | 0.695 | 0.83 | 3.5 | 3 | 0.994 |
| | 10m | 1.00 | 2.63 | 0.160 | 0.83 | 2.79 | 3 | 0.584 |
| | 15m | 1.00 | 2.01 | 0.385 | 0.75 | 2.27 | 1 | 0.793 |
| | 20m | 0.98 | 1.85 | 0.903 | 0.80 | 2.07 | 4 | 0.984 |
| | 30m | 0.99 | 1.55 | 0.165 | 0.72 | 1.71 | 4 | 0.848 |
| | 1h | 0.99 | 1.28 | 0.140 | 0.52 | 1.3 | 3 | 0.179 |
| $B$ | 30s | 0.99 | 5 | 0.177 | 0.84 | 6.96 | 4 | 0.724 |
| | 1m | 0.99 | 4.48 | 0.512 | 0.86 | 6.08 | 6 | 0.834 |
| | 2m | 1.00 | 3.52 | 0.104 | 0.84 | 5.1 | 11 | 0.780 |
| | 3m | 1.00 | 3.54 | 0.822 | 0.86 | 4.51 | 3 | 0.960 |
| | 4m | 0.99 | 3.05 | 0.331 | 0.83 | 4.07 | 14 | 0.884 |
| | 5m | 1.01 | 2.28 | 0.380 | 0.83 | 3.46 | 3 | 0.712 |
| | 6m | 1.00 | 2.27 | 0.779 | 0.83 | 3.36 | 7 | 0.974 |
| | 10m | 1.00 | 2.56 | 0.385 | 0.81 | 2.68 | 6 | 0.729 |
| | 15m | 0.99 | 1.95 | 0.242 | 0.76 | 2.14 | 6 | 0.838 |
| | 20m | 0.99 | 1.65 | 0.879 | 0.79 | 1.94 | 3 | 0.965 |
| | 30m | 0.99 | 1.42 | 0.802 | 0.74 | 1.55 | 5 | 0.795 |
| | 1h | 1.00 | 1.18 | 0.073 | 0.85 | 1.15 | 4 | 0.112 |
| $TS$ | 30s | 0.99 | 5 | 0.342 | 0.84 | 7.16 | 5 | 0.923 |
| | 1m | 0.99 | 4.08 | 0.608 | 0.84 | 6.14 | 8 | 0.919 |
| | 2m | 0.99 | 3.41 | 0.694 | 0.84 | 5.17 | 8 | 0.877 |
| | 3m | 0.99 | 3.22 | 0.449 | 0.84 | 4.61 | 7 | 0.867 |
| | 4m | 0.96 | 2.68 | 0.052 | 0.83 | 4.15 | 7 | 0.880 |
| | 5m | 1.00 | 3.22 | 0.305 | 0.83 | 3.85 | 3 | 0.913 |
| | 6m | 0.97 | 2.46 | 0.002 | 0.83 | 3.62 | 3 | 0.934 |
| | 10m | 1.00 | 2.57 | 0.481 | 0.83 | 3.11 | 9 | 0.973 |
| | 15m | 1.00 | 2.31 | 0.922 | 0.81 | 2.74 | 9 | 0.995 |
| | 20m | 1.00 | 1.89 | 0.714 | 0.81 | 2.45 | 8 | 0.995 |
| | 30m | 0.98 | 1.97 | 0.820 | 0.75 | 2.13 | 4 | 0.989 |
| | 1h | 1.01 | 1.49 | 0.693 | 0.72 | 1.56 | 2 | 0.987 |
| $R$ | | 0.99 | 1.7 | 0.485 | 0.81 | 1.76 | 18 | 0.870 |

Table 3: Estimation results for the volatility models. For each volatility measures, sampling frequency (when applicable) and volatility model (Base or P-Spline) the table reports the estimated persistence ($\hat{\alpha} + \hat{\beta} + \hat{\alpha}^{-}/2$), shape parameter $\hat{\phi}$ and the p–value of the Ljung–Box test of the residuals $rv_{(m,\delta)\,t}/\hat{\sigma}^2_{(m,\delta)\,t}$. Moreover the table reports the selected shrinkage coefficients $\hat{\lambda}$ for the P-Spline MEM.

with $\{B_i(t)\}_i$ some linear basis expansion in $t$, captures the *long run trend* in volatility; and

$$g_{(m,\delta)\ t} \equiv (1-\alpha-\beta-\alpha^-/2) + \alpha \frac{rv_{(m,\delta)\ t-1}}{\tau_{(m,\delta)\ t-1}} + \beta g_{(m,\delta)\ t-1} + \alpha^- \frac{rv^-_{(m,\delta)\ t-1}}{\tau_{(m,\delta)\ t-1}}, \qquad (5)$$

captures the *short run persistence*.

To fully specify the model of Equations 3–5 some appropriate choice of the basis functions $B_i(\cdot)$ in Equation 4 has to be made. The spline volatility modeling approach *à la* Engle & Rangel (2007) fully specifies the spline model by using a quadratic splines basis, that is

$$\{B_i(t)\} = \left\{1, t, t^2, (t-\xi_i)^2_+, ..., (t-\xi_n)^2_+\right\},$$

where $(u)_+ \equiv \max\{0, u\}$ and $\xi_1, ..., \xi_n$ are some (equally) spaced knots. The degrees of smoothness of the estimated trend will depend on the number of knots. Hence Engle & Rangel (2007) resort to the BIC to estimate the optimal number of knots.

In practice, this modeling approach might have some drawbacks. Quadratic splines have very poor numerical properties that are expected to tangle nonlinear estimation. Choosing the knots via some model selection criterion is often not appealing in that it is usually not feasible to search over all the $2^K - 1$ knots combinations and some subjective ordering of possible combinations has to be chosen. Lastly, the BIC is an information criterion with very poor forecasting properties as the maximum asymptotic forecasting MSE implied by a BIC estimation strategy is infinite (Leeb & Pötscher (2005)).

In light of these consideration and building on the proposal of Eilers & Marx (1996), we propose a novel approach for the flexible modeling of volatility in the presence of trends that we name P-Spline MEM. The term P-Splines is short notation for Penalized B-splines. This modeling strategy consist of using a basis of B-splines with equidistant knots in Equation 4 and fitting the model by a penalized maximum likelihood estimation procedure depending on a shrinkage coefficient that controls the degree of smoothness of the estimate trend.

B-splines are a common basis of functions used for smoothing and nonlinear approximation in the linear regression framework (Eilers & Marx (1996), White (2006)). They consists of a basis of functions made up of polynomial pieces indexed by a set of knots.

There are at least two properties of B-splines that turn out to be useful in this context. First, B-splines allow to simplify the numerical nonlinear estimation relative to Splines. Second, the derivatives of the log trend $\sum \gamma_i B_i(t)$ can be expressed as a linear combination of the finite differences of adjacent B-splines coefficients $\gamma_i$, and it is hence possible to control the degree of smoothness of the trend by appropriately constraining the model parameters.

The connection between the finite differences of the B-spline coefficients and the smoothness of the trend suggest a penalized maximum likelihood (PML) estimation strategy. Let $\theta \equiv (\gamma_1, ..., \gamma_k, \omega, \alpha, \alpha^-, \beta, \phi)'$ denote the model parameters and let $\gamma \equiv (\gamma_1, ..., \gamma_k)'$ denote the B-splines parameters. Then the penalized maximum likelihood estimator is defined as

$$\widehat{\theta}_\lambda \equiv \arg\max \{L_T(\theta) - \lambda \gamma' D_r' D_r \gamma\}$$

where $L_T(.)$ is the log–likelihood function of the sample, $D_r$ is the matrix representation of the difference operator of order $r$, and $\lambda$ is the shrinkage coefficient. The shrinkage coefficients $\lambda$ governs the bias/variance trade–off of the estimator: when $\lambda$ is 0 the PML estimator coincides with the ML estimator and, on the other extreme, as the shrinkage coefficient $\lambda$ grows to infinity the estimated log trend collapses to a polynomial of degree $r-1$.

We do not attempt to derive the the large sample properties of the PML estimator in this work. Typically this can be done resorting a large sample framework under local alternatives for the biased parameters as in Knight & Fu (2000) and Hjort & Claeskens (2003)[4].

PML estimation techniques are not very common in the financial econometrics time series literature but have a long tradition in statistics since the seminal contribution of Hoerl & Kennard (1970). From a forecasting perspective an appealing feature of PML strategies is that the estimated trend tends not to be too sensitive to small changes in the data. In fact, shrinkage estimation strategies are called *stable* regularizing procedures as opposed to model selection strategies that are *unstable* (Breiman (1996)). This property is important in rolling or recursive prediction exercises in that the sequence of predicted values of the trend will not tend to change abruptly from one period to another.

In order to use the PML estimator in real applications, we need to determine some data–driven method to choose the amount of shrinkage $\lambda$ to impose on the estimates, We resort to an Corrected AIC type information criterion (Hurvich & Tsai (1989)). The $\mathsf{AIC_C}$ for the P–Spline MEM is defined as

$$\mathsf{AIC_C}(\lambda) = -2L_T(\widehat{\theta}_\lambda) + 2\widehat{\mathsf{d}}_\lambda + \frac{2\widehat{\mathsf{d}}_\lambda(\widehat{\mathsf{d}}_\lambda + 1)}{T - \widehat{\mathsf{d}}_\lambda - 1}$$

where

$$\widehat{\mathsf{d}}_\lambda = \mathsf{tr}\left\{ \left( \mathcal{I}(\widehat{\theta}_{ML}) + 2\lambda \mathbf{P}_r \right)^{-1} \mathcal{I}(\widehat{\theta}_{ML}) \right\},$$

with

$$\mathbf{P}_r = \left[ \begin{array}{cc} D_r' D_r & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{array} \right].$$

This criterion uses as penalty for model complexity a function that is inversely proportional to the shrinkage coefficient $\lambda$ in analogy to the effective dimension of a linear smoother proposed in Hastie & Tibshirani (1990). We find this criterion appealing in that it leads to more parsimonious specifications in comparison to an AIC type criterion when the number of knots (hence parameters) is large with respect to the sample size.

Figure 1 about here.

Figure 1 shows the estimated trends obtained using the Engle & Rangel (2007) modeling approach and the P-Spline approach. The visual inspection of the graphs suggest that P–spline are able to better capture the features of the data. The condition number of the Hessian of the wide models (that is all knots or no shrinkage) drops from $10^{19}$ to $10^{6}$ using B-splines. As a result of the ill–conditioned Hessian, cubic splines tend to adapt

[4]As with other parameter reduction techniques there are problems with inference as Leeb & Pötscher (2006) pointed out that the risk of PML estimator cannot be uniformly consistently estimated (see also Hurvich & Tsai (1990)). This however does not have any consequences if one is interested in point estimation and forecasting, as it is the case in this work.

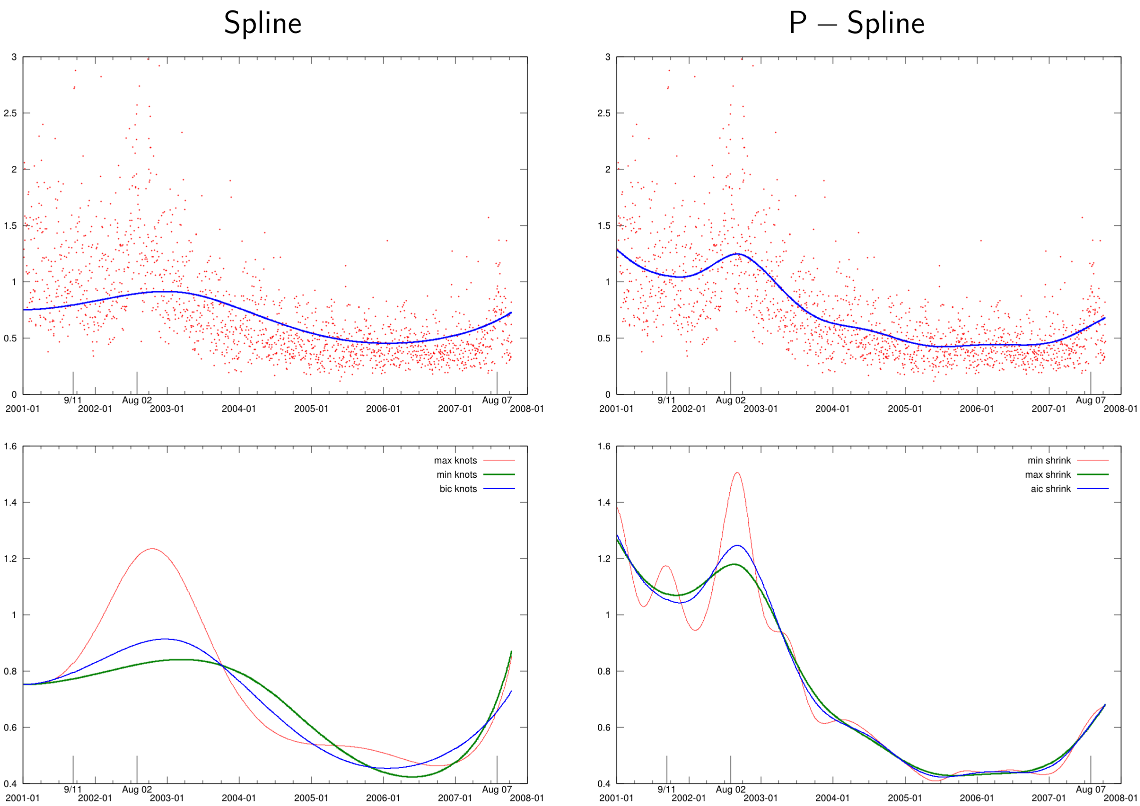


Figure 1: Spline fit comparison: Cubic Splines with BIC P–Splines with $\mathrm{AIC}_C$.

very slowly to the data during the nonlinear estimation while B-splines are much better behaved. The BIC over penalizes (it only selects two 2 knots) while our proposed AIC appears to behave satisfactorily.

Figure 2 about here.

The P-Spline MEM model is estimated over the full sample using 20 equidistant knots. The right hand side of Tables 1, 2 and 3 report the parameter estimates and residual diagnostics of the model. Figure 2 reports the graphs of the annualized realized volatility series (5 minute frequency) together with their corresponding estimated trend. The specification always appears to be able to capture the dynamics of the series satisfactorily as the Ljung–Box test statistic appears to be always nonsignificant at a 5% level. The persistence and shape of the innovation distribution depend of the sampling frequency in a similar way across measures and stocks. The persistence varies between 0.90 and 0.60 and tends to be higher at higher frequencies, in accordance to the mentioned stylized facts. The shape of the shocks distribution appears to be more mound-shaped at higher frequencies, in accordance to the Base model results.

# 5 Modeling Returns

In order to be able to make VaR forecasts using the volatility measure we need to define a model for the returns dynamics.

## 5.1 A Conditional Heteroskedastic Model for Returns Based on Volatility Measures Predictions

Let $r_t$ denote the daily return, let $h_t$ be the conditional variance of the returns and let $rv_{(m,\delta)\ t|t-1}$ be the conditional expectation of the volatility measure at day $t$. We assume that the conditional variance $h_t$ is a linear function of the volatility measures conditional expectation

$$h_t = c + m\ rv_{(m,\delta)\ t|t-1}, \tag{6}$$

and we assume that the return standardized by their conditional variance are well described by a standardized Student's $t$ distribution, that is

$$r_t = \sqrt{h_t}\, z_t, \qquad z_t \sim t_{1/\nu}, \tag{7}$$

where $t_{1/\nu}$ is a standardized (unit variance) $t$ distribution with $1/\nu$ dof (Fiorentini, Sentana & Calzolari (2003)). In other words the model for the returns of Equations 6 and 7 implies that the conditional heteroskedasticity of the returns series is captured by the conditional expectation of the volatility measures. The specification, however, does not require the volatility measures forecasts to be unbiased predictors of the returns' variance. The model allows us to test the Unbiased return Volatility Predictor hypothesis $H_0 : c = 0 \quad m = 1$ (UVP test).

Tables 4, 5, 6 about here.

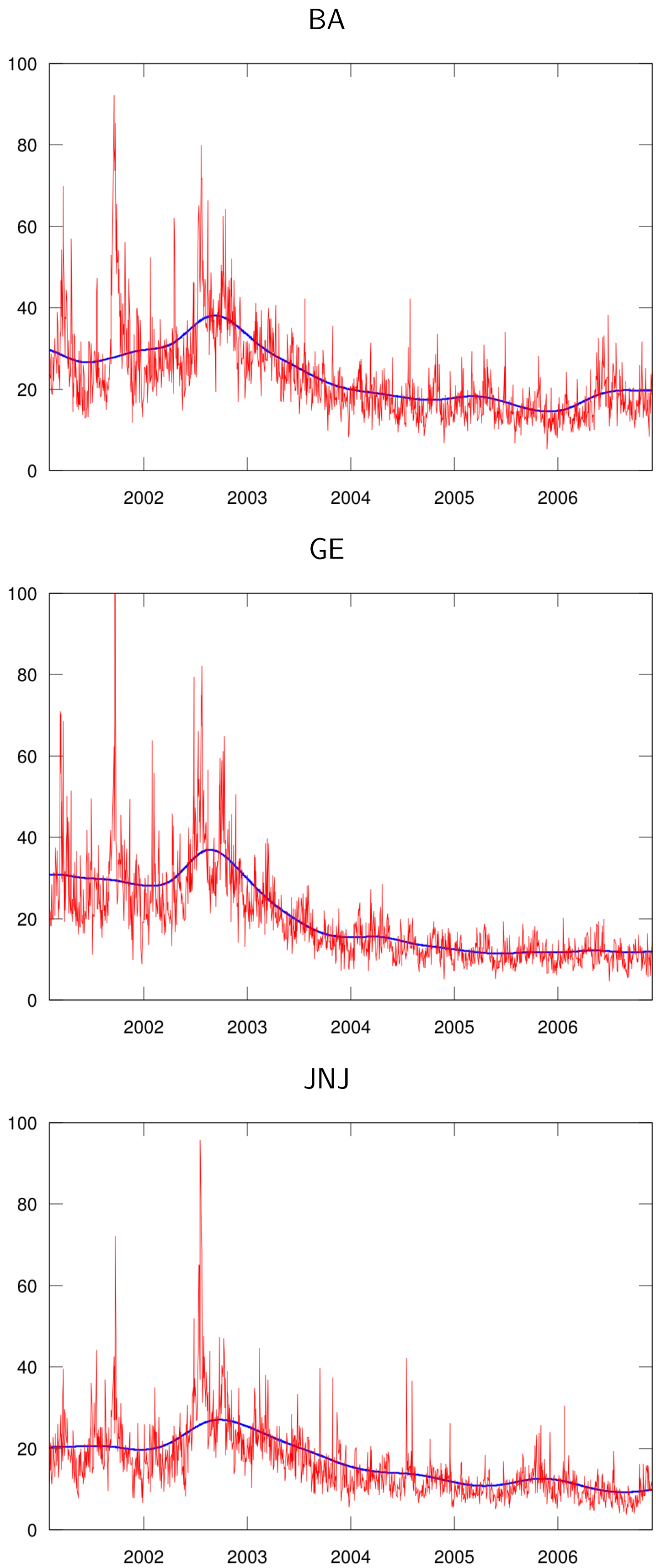


Figure 2: Annualized volatility Feb. 2001 – Dec 2006. The graphs displays the plot of (annualized) realized volatility computed at a 5 min. frequency and the estimated volatility trend of the series

# BA

| | Freq. | Base $\hat{\omega}$ | Base $\hat{\gamma}$ | Base $\hat{\nu}$ | Base UVP | Base $Q_{10}(\hat{z}_t^2)$ | P-Spline $\hat{\omega}$ | P-Spline $\hat{\gamma}$ | P-Spline $\hat{\nu}$ | P-Spline UVP | P-Spline $Q_{10}(\hat{z}_t^2)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $V$ | 30s | 0.44 (0.21) | 1.2 (0.12) | 0.11 (0.02) | 0.000 | 0.231 | 0.36 (0.21) | 1.24 (0.12) | 0.1 (0.02) | 0.000 | 0.234 |
| | 1m | 0.48 (0.21) | 1.11 (0.12) | 0.11 (0.02) | 0.000 | 0.235 | 0.41 (0.2) | 1.14 (0.11) | 0.1 (0.02) | 0.000 | 0.231 |
| | 2m | 0.48 (0.2) | 1.06 (0.11) | 0.1 (0.02) | 0.000 | 0.222 | 0.43 (0.19) | 1.08 (0.11) | 0.1 (0.02) | 0.000 | 0.209 |
| | 3m | 0.48 (0.2) | 1.04 (0.11) | 0.1 (0.02) | 0.000 | 0.221 | 0.43 (0.19) | 1.07 (0.1) | 0.1 (0.02) | 0.000 | 0.213 |
| | 4m | 0.47 (0.2) | 1.05 (0.11) | 0.1 (0.02) | 0.000 | 0.187 | 0.41 (0.19) | 1.08 (0.1) | 0.1 (0.02) | 0.000 | 0.183 |
| | 5m | 0.47 (0.2) | 1.08 (0.11) | 0.1 (0.02) | 0.000 | 0.182 | 0.41 (0.19) | 1.1 (0.11) | 0.1 (0.02) | 0.000 | 0.181 |
| | 6m | 0.45 (0.2) | 1.09 (0.11) | 0.1 (0.02) | 0.000 | 0.188 | 0.38 (0.18) | 1.12 (0.11) | 0.1 (0.02) | 0.000 | 0.187 |
| | 10m | 0.39 (0.2) | 1.17 (0.11) | 0.09 (0.02) | 0.000 | 0.154 | 0.35 (0.18) | 1.2 (0.11) | 0.09 (0.02) | 0.000 | 0.142 |
| | 15m | 0.35 (0.2) | 1.21 (0.11) | 0.09 (0.02) | 0.000 | 0.178 | 0.34 (0.17) | 1.23 (0.11) | 0.09 (0.02) | 0.000 | 0.174 |
| | 20m | 0.35 (0.2) | 1.25 (0.12) | 0.09 (0.02) | 0.000 | 0.153 | 0.34 (0.18) | 1.26 (0.11) | 0.09 (0.02) | 0.000 | 0.161 |
| | 30m | 0.27 (0.21) | 1.33 (0.13) | 0.1 (0.02) | 0.000 | 0.166 | 0.31 (0.18) | 1.31 (0.12) | 0.09 (0.02) | 0.000 | 0.176 |
| | 1h | 0.11 (0.21) | 1.69 (0.16) | 0.09 (0.02) | 0.000 | 0.118 | 0.22 (0.18) | 1.64 (0.14) | 0.09 (0.02) | 0.000 | 0.125 |
| $B$ | 30s | 0.37 (0.23) | 1.53 (0.16) | 0.11 (0.02) | 0.000 | 0.232 | 0.27 (0.22) | 1.59 (0.16) | 0.11 (0.02) | 0.000 | 0.235 |
| | 1m | 0.48 (0.22) | 1.24 (0.13) | 0.11 (0.02) | 0.000 | 0.242 | 0.37 (0.21) | 1.3 (0.13) | 0.11 (0.02) | 0.000 | 0.228 |
| | 2m | 0.51 (0.21) | 1.11 (0.12) | 0.11 (0.02) | 0.000 | 0.222 | 0.42 (0.2) | 1.15 (0.11) | 0.1 (0.02) | 0.000 | 0.209 |
| | 3m | 0.49 (0.2) | 1.09 (0.11) | 0.1 (0.02) | 0.000 | 0.239 | 0.42 (0.19) | 1.12 (0.11) | 0.1 (0.02) | 0.000 | 0.232 |
| | 4m | 0.46 (0.2) | 1.1 (0.11) | 0.1 (0.02) | 0.000 | 0.198 | 0.4 (0.19) | 1.13 (0.11) | 0.1 (0.02) | 0.000 | 0.195 |
| | 5m | 0.48 (0.2) | 1.1 (0.11) | 0.1 (0.02) | 0.000 | 0.186 | 0.4 (0.19) | 1.13 (0.11) | 0.1 (0.02) | 0.000 | 0.188 |
| | 6m | 0.49 (0.2) | 1.1 (0.11) | 0.1 (0.02) | 0.000 | 0.215 | 0.41 (0.19) | 1.14 (0.11) | 0.1 (0.02) | 0.000 | 0.215 |
| | 10m | 0.38 (0.2) | 1.21 (0.12) | 0.09 (0.02) | 0.000 | 0.158 | 0.35 (0.18) | 1.22 (0.11) | 0.09 (0.02) | 0.000 | 0.147 |
| | 15m | 0.38 (0.19) | 1.25 (0.12) | 0.09 (0.02) | 0.000 | 0.205 | 0.38 (0.17) | 1.25 (0.11) | 0.09 (0.02) | 0.000 | 0.198 |
| | 20m | 0.39 (0.2) | 1.27 (0.12) | 0.1 (0.02) | 0.000 | 0.183 | 0.37 (0.18) | 1.29 (0.12) | 0.09 (0.02) | 0.000 | 0.217 |
| | 30m | 0.35 (0.21) | 1.35 (0.14) | 0.1 (0.02) | 0.000 | 0.176 | 0.35 (0.18) | 1.36 (0.12) | 0.1 (0.02) | 0.000 | 0.204 |
| | 1h | 0.12 (0.21) | 1.76 (0.16) | 0.09 (0.02) | 0.000 | 0.110 | 0.24 (0.18) | 1.71 (0.15) | 0.09 (0.02) | 0.000 | 0.123 |
| $TS$ | 30s | 0.39 (0.22) | 0.66 (0.07) | 0.11 (0.02) | 0.000 | 0.228 | 0.3 (0.21) | 0.68 (0.07) | 0.11 (0.02) | 0.000 | 0.227 |
| | 1m | 0.46 (0.21) | 0.88 (0.09) | 0.11 (0.02) | 0.052 | 0.219 | 0.39 (0.2) | 0.9 (0.09) | 0.1 (0.02) | 0.084 | 0.217 |
| | 2m | 0.48 (0.2) | 0.95 (0.1) | 0.1 (0.02) | 0.001 | 0.204 | 0.42 (0.19) | 0.97 (0.09) | 0.1 (0.02) | 0.001 | 0.199 |
| | 3m | 0.43 (0.2) | 1 (0.1) | 0.1 (0.02) | 0.000 | 0.202 | 0.39 (0.19) | 1.01 (0.1) | 0.1 (0.02) | 0.000 | 0.196 |
| | 4m | 0.42 (0.2) | 1.02 (0.1) | 0.1 (0.02) | 0.000 | 0.194 | 0.37 (0.19) | 1.04 (0.1) | 0.1 (0.02) | 0.000 | 0.188 |
| | 5m | 0.41 (0.2) | 1.04 (0.1) | 0.1 (0.02) | 0.000 | 0.195 | 0.35 (0.19) | 1.07 (0.1) | 0.1 (0.02) | 0.000 | 0.190 |
| | 6m | 0.39 (0.2) | 1.06 (0.1) | 0.1 (0.02) | 0.000 | 0.197 | 0.34 (0.18) | 1.08 (0.1) | 0.09 (0.02) | 0.000 | 0.189 |
| | 10m | 0.36 (0.2) | 1.11 (0.11) | 0.09 (0.02) | 0.000 | 0.187 | 0.3 (0.18) | 1.13 (0.1) | 0.09 (0.02) | 0.000 | 0.179 |
| | 15m | 0.33 (0.2) | 1.14 (0.11) | 0.09 (0.02) | 0.000 | 0.172 | 0.29 (0.18) | 1.16 (0.1) | 0.09 (0.02) | 0.000 | 0.164 |
| | 20m | 0.32 (0.2) | 1.17 (0.11) | 0.09 (0.02) | 0.000 | 0.169 | 0.29 (0.18) | 1.18 (0.11) | 0.09 (0.02) | 0.000 | 0.165 |
| | 30m | 0.26 (0.21) | 1.22 (0.12) | 0.09 (0.02) | 0.000 | 0.179 | 0.28 (0.18) | 1.21 (0.11) | 0.09 (0.02) | 0.000 | 0.183 |
| | 1h | 0.02 (0.23) | 7.84 (0.76) | 0.1 (0.02) | 0.000 | 0.176 | 0.14 (0.19) | 7.53 (0.66) | 0.1 (0.02) | 0.000 | 0.209 |
| $R$ | | 0.14 (0.22) | 1.22 (0.12) | 0.1 (0.02) | 0.000 | 0.273 | 0.16 (0.19) | 1.21 (0.11) | 0.09 (0.02) | 0.000 | 0.242 |

Table 4: Estimation results for the return model. For each measure, sampling frequencies (when applicable) and volatility model (Base or P-Spline) the table reports the estimates of the model parameters (standard errors in parenthesis) the p–value of the UVP test and the p–value of the Ljung–Box test on the squared residuals $r_t^2/\hat{h}_t$.

# GE

| | | Base | | | | | P-Spline | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Freq. | $\hat{\omega}$ | $\hat{\gamma}$ | $\hat{\nu}$ | UVP | $Q_{10}(\hat{z}_t^2)$ | $\hat{\omega}$ | $\hat{\gamma}$ | $\hat{\nu}$ | UVP | $Q_{10}(\hat{z}_t^2)$ |
| $V$ | 30s | −0.23 (0.08) | 1.62 (0.11) | 0.1 (0.02) | 0.000 | 0.977 | −0.26 (0.08) | 1.67 (0.12) | 0.09 (0.02) | 0.000 | 0.976 |
| | 1m | −0.14 (0.08) | 1.47 (0.11) | 0.1 (0.02) | 0.000 | 0.980 | −0.18 (0.08) | 1.49 (0.11) | 0.09 (0.02) | 0.000 | 0.978 |
| | 2m | −0.11 (0.08) | 1.37 (0.1) | 0.1 (0.02) | 0.000 | 0.985 | −0.13 (0.08) | 1.37 (0.1) | 0.09 (0.02) | 0.000 | 0.983 |
| | 3m | −0.1 (0.07) | 1.33 (0.09) | 0.1 (0.02) | 0.000 | 0.991 | −0.13 (0.08) | 1.36 (0.09) | 0.09 (0.02) | 0.000 | 0.989 |
| | 4m | −0.09 (0.08) | 1.33 (0.09) | 0.1 (0.02) | 0.000 | 0.988 | −0.11 (0.07) | 1.33 (0.09) | 0.09 (0.02) | 0.000 | 0.985 |
| | 5m | −0.07 (0.08) | 1.31 (0.09) | 0.1 (0.02) | 0.000 | 0.988 | −0.11 (0.07) | 1.33 (0.09) | 0.09 (0.02) | 0.000 | 0.985 |
| | 6m | −0.07 (0.07) | 1.32 (0.09) | 0.1 (0.02) | 0.000 | 0.988 | −0.12 (0.07) | 1.35 (0.09) | 0.09 (0.02) | 0.000 | 0.985 |
| | 10m | −0.07 (0.07) | 1.37 (0.1) | 0.1 (0.02) | 0.000 | 0.987 | −0.12 (0.07) | 1.4 (0.1) | 0.09 (0.02) | 0.000 | 0.981 |
| | 15m | −0.08 (0.07) | 1.4 (0.1) | 0.1 (0.02) | 0.000 | 0.989 | −0.11 (0.07) | 1.43 (0.1) | 0.09 (0.02) | 0.000 | 0.985 |
| | 20m | −0.11 (0.08) | 1.43 (0.1) | 0.1 (0.02) | 0.000 | 0.988 | −0.12 (0.07) | 1.44 (0.1) | 0.08 (0.02) | 0.000 | 0.978 |
| | 30m | −0.05 (0.07) | 1.44 (0.1) | 0.1 (0.02) | 0.000 | 0.986 | −0.09 (0.07) | 1.47 (0.1) | 0.09 (0.02) | 0.000 | 0.976 |
| | 1h | −0.06 (0.08) | 1.61 (0.12) | 0.1 (0.02) | 0.000 | 0.990 | −0.1 (0.07) | 1.67 (0.11) | 0.08 (0.02) | 0.000 | 0.976 |
| $B$ | 30s | −0.14 (0.08) | 1.86 (0.13) | 0.1 (0.02) | 0.000 | 0.975 | −0.16 (0.08) | 1.89 (0.13) | 0.09 (0.02) | 0.000 | 0.972 |
| | 1m | −0.1 (0.08) | 1.56 (0.11) | 0.1 (0.02) | 0.000 | 0.983 | −0.12 (0.08) | 1.58 (0.11) | 0.09 (0.02) | 0.000 | 0.981 |
| | 2m | −0.09 (0.08) | 1.42 (0.1) | 0.1 (0.02) | 0.000 | 0.986 | −0.11 (0.07) | 1.42 (0.1) | 0.09 (0.02) | 0.000 | 0.983 |
| | 3m | −0.09 (0.07) | 1.38 (0.1) | 0.1 (0.02) | 0.000 | 0.993 | −0.12 (0.07) | 1.4 (0.1) | 0.09 (0.02) | 0.000 | 0.990 |
| | 4m | −0.06 (0.08) | 1.32 (0.1) | 0.1 (0.02) | 0.000 | 0.986 | −0.09 (0.07) | 1.36 (0.1) | 0.09 (0.02) | 0.000 | 0.983 |
| | 5m | −0.06 (0.08) | 1.35 (0.1) | 0.1 (0.02) | 0.000 | 0.988 | −0.09 (0.07) | 1.37 (0.1) | 0.09 (0.02) | 0.000 | 0.984 |
| | 6m | −0.05 (0.07) | 1.34 (0.1) | 0.1 (0.02) | 0.000 | 0.986 | −0.09 (0.07) | 1.38 (0.1) | 0.09 (0.02) | 0.000 | 0.984 |
| | 10m | −0.07 (0.07) | 1.43 (0.1) | 0.1 (0.02) | 0.000 | 0.984 | −0.1 (0.07) | 1.45 (0.1) | 0.09 (0.02) | 0.000 | 0.981 |
| | 15m | −0.08 (0.08) | 1.49 (0.11) | 0.1 (0.02) | 0.000 | 0.985 | −0.1 (0.07) | 1.49 (0.1) | 0.08 (0.02) | 0.000 | 0.976 |
| | 20m | −0.09 (0.08) | 1.42 (0.1) | 0.1 (0.02) | 0.000 | 0.985 | −0.11 (0.07) | 1.48 (0.1) | 0.08 (0.02) | 0.000 | 0.967 |
| | 30m | −0.08 (0.08) | 1.58 (0.12) | 0.11 (0.02) | 0.000 | 0.979 | −0.09 (0.07) | 1.59 (0.11) | 0.09 (0.02) | 0.000 | 0.965 |
| | 1h | −0.03 (0.07) | 1.72 (0.12) | 0.11 (0.02) | 0.000 | 0.989 | −0.09 (0.07) | 1.81 (0.13) | 0.09 (0.02) | 0.000 | 0.979 |
| $TS$ | 30s | −0.24 (0.08) | 0.87 (0.06) | 0.1 (0.02) | 0.000 | 0.977 | −0.27 (0.08) | 0.88 (0.06) | 0.09 (0.02) | 0.000 | 0.977 |
| | 1m | −0.14 (0.08) | 1.12 (0.08) | 0.1 (0.02) | 0.213 | 0.981 | −0.18 (0.08) | 1.14 (0.08) | 0.09 (0.02) | 0.083 | 0.981 |
| | 2m | −0.1 (0.08) | 1.2 (0.09) | 0.1 (0.02) | 0.040 | 0.985 | −0.14 (0.08) | 1.22 (0.09) | 0.09 (0.02) | 0.029 | 0.984 |
| | 3m | −0.1 (0.08) | 1.24 (0.09) | 0.1 (0.02) | 0.008 | 0.988 | −0.14 (0.08) | 1.25 (0.09) | 0.09 (0.02) | 0.008 | 0.987 |
| | 4m | −0.1 (0.08) | 1.25 (0.09) | 0.1 (0.02) | 0.003 | 0.988 | −0.13 (0.08) | 1.27 (0.09) | 0.09 (0.02) | 0.003 | 0.986 |
| | 5m | −0.1 (0.08) | 1.27 (0.09) | 0.1 (0.02) | 0.001 | 0.989 | −0.13 (0.07) | 1.28 (0.09) | 0.08 (0.02) | 0.002 | 0.987 |
| | 6m | −0.1 (0.08) | 1.29 (0.09) | 0.1 (0.02) | 0.000 | 0.990 | −0.13 (0.07) | 1.3 (0.09) | 0.08 (0.02) | 0.001 | 0.987 |
| | 10m | −0.09 (0.08) | 1.3 (0.09) | 0.1 (0.02) | 0.000 | 0.989 | −0.12 (0.07) | 1.33 (0.09) | 0.09 (0.02) | 0.000 | 0.983 |
| | 15m | −0.07 (0.07) | 1.33 (0.09) | 0.1 (0.02) | 0.000 | 0.987 | −0.11 (0.07) | 1.36 (0.09) | 0.08 (0.02) | 0.000 | 0.981 |
| | 20m | −0.07 (0.07) | 1.35 (0.1) | 0.1 (0.02) | 0.000 | 0.987 | −0.11 (0.07) | 1.38 (0.1) | 0.08 (0.02) | 0.000 | 0.979 |
| | 30m | −0.07 (0.07) | 1.4 (0.1) | 0.1 (0.02) | 0.000 | 0.988 | −0.1 (0.07) | 1.41 (0.1) | 0.08 (0.02) | 0.000 | 0.975 |
| | 1h | −0.07 (0.08) | 2.12 (0.16) | 0.11 (0.02) | 0.000 | 0.988 | −0.1 (0.07) | 2.17 (0.15) | 0.09 (0.02) | 0.000 | 0.966 |
| $R$ | | −0.03 (0.08) | 1.17 (0.08) | 0.1 (0.02) | 0.010 | 0.986 | −0.06 (0.07) | 1.22 (0.08) | 0.08 (0.02) | 0.003 | 0.975 |

Table 5: Estimation results for the return model. For each measure, sampling frequencies (when applicable) and volatility model (Base or P-Spline) the table reports the estimates of the model parameters (standard errors in parenthesis) the p–value of the UVP test and the p–value of the Ljung–Box test on the squared residuals $r_t^2/\hat{h}_t$.

# JNJ

| | | Base | | | | | P-Spline | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Freq. | $\hat{\omega}$ | $\hat{\gamma}$ | $\hat{\nu}$ | UVP | $Q_{10}(\hat{z}_t^2)$ | $\hat{\omega}$ | $\hat{\gamma}$ | $\hat{\nu}$ | UVP | $Q_{10}(\hat{z}_t^2)$ |
| $V$ | 30s | $-0.09$ (0.07) | 1.4 (0.13) | 0.17 (0.02) | 0.000 | 0.199 | $-0.08$ (0.07) | 1.33 (0.11) | 0.15 (0.02) | 0.000 | 0.262 |
| | 1m | $-0.05$ (0.07) | 1.25 (0.11) | 0.17 (0.02) | 0.002 | 0.194 | $-0.06$ (0.06) | 1.26 (0.11) | 0.17 (0.02) | 0.003 | 0.266 |
| | 2m | 0.04 (0.06) | 1.07 (0.1) | 0.17 (0.03) | 0.062 | 0.202 | 0 (0.06) | 1.12 (0.1) | 0.17 (0.02) | 0.059 | 0.282 |
| | 3m | 0.03 (0.06) | 1.08 (0.1) | 0.17 (0.03) | 0.078 | 0.226 | 0.01 (0.06) | 1.1 (0.1) | 0.17 (0.02) | 0.099 | 0.301 |
| | 4m | 0.06 (0.06) | 1.03 (0.1) | 0.17 (0.03) | 0.078 | 0.232 | 0.02 (0.06) | 1.08 (0.1) | 0.16 (0.02) | 0.096 | 0.301 |
| | 5m | 0.07 (0.06) | 1.04 (0.1) | 0.17 (0.03) | 0.028 | 0.185 | 0.04 (0.06) | 1.07 (0.1) | 0.17 (0.02) | 0.047 | 0.297 |
| | 6m | 0.06 (0.06) | 1.06 (0.1) | 0.17 (0.03) | 0.018 | 0.210 | 0.04 (0.06) | 1.09 (0.1) | 0.16 (0.02) | 0.017 | 0.288 |
| | 10m | 0.13 (0.06) | 1.05 (0.1) | 0.18 (0.03) | 0.000 | 0.292 | 0.09 (0.06) | 1.1 (0.1) | 0.17 (0.02) | 0.000 | 0.352 |
| | 15m | 0.13 (0.06) | 1.09 (0.11) | 0.18 (0.03) | 0.000 | 0.262 | 0.09 (0.05) | 1.13 (0.1) | 0.16 (0.02) | 0.000 | 0.330 |
| | 20m | 0.13 (0.06) | 1.14 (0.12) | 0.18 (0.03) | 0.000 | 0.353 | 0.09 (0.06) | 1.15 (0.1) | 0.17 (0.02) | 0.000 | 0.384 |
| | 30m | 0.12 (0.06) | 1.18 (0.12) | 0.18 (0.03) | 0.000 | 0.282 | 0.07 (0.05) | 1.24 (0.11) | 0.16 (0.02) | 0.000 | 0.307 |
| | 1h | 0.09 (0.06) | 1.55 (0.15) | 0.18 (0.03) | 0.000 | 0.325 | 0.08 (0.06) | 1.54 (0.14) | 0.17 (0.02) | 0.000 | 0.608 |
| $B$ | 30s | $-0.08$ (0.07) | 1.64 (0.14) | 0.16 (0.02) | 0.000 | 0.220 | $-0.07$ (0.07) | 1.59 (0.13) | 0.15 (0.02) | 0.000 | 0.296 |
| | 1m | $-0.05$ (0.07) | 1.37 (0.12) | 0.17 (0.02) | 0.000 | 0.190 | $-0.05$ (0.06) | 1.37 (0.12) | 0.16 (0.02) | 0.000 | 0.265 |
| | 2m | 0.03 (0.06) | 1.14 (0.11) | 0.17 (0.03) | 0.003 | 0.238 | 0.01 (0.06) | 1.17 (0.1) | 0.16 (0.02) | 0.004 | 0.291 |
| | 3m | 0.04 (0.06) | 1.11 (0.11) | 0.17 (0.03) | 0.007 | 0.228 | 0.01 (0.06) | 1.13 (0.1) | 0.17 (0.02) | 0.015 | 0.287 |
| | 4m | 0.05 (0.06) | 1.09 (0.1) | 0.17 (0.03) | 0.013 | 0.212 | 0.02 (0.06) | 1.12 (0.1) | 0.16 (0.02) | 0.022 | 0.295 |
| | 5m | 0.11 (0.06) | 1 (0.1) | 0.17 (0.03) | 0.009 | 0.169 | 0.04 (0.06) | 1.11 (0.1) | 0.16 (0.02) | 0.010 | 0.282 |
| | 6m | 0.07 (0.06) | 1.11 (0.11) | 0.17 (0.03) | 0.001 | 0.194 | 0.04 (0.06) | 1.12 (0.1) | 0.16 (0.02) | 0.002 | 0.267 |
| | 10m | 0.12 (0.06) | 1.1 (0.11) | 0.18 (0.03) | 0.000 | 0.280 | 0.08 (0.06) | 1.15 (0.11) | 0.17 (0.02) | 0.000 | 0.356 |
| | 15m | 0.13 (0.06) | 1.13 (0.11) | 0.18 (0.03) | 0.000 | 0.258 | 0.09 (0.05) | 1.17 (0.1) | 0.16 (0.02) | 0.000 | 0.312 |
| | 20m | 0.14 (0.06) | 1.15 (0.12) | 0.18 (0.03) | 0.000 | 0.267 | 0.1 (0.05) | 1.19 (0.11) | 0.17 (0.02) | 0.000 | 0.393 |
| | 30m | 0.13 (0.06) | 1.25 (0.12) | 0.18 (0.03) | 0.000 | 0.180 | 0.1 (0.05) | 1.27 (0.11) | 0.17 (0.02) | 0.000 | 0.292 |
| | 1h | 0.1 (0.06) | 1.62 (0.16) | 0.19 (0.03) | 0.000 | 0.275 | 0.06 (0.05) | 1.69 (0.15) | 0.17 (0.02) | 0.000 | 0.444 |
| $TS$ | 30s | $-0.09$ (0.07) | 0.74 (0.07) | 0.17 (0.02) | 0.000 | 0.228 | $-0.08$ (0.07) | 0.72 (0.06) | 0.16 (0.02) | 0.000 | 0.283 |
| | 1m | $-0.05$ (0.07) | 0.97 (0.09) | 0.17 (0.03) | 0.120 | 0.232 | $-0.06$ (0.06) | 0.96 (0.08) | 0.16 (0.02) | 0.050 | 0.292 |
| | 2m | 0.01 (0.07) | 0.99 (0.09) | 0.17 (0.03) | 0.994 | 0.221 | $-0.01$ (0.06) | 1.01 (0.09) | 0.17 (0.02) | 0.992 | 0.303 |
| | 3m | 0.02 (0.07) | 1.02 (0.1) | 0.17 (0.03) | 0.655 | 0.218 | 0.01 (0.06) | 1.02 (0.09) | 0.17 (0.02) | 0.792 | 0.315 |
| | 4m | 0.03 (0.07) | 1.06 (0.1) | 0.18 (0.03) | 0.110 | 0.418 | 0.02 (0.06) | 1.04 (0.09) | 0.16 (0.02) | 0.497 | 0.329 |
| | 5m | 0.05 (0.06) | 1.02 (0.1) | 0.17 (0.03) | 0.187 | 0.291 | 0.02 (0.06) | 1.05 (0.09) | 0.16 (0.02) | 0.275 | 0.330 |
| | 6m | 0.06 (0.07) | 1.05 (0.1) | 0.18 (0.03) | 0.042 | 0.471 | 0.03 (0.06) | 1.07 (0.09) | 0.16 (0.02) | 0.134 | 0.341 |
| | 10m | 0.09 (0.06) | 1.05 (0.1) | 0.17 (0.03) | 0.003 | 0.301 | 0.05 (0.06) | 1.09 (0.1) | 0.16 (0.02) | 0.009 | 0.345 |
| | 15m | 0.09 (0.06) | 1.08 (0.1) | 0.17 (0.03) | 0.000 | 0.233 | 0.06 (0.05) | 1.12 (0.1) | 0.16 (0.02) | 0.001 | 0.328 |
| | 20m | 0.12 (0.06) | 1.08 (0.1) | 0.17 (0.03) | 0.000 | 0.221 | 0.07 (0.05) | 1.13 (0.1) | 0.16 (0.02) | 0.000 | 0.333 |
| | 30m | 0.1 (0.06) | 1.2 (0.12) | 0.18 (0.03) | 0.000 | 0.407 | 0.07 (0.05) | 1.17 (0.1) | 0.16 (0.02) | 0.000 | 0.362 |
| | 1h | 0.14 (0.06) | 3.92 (0.39) | 0.18 (0.03) | 0.000 | 0.474 | 0.07 (0.05) | 4.23 (0.36) | 0.16 (0.02) | 0.000 | 0.550 |
| $R$ | | 0.05 (0.06) | 1.13 (0.1) | 0.17 (0.03) | 0.001 | 0.281 | 0.02 (0.05) | 1.14 (0.09) | 0.15 (0.02) | 0.004 | 0.354 |

Table 6: Estimation results for the return model. For each measure, sampling frequencies (when applicable) and volatility model (Base or P-Spline) the table reports the estimates of the model parameters (standard errors in parenthesis) the p–value of the UVP test and the p–value of the Ljung–Box test on the squared residuals $r_t^2/\hat{h}_t$.

Equations 6 and 7 are estimated over the full sample using the series of 1 day ahead prediction of the volatility measures obtained by both the estimated Base and P-Spline specifications. Tables 4, 5, 6 report parameter estimates and diagnostics. The model and both series of volatility predictions always appear to be able to capture the squared returns dynamics satisfactorily as the Ljung–Box test statistic appears to be always nonsignificant at standard significance levels. However, the volatility predictions do not appear to provide unbiased forecasts of the variance of returns in the great majority of cases as the p–value of the UVP test is almost always significant. The null of unbiasedness is convincingly not rejected only for two scales realized volatility when sampled at frequencies around 1 minute. The standardized returns exhibit a pronounced leptokurtosis. Interestingly, the P-Spline based predictors systematically lead to thinner tails than the Base.

The UVP test may be a little bit too crude to evaluate the precision of the volatility measures predictions as the volatility measures are expected to be downward biased. Straightforward calculations allow us to use the return specification to compute the MSE of the volatility measures forecasts as predictors of the returns' variance. Consider

$$\mathsf{MSE}\big(rv_{(m,\delta)\ t|t-1}\big) \equiv \mathsf{E}\big(h_t - rv_{(m,\delta)\ t|t-1}\big)^2 ,$$

simple algebra leads to

$$\begin{aligned}\mathsf{E}\big(h_t - rv_{(m,\delta)\ t|t-1}\big)^2 &= \mathsf{E}\big(c + m\, rv_{(m,\delta)\ t|t-1} - rv_{(m,\delta)\ t|t-1}\big)^2 \\ &= \big(c + (m-1)\ \mathsf{E}(rv_{(m,\delta)\ t|t-1})\big)^2 + (m-1)^2\ \mathsf{Var}(rv_{(m,\delta)\ t|t-1}).\end{aligned}$$

Figure 3 about here.

For diagnostic purposes we estimate such a quantity by plugging in the sample counterparts of the population parameters and parameter estimates using the estimation results over the full sample. Figure 3 displays the plots of the estimated MSE as a function of the sampling frequency for each volatility measures, in the spirit of the volatility signature plot (Andersen, Bollerslev, Christoffersen & Diebold (2006)). Interestingly, the graphs appear to be remarkably similar across stocks and forecasting method with the only exception of the range whose relative position appears to be different from stock to stock. The MSE of realized volatility appears to initially decrease as the sampling frequency increases and to then to steadily increase as the sampling frequency is higher than a couple of minutes. The MSE of bipower realized volatility follows exactly the same pattern but is systematically higher. The MSE of two scales realized volatility steeply decreases as the sampling frequency increases and appears to be much smaller than the MSE of the other UHFD measures. At a 30 seconds frequency the MSE of two scales realized volatility does appear to increase abruptly but this is probably a consequence of the failure in meeting the requirements for the validity of the asymptotic results. The ranking between UHFD volatility measures is rather clear: provided that the sampling frequency is sufficiently high two scales realized volatility achieves the best performance followed by realized volatility and bipower realized volatility. However, it has to be stressed that two scales realized volatility uses much more data than its competitors and was expected not to behave worse. Importantly, it appears that the simple range benchmark is difficult to beat. Interestingly, the range appears to be convincingly beaten according to this metric only when sampling at frequencies higher than 15 minutes.

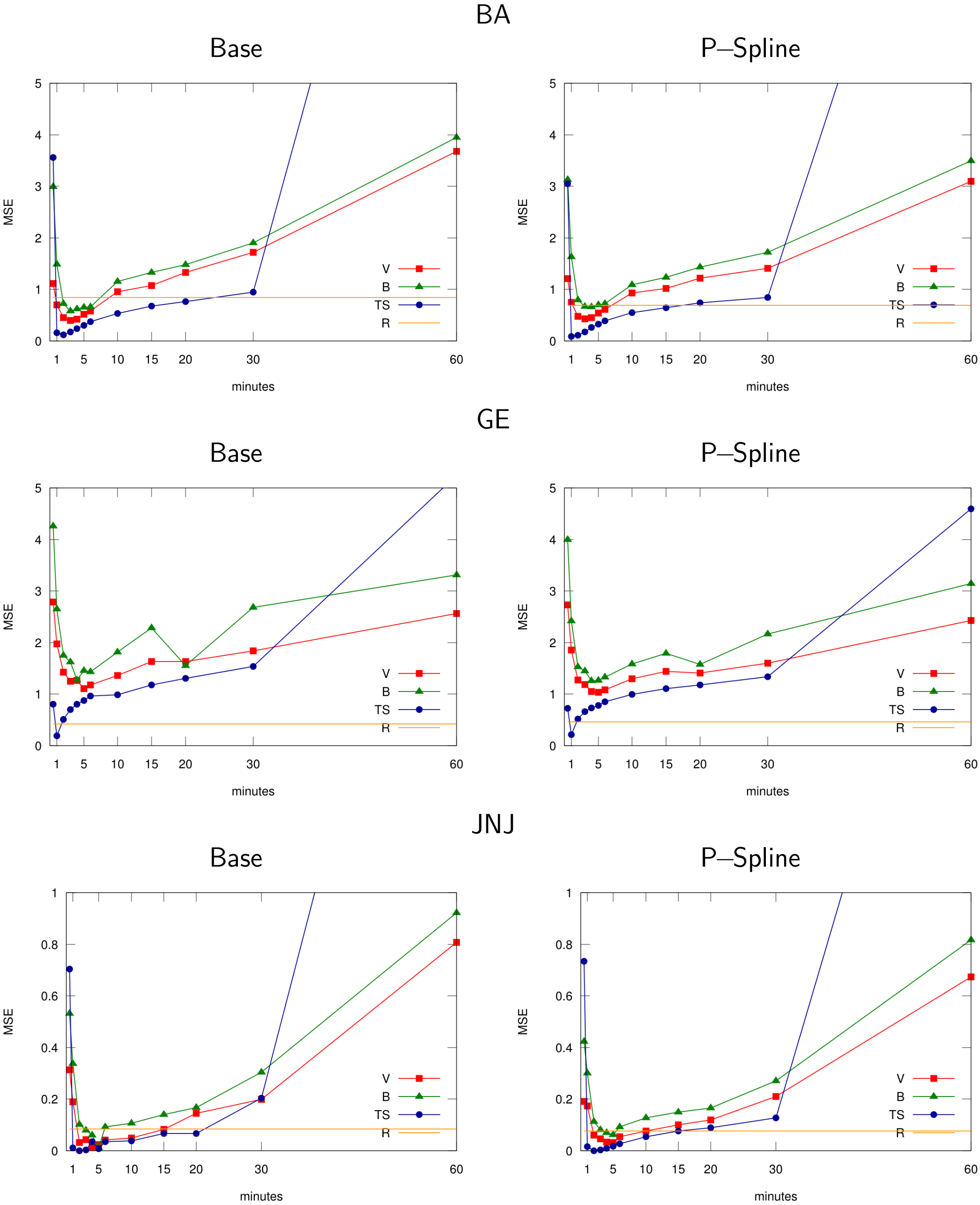


Figure 3: In–sample volatility MSE of the volatility measures. The graphs display the estimated MSE of the volatility measures as functions of the sampling frequency.

# 6 Forecasting Value–at–Risk

The literature on VaR forecasting has developed evaluation tools which explicitly undertake a risk-management viewpoint. This has emerged as a consequence of the fact that, despite the significant improvements in the evaluation of volatility predictions (e.g. Andersen & Bollerslev (1998)), a volatility evaluation metric might fail to asses the usefulness of volatility forecasts form a risk-management point of view (Brooks & Persand (2003)).

The performance of the VaR forecasts is assessed using a two stage procedure. The first stage consist of the assessment of the *adequacy* of the VaR forecasts using a battery of tests on the binary indicator of VaR failure. The second stage consists of an assessment of the *accuracy* of the VaR forecasts using a loss function measuring the goodness of fit of the predicted returns' tails. Such an approach follows the lines of the methodology proposed by Sarma et al. (2003).

The out–of–sample VaR forecasting exercise is performed using approximately the last 3 years of data in the sample. For each day in the out–of–sample period we estimate the Base and P-Spline models using approximately the last 900 days of data. We estimate the model for the returns using the series of 1 day ahead predictions obtained by the two specifications. The P-Spline model is estimated using 10 knots and the choice of the shrinkage coefficient $\lambda$ is performed via the AIC on the first rolling sample and then kept fixed for the rest of the prediction exercise. We then construct the 1 day ahead VaR predictions as

$$\widehat{\mathsf{VaR}}^{p}_{t+1|t} = -\mathsf{F}^{-1}_{t_{1/\hat{\nu}}}(p)\,\sqrt{\hat{c} + \hat{m}\;\widehat{rv}_{(m,\delta)\;t+1|t}},$$

where $\widehat{rv}_{(m,\delta)\;t+1|t}$ is the one–step ahead volatility measure prediction obtained by either the Base and P-Spline methods and $(\hat{c}, \hat{m}, \hat{\nu})'$ are the parameter estimates obtained from the corresponding model for the returns. The VaR forecasting exercise is also performed using a GARCH(1,1) model with leverage effects and Student's $t$ innovations for comparison purposes.

## 6.1 VaR Forecasting Adequacy

Let $\left\{\widehat{\mathsf{VaR}}^{p}_{t|t-1}\right\}$ be a sequence of 1 day ahead $(1-p)$ VaR forecasts and define the failure process $\{H_t\}$ as

$$\left\{H_t \equiv \left(r_t < -\widehat{\mathsf{VaR}}^{p}_{t|t-1}\right)\right\}$$

If the sequence of VaR prediction is adequate, then the VaR conditional coverage should be equal to $p$ for any $t$, that is

$$\mathsf{E}(H_t|\mathcal{F}_{t-1}) = p. \tag{8}$$

Many of the VaR evaluation tests proposed in the literature attempt at assessing the adequacy of VaR predictions by testing against different types of departures from Equation 8. In what follows we resort to the adequacy test proposed by Christoffersen (1998) and Engle & Manganelli (2004)[5].

[5]Berkowitz, Christoffersen & Pelletier (2006) contains a finite sample comparison of several VaR adequacy tests.

**Unconditional Coverage test.** Assuming that $\{H_t\}$ is an independently distributed failure process, the null hypothesis of the unconditional coverage test is that the failure probability is equal to $p$, and it is tested against the alternative of a failure rate different from $p$. Under the null, the test statistic is

$$LR_{\mathsf{uc}} = -2\log\frac{p^{n_1}(1-p)^{n_0}}{\hat{\pi}^{n_1}(1-\hat{\pi})^{n_0}} \sim \chi^2_{(1)},$$

where $n_0$ and $n_1$ are, respectively, the number of 0's and 1's in the series and $\hat{\pi} = n_1/(n_0+n_1)$.

**Independence test.** The null hypothesis of the independence test is that the failure process $\{H_t\}$ is independently distributed, and it is tested against the alternative of a first order Markov process. Under the null, the test statistic is

$$LR_{\mathsf{ind}} = -2\log\frac{(1-\hat{\pi}_2)^{(n_{00}+n_{10})}\hat{\pi}_2^{(n_{01}+n_{11})}}{(1-\hat{\pi}_{01})^{n_{00}}\hat{\pi}_{01}^{n_{01}}(1-\hat{\pi}_{11})^{n_{10}}\hat{\pi}_{11}^{n_{11}}} \sim \chi^2_{(1)},$$

where $n_{ij}$ is the number of $i$ values followed by a $j$ in the $H_t$ series, $\hat{\pi}_{01} = n_{01}/(n_{00}+n_{01})$, $\hat{\pi}_{11} = n_{11}/(n_{10}+n_{11})$ and $\hat{\pi}_2 = (n_{01}+n_{11})/(n_{00}+n_{01}+n_{10}+n_{11})$.

**Conditional Coverage test.** The null hypothesis of the conditional coverage test is that the failure process $\{H_t\}$ is an independent failure process with failure probability $p$, and it is tested against the alternative of a first–order Markov failure process with a different transition probability matrix. Under the null, the test statistic is

$$LR_{\mathsf{cc}} = -2\log\frac{p^{n_1}(1-p)^{n_0}}{(1-\hat{\pi}_{01})^{n_{00}}\hat{\pi}_{01}^{n_{01}}(1-\hat{\pi}_{11})^{n_{10}}\hat{\pi}_{11}^{n_{11}}} \sim \chi^2_{(2)}.$$

Note that conditionally on the first observation $LR_{\text{cc}} = LR_{\text{uc}} + LR_{\text{ind}}$.

**Dynamic Quantile test.** The null of the Dynamic Quantile test is that there is no correlation between $H_t$ and some appropriately chosen $\mathcal{F}_{t-1}$ measurable variable $x_{t-1}$ of dimension $q$. Let $X$ and $H$ denote respectively the matrix of $x_t$ and vector of $H_{t+1}$ observations and consider the LS estimator of the regression of $H - p\mathbf{1}$ on $X$, that is

$$\hat{\beta}_{\mathsf{LS}} = (X'X)^{-1}X'(H - p\mathbf{1});$$

the Dynamic Quantile test correspond to testing the hypothesis $H_0 : \beta = 0$, whose test statistic is

$$DQ_X = \frac{\hat{\beta}'_{\mathsf{LS}}X'X\hat{\beta}_{\mathsf{LS}}}{p(1-p)} \sim \chi^2_q$$

under the null. In our exercise we consider the $DQ_{\mathsf{hit}}$ test, where it is tested weather the last past 4 VaR failure indicators $H_t$ are able predict the current VaR failure.

Tables 7, 8 and 9 about here.

# BA

| | | Base | | | | | | P-Spline | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Meas. | Freq. | $\overline{H}$ | $\overline{\text{VaR}}$ | $\text{LR}_{\text{uc}}$ | $\text{LR}_{\text{ind}}$ | $\text{LR}_{\text{cc}}$ | $\text{DQ}_{\text{Hit}}$ | $\overline{H}$ | $\overline{\text{VaR}}$ | $\text{LR}_{\text{uc}}$ | $\text{LR}_{\text{ind}}$ | $\text{LR}_{\text{cc}}$ | $\text{DQ}_{\text{Hit}}$ |
| $V$ | 30s | 0.36 | 334 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 326.83 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 1m | 0.36 | 334.94 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 328.02 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 2m | 0.18 | 333.31 | 0.017 | 0.952 | 0.057 | 0.589 | 0.54 | 325.99 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 3m | 0.18 | 332.52 | 0.017 | 0.952 | 0.057 | 0.589 | 0.36 | 325.62 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 4m | 0.18 | 330.41 | 0.017 | 0.952 | 0.057 | 0.589 | 0.54 | 322.56 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 5m | 0.18 | 330.57 | 0.017 | 0.952 | 0.057 | 0.589 | 0.36 | 320.92 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 6m | 0.36 | 326.75 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 315.98 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 10m | 0.18 | 327.41 | 0.017 | 0.952 | 0.057 | 0.589 | 0.54 | 316.59 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 15m | 0.18 | 324.84 | 0.017 | 0.952 | 0.057 | 0.589 | 0.36 | 313.44 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 20m | 0.36 | 326.87 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 317.32 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 30m | 0.36 | 323.72 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 318.23 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 1h | 0.37 | 328.26 | 0.090 | 0.903 | 0.236 | 0.827 | 0.55 | 315.33 | 0.254 | 0.855 | 0.513 | 0.954 |
| $B$ | 30s | 0.36 | 342.74 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 334.19 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 1m | 0.18 | 340.49 | 0.017 | 0.952 | 0.057 | 0.589 | 0.36 | 332.64 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 2m | 0.18 | 336.76 | 0.017 | 0.952 | 0.057 | 0.589 | 0.36 | 329.13 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 3m | 0.18 | 333.67 | 0.017 | 0.952 | 0.057 | 0.589 | 0.54 | 327.36 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 4m | 0.18 | 331.68 | 0.017 | 0.952 | 0.057 | 0.589 | 0.36 | 323.38 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 5m | 0.18 | 331.96 | 0.017 | 0.952 | 0.057 | 0.589 | 0.36 | 322.24 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 6m | 0.18 | 328.43 | 0.017 | 0.952 | 0.057 | 0.589 | 0.36 | 318.78 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 10m | 0.18 | 328.26 | 0.017 | 0.952 | 0.057 | 0.589 | 0.36 | 318.21 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 15m | 0.18 | 325.27 | 0.017 | 0.952 | 0.057 | 0.589 | 0.36 | 313.48 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 20m | 0.36 | 327.31 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 314.30 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 30m | 0.36 | 326.1 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 313.34 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 1h | 0.37 | 328.64 | 0.090 | 0.903 | 0.236 | 0.827 | 0.54 | 315.47 | 0.233 | 0.857 | 0.484 | 0.946 |
| $TS$ | 30s | 0.36 | 336.77 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 329.48 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 1m | 0.36 | 336.18 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 328.45 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 2m | 0.18 | 334.3 | 0.017 | 0.952 | 0.057 | 0.589 | 0.36 | 327.15 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 3m | 0.18 | 332.43 | 0.017 | 0.952 | 0.057 | 0.589 | 0.36 | 324.19 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 4m | 0.18 | 330.41 | 0.017 | 0.952 | 0.057 | 0.589 | 0.54 | 321.76 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 5m | 0.18 | 328.94 | 0.017 | 0.952 | 0.057 | 0.589 | 0.54 | 320.17 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 6m | 0.18 | 327.69 | 0.017 | 0.952 | 0.057 | 0.589 | 0.54 | 318.92 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 10m | 0.18 | 326.82 | 0.017 | 0.952 | 0.057 | 0.589 | 0.54 | 315.63 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 15m | 0.18 | 327.86 | 0.017 | 0.952 | 0.057 | 0.589 | 0.54 | 315.88 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 20m | 0.36 | 328.17 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 316.02 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 30m | 0.36 | 329.6 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 316.68 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 1h | 0.37 | 332.67 | 0.090 | 0.903 | 0.236 | 0.827 | 0.37 | 315.39 | 0.090 | 0.903 | 0.236 | 0.827 |
| $R$ | | 0.36 | 340.44 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 319.69 | 0.233 | 0.857 | 0.484 | 0.946 |
| GARCH | | 0.9 | 380.1 | 0.233 | 0.857 | 0.484 | 0.946 | | | | | | |

Table 7: 99% VaR forecasting adequacy. For each measure, sampling frequencies (when applicable) and volatility model (Base or P-Spline) the table reports the average number of VaR failures, the average VaR and the p–values of the adequacy tests.

# GE

| | | Base | | | | | | P-Spline | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Meas. | Freq. | $\overline{H}$ | $\overline{\text{VaR}}$ | $\text{LR}_{\text{uc}}$ | $\text{LR}_{\text{ind}}$ | $\text{LR}_{\text{cc}}$ | $\text{DQ}_{\text{Hit}}$ | $\overline{H}$ | $\overline{\text{VaR}}$ | $\text{LR}_{\text{uc}}$ | $\text{LR}_{\text{ind}}$ | $\text{LR}_{\text{cc}}$ | $\text{DQ}_{\text{Hit}}$ |
| $V$ | 30s | 0.36 | 231.91 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 222.86 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 1m | 0.36 | 230.39 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 220.63 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 2m | 0.36 | 228.77 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 217.61 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 3m | 0.36 | 229.74 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 216.54 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 4m | 0.36 | 229.28 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 217.77 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 5m | 0.36 | 229.09 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 215.58 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 6m | 0.36 | 228.28 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 214.53 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 10m | 0.36 | 229.55 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 215.4 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 15m | 0.36 | 226.4 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 211.11 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 20m | 0.36 | 224.93 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 210.39 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 30m | 0.36 | 232.53 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 215.26 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 1h | 0.36 | 232.93 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 213.25 | 0.081 | 0.904 | 0.217 | 0.811 |
| $B$ | 30s | 0.36 | 232.03 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 223.08 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 1m | 0.36 | 229.96 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 220.04 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 2m | 0.36 | 228.25 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 216.64 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 3m | 0.36 | 229 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 214.92 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 4m | 0.36 | 230.08 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 217.82 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 5m | 0.36 | 231.35 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 217.7 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 6m | 0.36 | 229.12 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 215.03 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 10m | 0.36 | 229.84 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 215.82 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 15m | 0.36 | 226.81 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 211.11 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 20m | 0.36 | 221.16 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 209.17 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 30m | 0.36 | 230.77 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 213.46 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 1h | 0.37 | 237.83 | 0.092 | 0.903 | 0.240 | 0.831 | 0.37 | 218.25 | 0.092 | 0.903 | 0.240 | 0.831 |
| $TS$ | 30s | 0.36 | 232.65 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 223.5 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 1m | 0.36 | 231.64 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 221.52 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 2m | 0.36 | 230.28 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 219.75 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 3m | 0.36 | 229.69 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 218.03 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 4m | 0.36 | 229.55 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 217.17 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 5m | 0.36 | 229.56 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 216.55 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 6m | 0.36 | 229.44 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 216.88 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 10m | 0.36 | 230.08 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 216.94 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 15m | 0.36 | 228.59 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 215.13 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 20m | 0.36 | 226.74 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 212.07 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 30m | 0.36 | 226.81 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 213.21 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 1h | 0.36 | 232.7 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 212.39 | 0.081 | 0.904 | 0.217 | 0.811 |
| $R$ | | 0.36 | 224.53 | 0.081 | 0.904 | 0.217 | 0.811 | 0.72 | 204.32 | 0.486 | 0.809 | 0.762 | 0.992 |
| GARCH | | 0.36 | 240.8 | 0.081 | 0.904 | 0.217 | 0.811 | | | | | | |

Table 8: 99% VaR forecasting adequacy. For each measure, sampling frequencies (when applicable) and volatility model (Base or P-Spline) the table reports the average number of VaR failures, the average VaR and the p–values of the adequacy tests.

# JNJ

| Meas. | Freq. | Base $\overline{H}$ | Base $\overline{\text{VaR}}$ | Base $\text{LR}_{\text{uc}}$ | Base $\text{LR}_{\text{ind}}$ | Base $\text{LR}_{\text{cc}}$ | Base $\text{DQ}_{\text{Hit}}$ | P-Spline $\overline{H}$ | P-Spline $\overline{\text{VaR}}$ | P-Spline $\text{LR}_{\text{uc}}$ | P-Spline $\text{LR}_{\text{ind}}$ | P-Spline $\text{LR}_{\text{cc}}$ | P-Spline $\text{DQ}_{\text{Hit}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $V$ | 30s | 0.54 | 218.02 | 0.233 | 0.857 | 0.484 | 0.946 | 0.9 | 207.28 | 0.811 | 0.763 | 0.929 | 0.999 |
| | 1m | 0.9 | 213.54 | 0.811 | 0.763 | 0.929 | 0.999 | 0.9 | 203.24 | 0.811 | 0.763 | 0.929 | 0.999 |
| | 2m | 0.9 | 211.44 | 0.811 | 0.763 | 0.929 | 0.999 | 1.08 | 200.52 | 0.850 | 0.717 | 0.920 | 0.997 |
| | 3m | 1.08 | 212.08 | 0.850 | 0.717 | 0.920 | 0.997 | 1.08 | 201.32 | 0.850 | 0.717 | 0.920 | 0.997 |
| | 4m | 1.08 | 211.53 | 0.850 | 0.717 | 0.920 | 0.997 | 1.08 | 199.75 | 0.850 | 0.717 | 0.920 | 0.997 |
| | 5m | 0.9 | 213.37 | 0.811 | 0.763 | 0.929 | 0.999 | 1.26 | 200.63 | 0.552 | 0.672 | 0.766 | 0.020 |
| | 6m | 1.08 | 209.39 | 0.850 | 0.717 | 0.920 | 0.997 | 1.26 | 196.7 | 0.552 | 0.672 | 0.766 | 0.020 |
| | 10m | 0.9 | 215.36 | 0.811 | 0.763 | 0.929 | 0.999 | 1.26 | 199.18 | 0.552 | 0.672 | 0.766 | 0.020 |
| | 15m | 1.08 | 218.07 | 0.850 | 0.717 | 0.920 | 0.997 | 1.26 | 198.77 | 0.552 | 0.672 | 0.766 | 0.020 |
| | 20m | 0.9 | 220.65 | 0.811 | 0.763 | 0.929 | 0.999 | 1.26 | 202.1 | 0.552 | 0.672 | 0.766 | 0.020 |
| | 30m | 0.9 | 215.6 | 0.811 | 0.763 | 0.929 | 0.999 | 0.9 | 198.3 | 0.811 | 0.763 | 0.929 | 0.001 |
| | 1h | 0.75 | 213.26 | 0.547 | 0.805 | 0.810 | 0.995 | 0.94 | 199.38 | 0.888 | 0.758 | 0.944 | 0.999 |
| $B$ | 30s | 0.54 | 224.33 | 0.233 | 0.857 | 0.484 | 0.946 | 0.72 | 211.79 | 0.486 | 0.809 | 0.762 | 0.992 |
| | 1m | 0.54 | 215.2 | 0.233 | 0.857 | 0.484 | 0.946 | 0.9 | 204.39 | 0.811 | 0.763 | 0.929 | 0.999 |
| | 2m | 0.9 | 212.67 | 0.811 | 0.763 | 0.929 | 0.999 | 1.08 | 200.71 | 0.850 | 0.717 | 0.920 | 0.997 |
| | 3m | 1.08 | 211.61 | 0.850 | 0.717 | 0.920 | 0.997 | 1.08 | 200.92 | 0.850 | 0.717 | 0.920 | 0.997 |
| | 4m | 1.08 | 211.36 | 0.850 | 0.717 | 0.920 | 0.997 | 1.08 | 199.8 | 0.850 | 0.717 | 0.920 | 0.997 |
| | 5m | 0.9 | 213.07 | 0.811 | 0.763 | 0.929 | 0.999 | 1.26 | 201.12 | 0.552 | 0.672 | 0.766 | 0.020 |
| | 6m | 0.9 | 210.83 | 0.811 | 0.763 | 0.929 | 0.999 | 1.26 | 197.92 | 0.552 | 0.672 | 0.766 | 0.020 |
| | 10m | 0.9 | 215.99 | 0.811 | 0.763 | 0.929 | 0.999 | 1.26 | 199.85 | 0.552 | 0.672 | 0.766 | 0.020 |
| | 15m | 1.08 | 218.11 | 0.850 | 0.717 | 0.920 | 0.997 | 1.26 | 197.08 | 0.552 | 0.672 | 0.766 | 0.020 |
| | 20m | 0.9 | 220.22 | 0.811 | 0.763 | 0.929 | 0.999 | 1.26 | 202.16 | 0.552 | 0.672 | 0.766 | 0.020 |
| | 30m | 1.08 | 218.5 | 0.850 | 0.717 | 0.920 | 0.997 | 0.9 | 200.96 | 0.811 | 0.763 | 0.929 | 0.001 |
| | 1h | 0.75 | 215.63 | 0.547 | 0.805 | 0.810 | 0.995 | 0.94 | 201.24 | 0.888 | 0.758 | 0.944 | 0.999 |
| $TS$ | 30s | 0.54 | 219.39 | 0.233 | 0.857 | 0.484 | 0.946 | 0.72 | 208.76 | 0.486 | 0.809 | 0.762 | 0.992 |
| | 1m | 0.72 | 213.67 | 0.486 | 0.809 | 0.762 | 0.992 | 1.08 | 204.2 | 0.850 | 0.717 | 0.920 | 0.007 |
| | 2m | 0.9 | 212.45 | 0.811 | 0.763 | 0.929 | 0.999 | 1.08 | 201.63 | 0.850 | 0.717 | 0.920 | 0.997 |
| | 3m | 1.08 | 212.93 | 0.850 | 0.717 | 0.920 | 0.997 | 1.08 | 201.26 | 0.850 | 0.717 | 0.920 | 0.997 |
| | 4m | 1.08 | 213.78 | 0.850 | 0.717 | 0.920 | 0.997 | 1.08 | 201.08 | 0.850 | 0.717 | 0.920 | 0.997 |
| | 5m | 1.08 | 213.89 | 0.850 | 0.717 | 0.920 | 0.997 | 1.26 | 201.13 | 0.552 | 0.672 | 0.766 | 0.020 |
| | 6m | 1.08 | 214.18 | 0.850 | 0.717 | 0.920 | 0.997 | 1.26 | 200.29 | 0.552 | 0.672 | 0.766 | 0.020 |
| | 10m | 1.08 | 215.56 | 0.850 | 0.717 | 0.920 | 0.997 | 1.26 | 200.52 | 0.552 | 0.672 | 0.766 | 0.020 |
| | 15m | 1.08 | 215.89 | 0.850 | 0.717 | 0.920 | 0.997 | 1.26 | 198.93 | 0.552 | 0.672 | 0.766 | 0.020 |
| | 20m | 1.08 | 216.36 | 0.850 | 0.717 | 0.920 | 0.997 | 1.26 | 199.17 | 0.552 | 0.672 | 0.766 | 0.020 |
| | 30m | 0.9 | 218.5 | 0.811 | 0.763 | 0.929 | 0.999 | 1.08 | 199.51 | 0.850 | 0.717 | 0.920 | 0.007 |
| | 1h | 0.75 | 221.8 | 0.547 | 0.805 | 0.810 | 0.995 | 0.94 | 199.87 | 0.888 | 0.758 | 0.944 | 0.001 |
| $R$ | | 0.9 | 226.82 | 0.811 | 0.763 | 0.929 | 0.999 | 1.08 | 207.39 | 0.850 | 0.717 | 0.920 | 0.007 |
| GARCH | | 0.9 | 254.03 | 0.811 | 0.763 | 0.929 | 0.999 | | | | | | |

Table 9: 99% VaR forecasting adequacy. For each measure, sampling frequencies (when applicable) and volatility model (Base or P-Spline) the table reports the average number of VaR failures, the average VaR and the p–values of the adequacy tests.

Tables 7, 8 and 9 report the out–of–sample 99% VaR adequacy results of the prediction exercise. The tables report the average number of failures, the average VaR and the p–values of the adequacy tests. At a 1% significance level all the nulls of VaR adequacy are not rejected. In the BA and GE stock there is some mild evidence of over coverage that appears to be stronger in the BA case using the Base forecasts and becomes weaker using the P-Spline forecasts at lower frequencies. In the JNJ stock there is some mild evidence of dependence in the VaR failures using the P-Spline forecasts. The volatility measure systematically lead to smaller average VaR than a GARCH and the P-Spline predictions systematically lead to smaller average VaR than the corresponding Base predictions. Overall, the adequacy of the VaR forecasts appears to be quite similar across all forecasting methods and it is difficult to find evidence that UHFD volatility measure provide significantly more adequate VaR forecasts than the forecasts based on the range or GARCH.

## 6.2 VaR Forecasting Accuracy

Statistical adequacy is a necessary requirement that VaR forecasts must satisfy, but it does not provide information as to the accuracy of such predictions and it does not always help to discriminate among different VaR forecasting methods.

We evaluate the out–of–sample accuracy of the VaR forecast using the probability deviation loss functions proposed by Kuester et al. (2006), The loss function is computed using the series of probability integral transformations of the returns using their estimated one day ahead cdf, i.e. $\hat{u}_{t+1} = \widehat{\mathsf{F}}_{t+1|t}(r_{t+1})$. For each of such $\hat{u}_{t+1}$ in $(0, 0.10]$, the probability deviations $\hat{d}_u$ are defined as the difference between the empirical cdf of the $\hat{u}$'s and a uniform cdf. We can then construct measure of fit as the sum of squared and sum of absolute probability deviations, that is

$$\mathsf{MSE} \equiv \sum_{\hat{u} \in (0,0.10]} \hat{d}_u^2 \qquad \mathsf{MAE} \equiv \sum_{\hat{u} \in (0,0.10]} |\hat{d}_u|;$$

which measure the goodness of fit of the models on the left tail of the return distribution. Such loss function has interesting prequential appeal (Dawid (1984)) and is also reminiscent of previous work on density forecast evaluation like Diebold, Gunther & Tay (1998).

Tables 10, 11 and 12 about here.

Figure 4 about here.

Tables 10, 11 and 12 report the probability deviations MSE and MAE of the VaR forecasting exercise. Figure 4 displays the graphs of the probability deviations MSE of the volatility measures as functions of the sampling frequency. The out of sample performance of the UHFD volatility measures appears to behave rather similarly across stocks. The graphs suggest that it is very difficult to discriminate between different UHFD volatility measures in that they all have very similar MSE profiles. In most cases performance appears to increase as the sampling frequency decreases and the best out of sample performance is obtained around 20–30 minutes. Moreover, the P-Spline forecasts systematically increase the out–of–sample accuracy of the VaR forecasts over the Base counterparts. The UHFD measures always produce more accurate forecasts than the GARCH

## BA

| | Freq. | Base MSE | Base MAE | P-Spline MSE | P-Spline MAE |
|---|---|---|---|---|---|
| $V$ | 30s | 3.147 | 1.683 | 2.229 | 1.409 |
| | 1m | 3.343 | 1.73 | 2.492 | 1.469 |
| | 2m | 3.222 | 1.704 | 2.066 | 1.342 |
| | 3m | 3.024 | 1.656 | 1.968 | 1.306 |
| | 4m | 2.477 | 1.497 | 1.436 | 1.075 |
| | 5m | 2.707 | 1.551 | 1.454 | 1.084 |
| | 6m | 2.497 | 1.492 | 0.889 | 0.786 |
| | 10m | 2.057 | 1.379 | 0.551 | 0.68 |
| | 15m | 2.191 | 1.403 | 0.32 | 0.513 |
| | 20m | 2.282 | 1.459 | 0.298 | 0.478 |
| | 30m | 1.639 | 1.202 | 0.132 | 0.32 |
| | 1h | 1.933 | 1.288 | 0.03 | 0.143 |
| $B$ | 30s | 4.391 | 2.004 | 3.059 | 1.661 |
| | 1m | 4.51 | 2.012 | 3.308 | 1.705 |
| | 2m | 3.689 | 1.824 | 2.279 | 1.395 |
| | 3m | 3.417 | 1.739 | 2.135 | 1.348 |
| | 4m | 2.694 | 1.561 | 1.374 | 1.045 |
| | 5m | 2.921 | 1.602 | 1.498 | 1.075 |
| | 6m | 2.424 | 1.462 | 0.989 | 0.858 |
| | 10m | 2.156 | 1.416 | 0.652 | 0.771 |
| | 15m | 2.303 | 1.44 | 0.464 | 0.591 |
| | 20m | 2.275 | 1.463 | 0.295 | 0.496 |
| | 30m | 1.536 | 1.157 | 0.441 | 0.727 |
| | 1h | 1.571 | 1.188 | 0.792 | 0.851 |
| $TS$ | 30s | 3.725 | 1.829 | 2.547 | 1.504 |
| | 1m | 3.647 | 1.808 | 2.478 | 1.471 |
| | 2m | 3.461 | 1.761 | 2.309 | 1.377 |
| | 3m | 3.252 | 1.709 | 1.867 | 1.213 |
| | 4m | 2.753 | 1.581 | 1.287 | 0.98 |
| | 5m | 2.485 | 1.496 | 1.041 | 0.866 |
| | 6m | 2.376 | 1.459 | 0.995 | 0.854 |
| | 10m | 2.267 | 1.426 | 0.47 | 0.564 |
| | 15m | 2.23 | 1.426 | 0.356 | 0.468 |
| | 20m | 2.307 | 1.456 | 0.341 | 0.472 |
| | 30m | 2.477 | 1.504 | 0.197 | 0.395 |
| | 1h | 2.578 | 1.467 | 0.276 | 0.443 |
| $R$ | | 3.225 | 1.661 | 0.266 | 0.462 |
| GARCH | | 10.858 | 2.863 | | |

Table 10: VaR forecasting accuracy. For each measure, sampling frequencies (when applicable) and volatility model (Base or P-Spline) the table reports the probability deviations MSE and MAE

## GE

| | Freq. | Base MSE | Base MAE | P-Spline MSE | P-Spline MAE |
|---|---|---|---|---|---|
| $V$ | 30s | 10.492 | 3.131 | 7.334 | 2.628 |
| | 1m | 9.366 | 2.966 | 6.037 | 2.382 |
| | 2m | 7.994 | 2.741 | 4.433 | 2.032 |
| | 3m | 8.511 | 2.829 | 3.634 | 1.842 |
| | 4m | 8.213 | 2.777 | 3.423 | 1.795 |
| | 5m | 7.918 | 2.724 | 2.884 | 1.644 |
| | 6m | 7.877 | 2.721 | 2.062 | 1.388 |
| | 10m | 8.035 | 2.733 | 2.758 | 1.588 |
| | 15m | 7.596 | 2.666 | 1.617 | 1.199 |
| | 20m | 6.697 | 2.502 | 1.977 | 1.312 |
| | 30m | 8.602 | 2.835 | 1.35 | 1.109 |
| | 1h | 8.145 | 2.758 | 0.483 | 0.641 |
| $B$ | 30s | 10.537 | 3.147 | 7.471 | 2.654 |
| | 1m | 9.305 | 2.96 | 5.811 | 2.335 |
| | 2m | 7.801 | 2.707 | 4.138 | 1.96 |
| | 3m | 7.53 | 2.661 | 3.315 | 1.76 |
| | 4m | 8.355 | 2.801 | 3.03 | 1.687 |
| | 5m | 8.376 | 2.799 | 2.991 | 1.676 |
| | 6m | 7.721 | 2.691 | 1.993 | 1.345 |
| | 10m | 7.727 | 2.68 | 2.898 | 1.632 |
| | 15m | 8.11 | 2.749 | 2.352 | 1.447 |
| | 20m | 6.274 | 2.43 | 2.101 | 1.377 |
| | 30m | 7.523 | 2.649 | 1.087 | 0.963 |
| | 1h | 10.208 | 3.088 | 1.775 | 1.271 |
| $TS$ | 30s | 10.492 | 3.13 | 6.688 | 2.511 |
| | 1m | 10.108 | 3.08 | 5.711 | 2.314 |
| | 2m | 9.056 | 2.913 | 4.301 | 1.993 |
| | 3m | 8.3 | 2.785 | 3.619 | 1.839 |
| | 4m | 8.027 | 2.74 | 3.103 | 1.705 |
| | 5m | 7.909 | 2.721 | 3.035 | 1.687 |
| | 6m | 7.813 | 2.704 | 2.585 | 1.555 |
| | 10m | 7.902 | 2.716 | 2.518 | 1.532 |
| | 15m | 7.353 | 2.621 | 2.094 | 1.381 |
| | 20m | 7.405 | 2.63 | 2.1 | 1.393 |
| | 30m | 7.574 | 2.665 | 2.123 | 1.395 |
| | 1h | 9.977 | 3.055 | 3.068 | 1.646 |
| $R$ | | 5.367 | 2.257 | 0.314 | 0.428 |
| GARCH | | 7.791 | 2.7 | | |

Table 11: VaR forecasting accuracy. For each measure, sampling frequencies (when applicable) and volatility model (Base or P-Spline) the table reports the probability deviations MSE and MAE

## JNJ

| | | Base | | P-Spline | |
|---|---|---|---|---|---|
| | Freq. | MSE | MAE | MSE | MAE |
| $V$ | 30s | 2.374 | 1.385 | 0.503 | 0.609 |
| | 1m | 2.008 | 1.263 | 0.338 | 0.487 |
| | 2m | 2.133 | 1.288 | 0.47 | 0.544 |
| | 3m | 2.4 | 1.378 | 0.445 | 0.581 |
| | 4m | 2.307 | 1.352 | 0.341 | 0.488 |
| | 5m | 2.519 | 1.402 | 0.108 | 0.244 |
| | 6m | 1.768 | 1.145 | 0.18 | 0.314 |
| | 10m | 2.288 | 1.323 | 0.191 | 0.333 |
| | 15m | 3.082 | 1.57 | 0.07 | 0.222 |
| | 20m | 3.885 | 1.726 | 0.09 | 0.242 |
| | 30m | 1.993 | 1.248 | 0.09 | 0.242 |
| | 1h | 2.85 | 1.446 | 0.404 | 0.508 |
| $B$ | 30s | 3.563 | 1.697 | 0.712 | 0.763 |
| | 1m | 2.183 | 1.32 | 0.361 | 0.478 |
| | 2m | 2.405 | 1.371 | 0.449 | 0.529 |
| | 3m | 2.228 | 1.333 | 0.417 | 0.532 |
| | 4m | 2.493 | 1.396 | 0.382 | 0.52 |
| | 5m | 2.326 | 1.35 | 0.077 | 0.211 |
| | 6m | 1.939 | 1.205 | 0.189 | 0.327 |
| | 10m | 2.601 | 1.416 | 0.236 | 0.427 |
| | 15m | 3.175 | 1.584 | 0.07 | 0.224 |
| | 20m | 3.63 | 1.663 | 0.15 | 0.32 |
| | 30m | 2.261 | 1.357 | 0.102 | 0.26 |
| | 1h | 3.516 | 1.578 | 0.628 | 0.633 |
| $TS$ | 30s | 2.912 | 1.531 | 0.438 | 0.579 |
| | 1m | 2.26 | 1.332 | 0.343 | 0.501 |
| | 2m | 2.47 | 1.399 | 0.443 | 0.57 |
| | 3m | 2.644 | 1.449 | 0.419 | 0.553 |
| | 4m | 2.823 | 1.5 | 0.238 | 0.39 |
| | 5m | 2.773 | 1.48 | 0.144 | 0.295 |
| | 6m | 2.772 | 1.47 | 0.121 | 0.268 |
| | 10m | 3.136 | 1.54 | 0.153 | 0.332 |
| | 15m | 2.965 | 1.491 | 0.097 | 0.241 |
| | 20m | 2.881 | 1.481 | 0.076 | 0.226 |
| | 30m | 3.204 | 1.563 | 0.077 | 0.238 |
| | 1h | 5.658 | 2.143 | 0.426 | 0.568 |
| $R$ | | 5.188 | 1.954 | 0.431 | 0.54 |
| GARCH | | 9.36 | 2.783 | | |

Table 12: VaR forecasting accuracy. For each measure, sampling frequencies (when applicable) and volatility model (Base or P-Spline) the table reports the probability deviations MSE and MAE

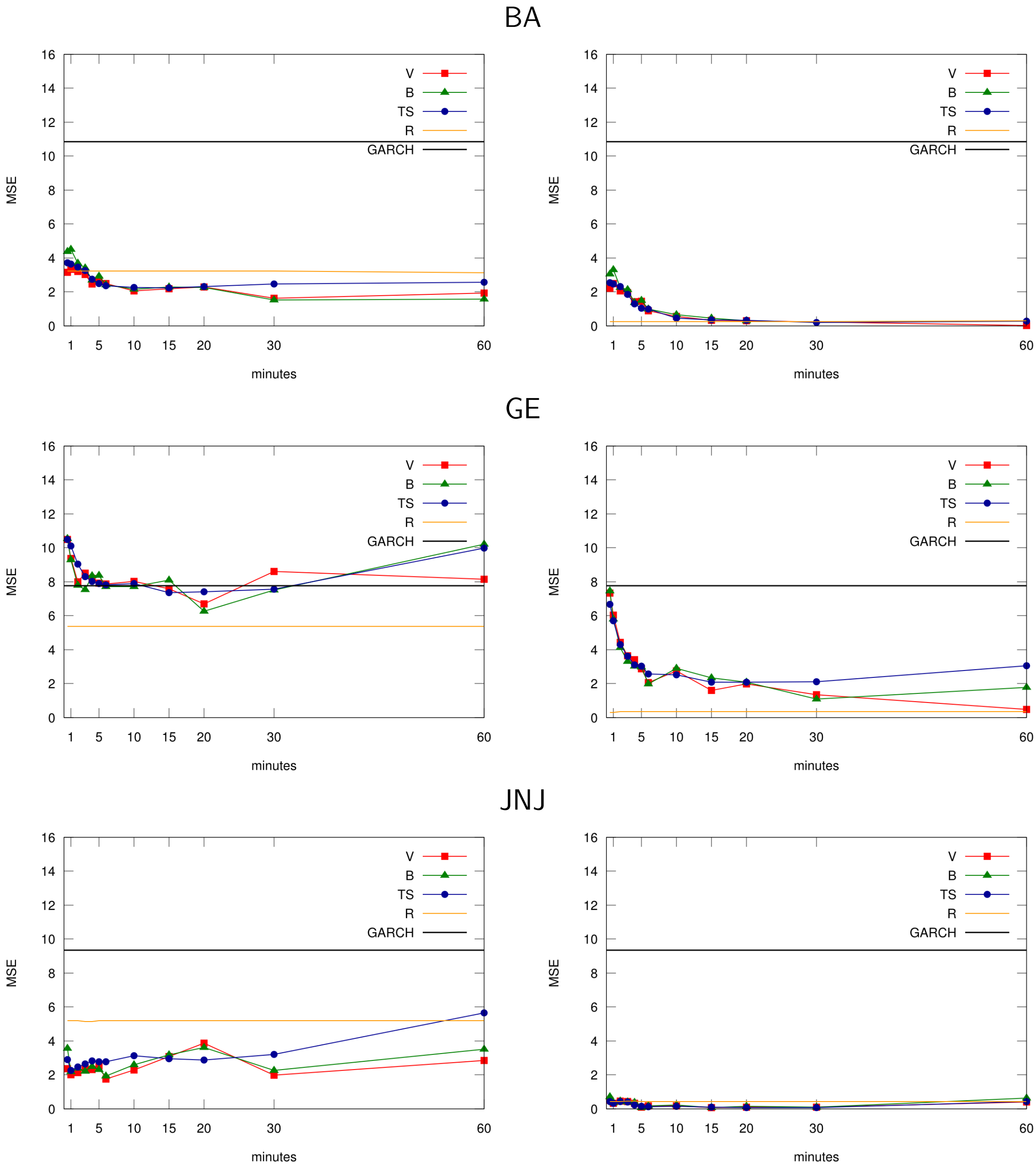


Figure 4: Out–of–sample VaR MSE of the volatility measures. The graphs display the estimated MSE of the volatility measures as functions of the sampling frequency.

benchmark, with the exception of the Base forecasts for the GE stock. However, the range appears hard to beat. In the GE stock the range forecasts systematically perform better than all the measures. In the BA and JNJ stock the Base range forecasts are beaten by the UHFD measures at most sampling frequency but the P-Spline range forecasts appear to have a substantially close performance to the UHFD measures.

# 7 Conclusion

We find that UHFD volatility measures perform similarly in terms of VaR forecasting, obtain the best forecasting results at "low" frequencies (20/30min), and do not appear to outperform range. In comparison to a standard GARCH, models for realized volatility measures produce VaR forecasts which are more accurate but yet as adequate. Modeling volatility trends using our novel P–Spline MEM systematically improves forecasting ability. The empirical evidence suggests that the range has a very good cost–to–quality ratio for VaR prediction.

The empirical evidence of this paper can be somehow counterintuitive. The UHFD volatility measures literature argues that by using *all* the data it is possible to construct arbitrarily precise estimates of volatility and it is not uncommon to find papers claiming that using UHFD volatility measures corresponds to "observe" volatility.

We believe that there are some straightforward arguments that explain our findings.

A contribution of Granger (Granger (1998)) on the advent of UHFD points out that asymptotic theory assumes that the amount of information increases with the amount of data, but there are many situations in which this will just not hold. Put concisely *"by observing earth movements more carefully we do not observe more large earthquakes"* (Granger (1998)).

The empirical findings suggest that microstructure dynamics seem to bias volatility dynamics at very high frequencies and this compromises the benefits of sampling at ultra high–frequencies. Suppose that the timing between price changes is irregularly spaced (c.f. Engle & Russell (1998)) and that the prices are sampled over a regular scale: Jordà & Marcellino (2003) have shown that the aggregated data will exhibit conditional heteroskedasticity even if such feature is absent in the data generating process. The recently proposed realized kernel (Barndorff-Nielsen et al. (2006*a*) and Barndorff-Nielsen, Hansen, Lunde & Shepard (2006*b*)) appear to be more robust to these type of microstructure dynamics and it will be interesting to see if such estimators will be able to improve upon the forecasting ability of the UHFD measures used in this work.

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# A Stylized Facts

All the series analyzed in this study are constructed using "cleaned" mid quotes from the NYSE between 9:30 and 16:05. The data is extracted from the NYSE-TAQ database. More detailed discussion on the features of the UHFD series can be found in Brownlees & Gallo (2006). This study highlights some features of the volatility measures series which do not appear to have been extensively documented in the literature yet.

Table 13, 14 and 15 about here.

Tables 13, 14 and 15 report a number of descriptive statistics on the volatility measures. The tables report for each measures and sampling frequency (when applicable) the sample moments, first autocorrelation and the correlation with realized volatility computed at the same frequency. The bottom row of each table reports the sample moments and first autocorrelation of squared returns. Almost all volatility measures tend to systematically underestimate the returns variance at almost all frequencies. This is due to fact that UHFD volatility measures are intra–daily measures of volatility while the daily close–to–close return is made up of an overnight component (close–to–open) and an intra-daily component (open–to–close). The sample moments suggest that the shape of the distribution of the volatility measures depends not only on the measure but also on the sampling frequency. The sample skewness and kurtosis indices also appear to be quite noisy at times. Judging from the realized volatility sample mean it is unclear if the impact of iid microstructure noise is substantial. In the presence of iid microstructure noise as the sampling frequency gets very high realized volatility diverges to infinity (e.g. Bandi & Russell (2003)). The weak evidence of this effect in the sample, at least up to a 30 seconds frequency, is probably due to the fact that the decimalization, using mid–quotes and the high liquidity of the stocks considered make the extent of some frictions less strong in comparison to earlier/other datasets. This is also in line with the empirical evidence on the magnitude of the microstructure noise in Barndorff-Nielsen et al. (2006*a*). The degree of persistence of the volatility measures is higher when sampling at higher frequencies and is always larger than the persistence in the squared returns. The correlation with realized volatility appears to be frequency dependent as well, and the higher the frequency the higher the correlation. The increase in the persistence and the correlation between UHFD measures at higher frequencies hint at the presence of a some sort dependent noise that appears to bias the UHFD volatility measures. Some analogous results are also found in the empirical analysis of Barndorff-Nielsen et al. (2006*a*) and Barndorff-Nielsen et al. (2006*b*).

Tables 16, 17, 18 about here.

Tables 16, 17, 18 report the sample skewness and kurtosis of the series of returns standardized by the square root of the volatility measures, as well as the p–value of the Ljung–Box test for autocorrelation. All the volatility measures are always capable of washing away the dependence in the squared returns as the Ljung–Box test is always not rejected at standard significance levels. However, it is unclear if the normality assumption of the standardized returns is always satisfactory. Focusing on the returns standardized by the UHFD volatility measures, it appears that the BA ticker does not exhibit a significant skewness and leptkurtosis when using data sampled at sufficiently high frequencies; the GE ticker exhibits a significant asymmetry at high frequencies while it exhibits evidence of leptokurtosis at lower frequencies and the JNJ ticker does not exhibit strong skewness but does exhibit a significant leptokurtosis. Interestingly, returns standardized by the range always appear to be symmetric and platikurtic.

## BA

| Meas. | Freq. | Mean | Std.Dev. | Skew. | Kurt. | $\hat{\rho}_1$ | $\hat{\rho}_{rv}$ |
|---|---|---|---|---|---|---|---|
| $V$ | 30s | 2.61 | 2.63 | 3.61 | 25.10 | 0.81 | |
| | 1m | 2.80 | 2.91 | 3.56 | 23.50 | 0.82 | |
| | 2m | 2.92 | 3.15 | 3.79 | 25.62 | 0.81 | |
| | 3m | 2.95 | 3.29 | 4.10 | 30.20 | 0.76 | |
| | 4m | 2.92 | 3.31 | 4.07 | 28.21 | 0.77 | |
| | 5m | 2.87 | 3.32 | 4.22 | 29.84 | 0.77 | |
| | 6m | 2.83 | 3.38 | 4.61 | 36.42 | 0.71 | |
| | 10m | 2.69 | 3.35 | 5.10 | 43.29 | 0.71 | |
| | 15m | 2.61 | 3.27 | 4.31 | 31.11 | 0.59 | |
| | 20m | 2.54 | 3.48 | 5.10 | 40.48 | 0.58 | |
| | 30m | 2.47 | 3.84 | 7.12 | 90.56 | 0.40 | |
| | 1h | 2.04 | 3.20 | 4.72 | 33.16 | 0.42 | |
| $B$ | 30s | 2.07 | 2.01 | 3.57 | 24.03 | 0.79 | 0.99 |
| | 1m | 2.48 | 2.57 | 3.51 | 21.99 | 0.80 | 0.99 |
| | 2m | 2.77 | 3 | 3.56 | 21.56 | 0.81 | 0.99 |
| | 3m | 2.82 | 3.14 | 3.65 | 22.42 | 0.78 | 0.99 |
| | 4m | 2.82 | 3.21 | 4.07 | 27.56 | 0.77 | 0.99 |
| | 5m | 2.80 | 3.26 | 4.14 | 27.94 | 0.77 | 0.99 |
| | 6m | 2.77 | 3.3 | 4.47 | 34.41 | 0.71 | 0.99 |
| | 10m | 2.64 | 3.43 | 5.47 | 49.93 | 0.66 | 0.99 |
| | 15m | 2.51 | 3.2 | 4.22 | 29.12 | 0.59 | 0.98 |
| | 20m | 2.44 | 3.32 | 5.08 | 40.99 | 0.53 | 0.98 |
| | 30m | 2.34 | 3.60 | 6.83 | 84.09 | 0.37 | 0.98 |
| | 1h | 1.94 | 3.09 | 4.74 | 34.06 | 0.37 | 0.97 |
| $TS$ | 30s | 4.83 | 4.69 | 2.97 | 15.24 | 0.82 | 0.99 |
| | 1m | 3.56 | 3.62 | 3.05 | 16.03 | 0.82 | 0.99 |
| | 2m | 3.26 | 3.46 | 3.41 | 20.15 | 0.80 | 0.98 |
| | 3m | 3.14 | 3.43 | 3.73 | 23.60 | 0.77 | 0.98 |
| | 4m | 3.06 | 3.40 | 3.88 | 25.18 | 0.76 | 0.98 |
| | 5m | 3.00 | 3.38 | 3.98 | 26.12 | 0.73 | 0.98 |
| | 6m | 2.96 | 3.41 | 4.14 | 27.92 | 0.73 | 0.97 |
| | 10m | 2.86 | 3.44 | 4.48 | 32.73 | 0.69 | 0.96 |
| | 15m | 2.79 | 3.46 | 5.10 | 48.47 | 0.63 | 0.94 |
| | 20m | 2.73 | 3.48 | 5.21 | 50.31 | 0.60 | 0.92 |
| | 30m | 2.67 | 3.52 | 4.85 | 40.74 | 0.54 | 0.91 |
| | 1h | 0.451 | 0.638 | 4.10 | 24.69 | 0.43 | 0.77 |
| $R$ | | 2.80 | 3.82 | 5.19 | 46.23 | 0.51 | |
| $r_t^2$ | | 3.74 | 12.3 | 20.27 | 575.78 | 0.17 | |

Table 13: Descriptive statistics of the volatility measures. For each volatility measures and sampling frequency (when applicable) the table reports mean, standard deviation, skewness, kurtosis, first autocorrelation and the correlation with realized volatility $rv_V$ computed at the same frequency

## GE

| Meas. | Freq. | Mean | Std.Dev. | Skew. | Kurt. | $\hat{\rho}_1$ | $\hat{\rho}_{rv}$ |
|---|---|---|---|---|---|---|---|
| $V$ | 30s | 1.94 | 2.38 | 3.87 | 31.55 | 0.79 | |
| | 1m | 2.13 | 2.72 | 3.72 | 26.54 | 0.78 | |
| | 2m | 2.27 | 3.08 | 4.36 | 36.16 | 0.74 | |
| | 3m | 2.29 | 3.25 | 5.37 | 58.05 | 0.72 | |
| | 4m | 2.30 | 3.4 | 5.50 | 54.99 | 0.68 | |
| | 5m | 2.29 | 3.44 | 6.08 | 69.25 | 0.65 | |
| | 6m | 2.27 | 3.42 | 5.55 | 52.61 | 0.65 | |
| | 10m | 2.17 | 3.27 | 5.81 | 57.76 | 0.59 | |
| | 15m | 2.13 | 3.67 | 9.32 | 160.59 | 0.46 | |
| | 20m | 2.12 | 3.45 | 6.22 | 68.60 | 0.46 | |
| | 30m | 2.05 | 3.62 | 9.72 | 182.23 | 0.43 | |
| | 1h | 1.80 | 3.2 | 5.75 | 52.94 | 0.38 | |
| $B$ | 30s | 1.67 | 2.09 | 3.48 | 23.95 | 0.81 | 1.00 |
| | 1m | 1.98 | 2.59 | 3.73 | 26.77 | 0.79 | 1.00 |
| | 2m | 2.18 | 2.99 | 4.34 | 35.82 | 0.75 | 1.00 |
| | 3m | 2.23 | 3.33 | 6.33 | 80.94 | 0.69 | 0.99 |
| | 4m | 2.25 | 3.47 | 5.99 | 65.13 | 0.65 | 0.99 |
| | 5m | 2.22 | 3.47 | 6.35 | 73.28 | 0.63 | 0.99 |
| | 6m | 2.21 | 3.5 | 5.95 | 58.14 | 0.63 | 0.99 |
| | 10m | 2.09 | 3.22 | 6.11 | 65.96 | 0.60 | 0.99 |
| | 15m | 2.05 | 3.79 | 10.67 | 203.66 | 0.43 | 0.99 |
| | 20m | 2.05 | 3.25 | 5.03 | 38.63 | 0.47 | 0.97 |
| | 30m | 1.91 | 3.40 | 8.71 | 146.51 | 0.43 | 0.98 |
| | 1h | 1.67 | 3.09 | 5.81 | 51.75 | 0.40 | 0.97 |
| $TS$ | 30s | 3.71 | 4.63 | 4.31 | 40.45 | 0.76 | 1.00 |
| | 1m | 2.78 | 3.64 | 4.20 | 35.39 | 0.75 | 0.99 |
| | 2m | 2.55 | 3.5 | 4.59 | 40.04 | 0.73 | 0.99 |
| | 3m | 2.48 | 3.48 | 4.88 | 43.53 | 0.71 | 0.99 |
| | 4m | 2.43 | 3.51 | 5.28 | 50.41 | 0.69 | 0.99 |
| | 5m | 2.4 | 3.56 | 5.82 | 61.93 | 0.66 | 0.99 |
| | 6m | 2.35 | 3.49 | 5.85 | 62.28 | 0.64 | 0.98 |
| | 10m | 2.27 | 3.45 | 6.42 | 76.37 | 0.59 | 0.97 |
| | 15m | 2.22 | 3.42 | 6.84 | 88.75 | 0.54 | 0.97 |
| | 20m | 2.19 | 3.41 | 6.65 | 83.24 | 0.53 | 0.96 |
| | 30m | 2.14 | 3.47 | 7.25 | 100.58 | 0.48 | 0.93 |
| | 1h | 1.41 | 2.61 | 9.24 | 150.75 | 0.35 | 0.74 |
| $R$ | | 2.43 | 4.64 | 8.15 | 103.03 | 0.41 | |
| $r_t^2$ | | 3.03 | 8.65 | 8.52 | 102.23 | 0.17 | |

Table 14: Descriptive statistics of the volatility measures. For each volatility measures and sampling frequency (when applicable) the table reports mean, standard deviation, skewness, kurtosis, first autocorrelation and the correlation with realized volatility $rv_V$ computed at the same frequency

# JNJ

| Meas. | Freq. | Mean | Std.Dev. | Skew. | Kurt. | $\hat{\rho}_1$ | $\hat{\rho}_{rv}$ |
|---|---|---|---|---|---|---|---|
| $V$ | 30s | 1.17 | 1.18 | 4.32 | 33.89 | 0.78 | |
| | 1m | 1.33 | 1.51 | 4.87 | 42.13 | 0.76 | |
| | 2m | 1.47 | 1.89 | 5.81 | 58.83 | 0.73 | |
| | 3m | 1.50 | 2.08 | 6.38 | 68.52 | 0.71 | |
| | 4m | 1.51 | 2.22 | 7.28 | 87.23 | 0.70 | |
| | 5m | 1.51 | 2.33 | 7.75 | 96.69 | 0.72 | |
| | 6m | 1.50 | 2.59 | 9.64 | 143.58 | 0.62 | |
| | 10m | 1.41 | 2.43 | 9.10 | 132.41 | 0.55 | |
| | 15m | 1.36 | 2.18 | 6.96 | 85.96 | 0.53 | |
| | 20m | 1.33 | 2.26 | 7.74 | 99.01 | 0.48 | |
| | 30m | 1.25 | 2.03 | 5.57 | 47.77 | 0.48 | |
| | 1h | 0.99 | 2.05 | 12.95 | 281.07 | 0.29 | |
| $B$ | 30s | 0.95 | 0.936 | 4.38 | 34.18 | 0.77 | 0.99 |
| | 1m | 1.2 | 1.38 | 5.14 | 48.43 | 0.77 | 0.99 |
| | 2m | 1.41 | 1.87 | 6.29 | 70.78 | 0.75 | 0.99 |
| | 3m | 1.46 | 2.08 | 6.71 | 75.72 | 0.70 | 0.99 |
| | 4m | 1.47 | 2.24 | 8.11 | 108.57 | 0.68 | 0.99 |
| | 5m | 1.46 | 2.37 | 8.99 | 133.29 | 0.66 | 0.99 |
| | 6m | 1.46 | 2.75 | 11.73 | 208.52 | 0.56 | 0.99 |
| | 10m | 1.36 | 2.47 | 10.42 | 178.58 | 0.52 | 0.99 |
| | 15m | 1.32 | 2.23 | 8.66 | 139.34 | 0.49 | 0.98 |
| | 20m | 1.29 | 2.26 | 8.43 | 115.30 | 0.45 | 0.98 |
| | 30m | 1.19 | 2.16 | 6.79 | 69.45 | 0.43 | 0.98 |
| | 1h | 0.91 | 1.79 | 10.25 | 180.22 | 0.29 | 0.97 |
| $TS$ | 30s | 2.20 | 2.24 | 4.39 | 34.63 | 0.78 | 1.00 |
| | 1m | 1.73 | 2.02 | 4.96 | 42.17 | 0.77 | 1.00 |
| | 2m | 1.65 | 2.20 | 5.98 | 58.31 | 0.75 | 0.99 |
| | 3m | 1.62 | 2.32 | 6.66 | 69.48 | 0.72 | 0.99 |
| | 4m | 1.59 | 2.40 | 7.27 | 80.96 | 0.70 | 0.99 |
| | 5m | 1.56 | 2.41 | 7.68 | 90.83 | 0.68 | 0.99 |
| | 6m | 1.53 | 2.43 | 8.02 | 98.89 | 0.66 | 0.98 |
| | 10m | 1.46 | 2.45 | 8.48 | 109.09 | 0.61 | 0.97 |
| | 15m | 1.41 | 2.28 | 7.46 | 83.74 | 0.59 | 0.94 |
| | 20m | 1.37 | 2.16 | 7.05 | 77.55 | 0.55 | 0.95 |
| | 30m | 1.30 | 2.03 | 6.97 | 81.28 | 0.52 | 0.92 |
| | 1h | 0.35 | 0.639 | 9.85 | 162.23 | 0.34 | 0.80 |
| $R$ | | 1.36 | 2.23 | 7.44 | 93.03 | 0.47 | |
| $r_t^2$ | | 1.68 | 8.4 | 30.35 | 1058.03 | 0.052 | |

Table 15: Descriptive statistics of the volatility measures. For each volatility measures and sampling frequency (when applicable) the table reports mean, standard deviation, skewness, kurtosis, first autocorrelation and the correlation with realized volatility $rv_V$ computed at the same frequency

## BA

| Meas | Freq | Skew. | Kurt | $Q_{10}(z_t^2)$ |
|---|---|---|---|---|
| $V$ | 30s | 0.05 | 2.99 | 0.219 |
| | 1m | 0.06 | 2.94 | 0.353 |
| | 2m | 0.09 | 2.84 | 0.734 |
| | 3m | 0.06 | 2.84 | 0.791 |
| | 4m | 0.06 | 2.78* | 0.883 |
| | 5m | 0.03 | 2.76** | 0.858 |
| | 6m | 0.07 | 2.77** | 0.956 |
| | 10m | 0.08 | 2.78* | 0.906 |
| | 15m | 0.09 | 2.96 | 0.920 |
| | 20m | 0.14 | 2.95 | 0.510 |
| | 30m | 0.09 | 3.36** | 0.447 |
| | 1h | 0.37*** | 5.38*** | 0.628 |
| $B$ | 30s | 0.04 | 3.00 | 0.013 |
| | 1m | 0.05 | 2.97 | 0.143 |
| | 2m | 0.08 | 2.84 | 0.561 |
| | 3m | 0.03 | 2.91 | 0.460 |
| | 4m | 0.04 | 2.83 | 0.670 |
| | 5m | 0.03 | 2.80* | 0.565 |
| | 6m | 0.06 | 2.83 | 0.877 |
| | 10m | 0.11 | 2.83 | 0.942 |
| | 15m | 0.07 | 3.05 | 0.832 |
| | 20m | 0.17* | 2.99 | 0.330 |
| | 30m | -0.03 | 4.80*** | 0.881 |
| | 1h | 0.55*** | 6.65*** | 0.655 |
| $TS$ | 30s | 0.04 | 3.05 | 0.063 |
| | 1m | 0.05 | 2.96 | 0.245 |
| | 2m | 0.05 | 2.87 | 0.572 |
| | 3m | 0.04 | 2.80 | 0.752 |
| | 4m | 0.04 | 2.78* | 0.826 |
| | 5m | 0.04 | 2.76** | 0.907 |
| | 6m | 0.04 | 2.71*** | 0.896 |
| | 10m | 0.04 | 2.67*** | 0.905 |
| | 15m | 0.03 | 2.71*** | 0.908 |
| | 20m | 0.04 | 2.80* | 0.853 |
| | 30m | 0.03 | 2.92 | 0.732 |
| | 1h | 0.08 | 3.12 | 0.608 |
| $R$ | | 0.07 | 2.46*** | 0.572 |

Table 16: Descriptive of returns standardized by the square root of the variance measures. For each volatility measures and sampling frequency (when applicable) the table reports skewness, kurtosis and p–value of Ljung–Box statistic.

## GE

| Meas | Freq | Skew. | Kurt | $Q_{10}(z_t^2)$ |
|---|---|---|---|---|
| $V$ | 30s | 0.21** | 3.44*** | 0.825 |
| | 1m | 0.22** | 3.28** | 0.795 |
| | 2m | 0.22** | 3.27** | 0.686 |
| | 3m | 0.22** | 3.26* | 0.611 |
| | 4m | 0.22** | 3.28** | 0.504 |
| | 5m | 0.22** | 3.19 | 0.407 |
| | 6m | 0.18* | 3.18 | 0.609 |
| | 10m | 0.19* | 3.29** | 0.142 |
| | 15m | 0.13 | 3.44*** | 0.114 |
| | 20m | 0.15 | 3.45*** | 0.130 |
| | 30m | 0.06 | 3.81*** | 0.659 |
| | 1h | 0.04 | 4.79*** | 0.490 |
| $B$ | 30s | 0.20** | 3.43*** | 0.860 |
| | 1m | 0.23** | 3.26* | 0.785 |
| | 2m | 0.21** | 3.25* | 0.590 |
| | 3m | 0.21** | 3.26* | 0.624 |
| | 4m | 0.22** | 3.25* | 0.536 |
| | 5m | 0.25** | 3.27** | 0.442 |
| | 6m | 0.19* | 3.23* | 0.524 |
| | 10m | 0.20** | 3.39*** | 0.152 |
| | 15m | 0.21** | 3.67*** | 0.162 |
| | 20m | 0.18* | 3.52*** | 0.097 |
| | 30m | -0.04 | 5.38*** | 0.051 |
| | 1h | 0.44*** | 9.91*** | 0.882 |
| $TS$ | 30s | 0.22** | 3.41*** | 0.759 |
| | 1m | 0.23** | 3.28** | 0.760 |
| | 2m | 0.25** | 3.23* | 0.722 |
| | 3m | 0.24** | 3.23* | 0.637 |
| | 4m | 0.24** | 3.21 | 0.582 |
| | 5m | 0.23** | 3.19 | 0.522 |
| | 6m | 0.22** | 3.18 | 0.483 |
| | 10m | 0.19* | 3.19 | 0.267 |
| | 15m | 0.14 | 3.12 | 0.178 |
| | 20m | 0.12 | 3.13 | 0.164 |
| | 30m | 0.08 | 3.23* | 0.192 |
| | 1h | 0.06 | 3.68*** | 0.360 |
| $R$ | | 0.15 | 2.67*** | 0.045 |

Table 17: Descriptive of returns standardized by the square root of the variance measures. For each volatility measures and sampling frequency (when applicable) the table reports skewness, kurtosis and p–value of Ljung–Box statistic.

# JNJ

| Meas | Freq | Skew. | Kurt | $Q_{10}(z_t^2)$ |
|---|---|---|---|---|
| $V$ | 30s | 0.11 | 4.20*** | 0.758 |
| | 1m | 0.10 | 3.98*** | 0.864 |
| | 2m | 0.11 | 3.72*** | 0.853 |
| | 3m | 0.11 | 3.67*** | 0.765 |
| | 4m | 0.10 | 3.49*** | 0.811 |
| | 5m | 0.08 | 3.34** | 0.819 |
| | 6m | 0.08 | 3.36** | 0.704 |
| | 10m | 0.05 | 3.61*** | 0.504 |
| | 15m | -0.01 | 3.70*** | 0.581 |
| | 20m | 0.02 | 3.66*** | 0.631 |
| | 30m | -0.08 | 4.26*** | 0.656 |
| | 1h | -0.10 | 4.92*** | 0.742 |
| $B$ | 30s | 0.09 | 4.20*** | 0.786 |
| | 1m | 0.12 | 3.97*** | 0.940 |
| | 2m | 0.14 | 3.83*** | 0.858 |
| | 3m | 0.12 | 3.67*** | 0.801 |
| | 4m | 0.10 | 3.40*** | 0.817 |
| | 5m | 0.06 | 3.34** | 0.825 |
| | 6m | 0.05 | 3.33** | 0.856 |
| | 10m | 0.08 | 3.55*** | 0.390 |
| | 15m | 0.06 | 3.93*** | 0.455 |
| | 20m | 0.01 | 3.58*** | 0.565 |
| | 30m | -0.09 | 4.48*** | 0.657 |
| | 1h | -0.58*** | 7.51*** | 0.869 |
| $TS$ | 30s | 0.11 | 4.13*** | 0.773 |
| | 1m | 0.10 | 3.86*** | 0.873 |
| | 2m | 0.09 | 3.61*** | 0.843 |
| | 3m | 0.08 | 3.51*** | 0.815 |
| | 4m | 0.07 | 3.45*** | 0.815 |
| | 5m | 0.07 | 3.36** | 0.854 |
| | 6m | 0.07 | 3.32** | 0.878 |
| | 10m | 0.05 | 3.27** | 0.889 |
| | 15m | 0.05 | 3.39*** | 0.883 |
| | 20m | 0.04 | 3.32** | 0.862 |
| | 30m | -0.02 | 3.47*** | 0.831 |
| | 1h | -0.04 | 3.89*** | 0.352 |
| $R$ | | 0.02 | 2.73** | 0.656 |

Table 18: Descriptive of returns standardized by the square root of the variance measures. For each volatility measures and sampling frequency (when applicable) the table reports skewness, kurtosis and p–value of Ljung–Box statistic.