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# Comparison of Volatility Measures: a Risk Management Perspective

Christian T. Brownlees,
Giampiero M. Gallo

Università degli Studi
di Firenze

Dipartimento di Statistica "G. Parenti" – Viale Morgagni 59 – 50134 Firenze - www.ds.unifi.it

# Comparison of Volatility Measures: a Risk Management Perspective

Christian T. Brownlees* Giampiero M. Gallo*

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**Abstract**

In this paper we address the issue of forecasting Value–at–Risk (VaR) using different volatility measures: realized volatility, bipower realized volatility, two scales realized volatility, realized kernel as well as the daily range. We propose a dynamic model with a flexible trend specification bonded with a penalized maximum likelihood estimation strategy: the P-Spline Multiplicative Error Model. Exploiting UHFD volatility measures, VaR predictive ability is considerably improved upon relative to a baseline GARCH but not so relative to the range; there are relevant gains from modeling volatility trends and using realized kernels that are robust to *dependent* microstructure noise.

**Keywords:** Volatility Measures, VaR Forecasting, GARCH, MEM, P-Spline.

**JEL:** C22, C51, C52, C53

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# 1 Introduction

Measurement and forecasting latent volatility has many important applications in many areas of finance including asset allocation, option pricing and risk management. The two tasks have been successfully accomplished within the same ARCH framework (Engle (1982), Bollerslev, Engle & Nelson (1994)) for the past 25 years. Alternative measurements based on different assumptions and different information sets have been in use for a while, such as historical variances, range, implied volatilities; in recent times the properties of volatility proxies derived from the availability of intra-daily data sampled at high frequency have been the object of a sizable strand of research (e.g. Andersen & Bollerslev (1998), Andersen, Bollerslev, Diebold & Ebens (2001) Barndorff-Nielsen & Shephard (2002), Andersen, Bollerslev, Diebold & Labys (2003)). Under suitable assumptions they converge (as the sampling frequency of the intra-daily data increases) to the integrated variance[1], that is the integral of instantaneous (or spot) volatility of an underlying continuous time process over a short period. While it is possible, in theory, to construct ex–post measures of return variability with arbitrary precision, their relationship to the latent underlying process (e.g. with or without jumps) and how to forecast volatility on the basis of existing information is still open to question.

Not knowing what latent process best describes the data generating process, in this work we address the forecasting issue from a pragmatic point of view, trying to establish to which extent different volatility measures improve upon the out–of–sample forecasting ability of standard methods. Several metrics can be used to evaluate the forecasting performance: a Mincer Zarnowitz type regression where each forecast is contrasted against a suitable 'target' (typically one of the measures themselves), implied volatility measures (such as VIX), or, within a risk management framework, the quality of the derived measures of Value at Risk (VaR) or Expected Shortfall (ES) which have emerged as a prominent measure of market risk. A VaR forecasting application is an interesting battleground (Andersen et al. (2003)), so to speak, for comparing different volatility measures. Here it is limited to a single asset, but it could be extended to a multivariate context.

In this work we compare the daily range (Parkinson (1980)) and a set of Ultra–High Frequency Data (UHFD) based volatility measures computed each day using data sampled at different frequen-

[1]In what follows we will study the dynamics of the variance of asset returns and we will follow the widespread convention (cf. Andersen, Bollerslev & Diebold (2007)) to refer to corresponding measures as *volatility measures*.

cies: realized volatility (Andersen & Bollerslev (1998), Andersen et al. (2003)), bipower realized volatility (Barndorff-Nielsen & Shephard (2004)), two scales realized volatility (Zhang, Mykland & Aït-Sahalia (2005)) and realized kernels (Barndorff-Nielsen, Hansen, Lunde & Shephard (2006)). We adopt a risk management framework using a two-step VaR prediction procedure. The first step consists of specifying the dynamics of the volatility measures with a Multiplicative Error Model (MEM) (cf. Engle (2002), Engle & Gallo (2006)) and the novel P–Spline MEM (building on Engle & Rangel (2008) and Eilers & Marx (1996)), which combines volatility clustering with a flexible specification of the volatility trend. The second step consists of modeling returns using a conditional heteroskedastic model based on the volatility predictions from different measures. We then evaluate the VaR performance assessing the accuracy and adequacy of VaR forecasts against a GARCH benchmark.

The out–of–sample VaR forecasting results on a sample of NYSE blue chips hint that UHFD volatility measures are more accurate than the benchmark model but they do not outperform the range. For realized volatility, bipower realized volatility and two scales realized volatility we find that in most cases the sampling frequency of the intra-daily data plays a bigger role in forecasting than the choice of the UHFD volatility measures and "low" frequencies (20/30 minutes) perform better than "high" frequencies (30 seconds/1 minute). The realized kernel paired with the P-Spline MEM on the other hand usually performs better than the other measures at high frequencies and is fairly insensitive to the choice of the sampling frequency. The P–Spline MEM captures satisfactorily the series dynamics and it systematically improves out–of–sample forecasting ability over simpler specifications. The in–sample volatility modeling results show that realized kernels provide the most precise estimate of the returns variance followed by two scales realized volatility, realized volatility and bipower realized volatility. Our findings are consistent with the claim that at high frequencies microstructure dynamics bias volatility dynamics; moreover, it is advantageous to use volatility measures robust to dependent microstructure noise such as realized kernels but there are limited gains in sampling at very high frequencies.

The closest contributions to our paper are the work by Andersen et al. (2003), Giot & Laurent (2004) and Clements, Galvao & Kim (2006) that contain VaR forecasting applications using realized volatility. Initial work on realized volatility includes Zhou (1996), Andersen & Bollerslev (1998), Andersen et al. (2001), Barndorff-Nielsen & Shephard (2002), Meddahi (2002) and Andersen et al. (2003). Recent extensions and refinement of the early results are found in, *inter alia*, Bandi & Russell (2007), Oomen (2005), Zhang (2006), Hansen & Lunde (2006), Barndorff-Nielsen et al. (2006), Christensen & Podolskij (2006). Stylized facts on equity UHFD are described in Andersen et al. (2001), Ebens (1999) and Hansen & Lunde (2006). MEMs are a generalization of the ARCH and ACD for modeling nonnegative time series proposed in Engle (2002) and have had a significant application in comparing different volatility indicators in Engle & Gallo (2006). Further extensions and applications are presented in Chou (2005), Cipollini, Engle & Gallo (2006), Lanne (2006) and Brunetti & Lilholdt (2007). Engle & Rangel (2008) proposed the spline modeling approach for capturing volatility trends. The P-Spline modeling approach was proposed in the context of smoothing in the GLM framework by Eilers & Marx (1996) and Marx & Eilers (1998). This type of modeling different frequencies of evolution of volatility is alternative to traditional approaches which take long-range dependence into account in the form of ARFIMA–type of models on the logarithm of realized volatility (e.g. Andersen et al. (2003), Martens & Zein (2004), Koopman, Jungbacker & Hol (2005), Deo, Hurvich & Lu (2006), Pong, Shacketon, Taylor & Xu (2004)) and regression models mixing information at different frequencies (e.g. the so called Heterogeneous AR (HAR) model of Corsi (2004) extended by Andersen et al. (2007) and Ghysels, Santa-Clara & Valkanov (2006) in a MIDAS framework). The literature on the evaluation of the VaR forecasts includes the contributions by Christoffersen (1998), Sarma, Thomas & Shah (2003), Engle & Manganelli (2004), Giacomini & Komunjer (2005) and Kuester, Mittnik & Paolella (2006).

The rest of the paper is organized as follows. Section 2 describes the VaR modeling framework based on volatility measures. Section 3 defines the volatility measures used in this work and summarizes the stylized fact of the series. Section 4 discusses the dynamic specifications for the volatility measures. Section 5 discusses a conditionally heteroskedastic model for returns based on the volatility measures. Section 6 presents the VaR forecasting results. Concluding remarks follow in Section 7.

## 2 A Value–at–Risk Framework for the Comparison

There is a wide variety of methods for forecasting VaR in the literature: Historical Simulation, Extreme Value Theory, Conditional Autoregressive Value at Risk (CAViaR) and so forth. Kuester et al. (2006) contains a review and comparison of many proposals.

Our VaR modeling approach builds up on the contribution of Giot & Laurent (2004) for forecasting VaR using realized volatility. Let $r_t$ be the daily (close–to–close) return at time $t$ on a single asset. We assume that

$$r_t = \sqrt{h_t}\, z_t, \qquad z_t \sim F,$$

where $h_t$ is the conditional variance of the daily return at time $t$ and $z_t$ is an i.i.d. unit variance and possibly skewed and leptokurtic random variable from some appropriate cumulative distribution $F$. The 1 day ahead 100(1-p)% VaR is defined as the maximum 1 day ahead loss, that is

$$\mathsf{VaR}^p_{t|t-1} \equiv -F^{-1}(p)\sqrt{h_t},$$

assuming that $h_t$ is known conditional on the information available at time $t-1$. In a GARCH framework one would model the conditional variance of returns, project it one day ahead and use some distributional assumption on $F$ (either parametric or empirical based) to provide the proper quantile of the distribution of the standardized residuals.

If a series for a return variance proxy is directly available, one can depart from this standard procedure. Let $rv_{(m,\delta)\,t}$ denote such a generic proxy computed according to definition $m$ using intra-daily data sampled at frequency $\delta$ on day $t$ and let $rv_{(m,\delta)\,t|t-1}$ denote its expectation conditional on the information available at time $t-1$, using some suitable model specification. Then we assume that the conditional variance of returns is some function of $rv_{(m,\delta)\,t|t-1}$ and a vector of unknown parameters $\varphi$:

$$h_t = f(\, rv_{(m,\delta)\,t|t-1} \mid \varphi \,).$$

In order to work within this framework we need to specify (i) a model that captures the dynamics of the volatility measures in order to obtain the conditional expectations of volatility, (ii) a model that connects the conditional variance of returns with the conditional expectation of the volatility measures and (iii) an appropriate distribution for the standardized return distribution.

# 3 Definitions and Stylized Facts

The intuition behind UHFD volatility measures dates at least back to Merton (1980). Authors including Andersen, Bollerslev, Diebold brought back the idea in the mid–90's in correspondence with the availability of large databases containing detailed information of financial transactions in several financial markets.

## 3.1 Volatility Measures

The building blocks of the UHFD volatility measures are intra–daily prices. Let $p_{i,t}$ denote the $i$-th intra–daily log–price of day $t$ sampled at frequency $\delta$ (thus $i = 1, \ldots, n(\delta) = n_{\mathsf{sec}}/\delta$ with $n_{\mathsf{sec}}$ denoting the number of seconds in a trading day). The intra–daily price series are constructed using either Calendar Time Sampling (CTS) or Tick Time Sampling (TTS)[2]. In CTS, one takes the last recorded tick-by-tick price every $\delta$ units of time starting from an initial time of the day (typically the opening) until the closing: for example, sampling every minute delivers $n(1\,\mathtt{min}) = 390$ for a market such as the NYSE open between 9:30am and 4:00pm. In TTS, the series is sampled every $d$ ticks. For the ease of comparison, we follow the convention to express the TTS frequency in terms of units of time like in CTS; following Hansen & Lunde (2006), one follows the sampling scheme

$$d_t = \left\lceil 1 + \frac{n_{\mathsf{tick},t}}{n(\delta)} \right\rceil,$$

where $n_{\mathsf{tick},t}$ denotes the number of ticks in day $t$ and $\lceil \cdot \rceil$ is the ceiling function. Note that overnight information is not included in these series and this has to be taken into account in the modeling of daily (close–to–close) returns (c.f. Gallo (2001), Martens (2002), Fleming, Kirby & Ostdiek (2003) and Hansen & Lunde (2005)).

[2] Recently, a number of researchers have claimed that sampling in tick time is more appropriate than sampling in calendar time, see also Renault & Werker (2004).

The **Realized Volatility** (Andersen et al. (2001)) has become the benchmark UHFD volatility measure, commonly used in applied work. It is defined as

$$rv_{(\mathsf{V},\delta)\,t} \equiv \sum_{i=2}^{n(\delta)} (p_{i,t} - p_{i-1,t})^2.$$

Under appropriate assumptions including the absence of jumps and microstructure noise, $rv_{(\mathsf{V},\delta),t}$ convergences to the latent volatility as the sampling frequency increases.

The **Bipower Realized Volatility** (Barndorff-Nielsen & Shephard (2004)) was proposed as a robust UHFD volatility measure in the presence of *infrequent jumps*. It is defined as

$$rv_{(\mathsf{B},\delta)\,t} \equiv \frac{\pi}{2} \sum_{i=3}^{n(\delta)} |p_{i,t} - p_{i-1,t}||p_{i-1,t} - p_{i-2,t}|.$$

Under appropriate assumptions including the absence of microstructure noise, bipower realized volatility converges to the latent volatility while realized volatility converges to the latent volatility plus a component depending on the jumps.

The **Two Scales Realized Volatility** (Zhang et al. (2005)) is the first consistent estimator in the presence of *independent* microstructure noise. The definition of this measure requires some further notation. Let $p^{\mathsf{f}}_{i,t}$ denote the $i$–th intra–daily log–price of day $t$ sampled at some "very high" fixed frequency $\delta_{\mathsf{f}}$ and let $p^{g}_{j,t} = p_{g+(\delta/\delta_{\mathsf{f}})(j-1),t}$, with $g = 1, ..., G$ and $G = \delta/\delta_{\mathsf{f}}$, denote the intra–daily log–price series obtained by sampling observations from $p_{i,t}$ at frequency $\delta$ starting from $G$ different initial times of day. Define $rv^{g}_{(\mathsf{V},\delta)\,t} \equiv \sum_{j=2}^{n(\delta)_g} (p^{g}_{j,t} - p^{g}_{j-1,t})^2$ and $rv_{(\mathsf{V},\delta_{\mathsf{f}})\,t} \equiv \sum_{i=1}^{n} (p^{\mathsf{f}}_{i,t} - p^{\mathsf{f}}_{i-1,t})^2$. Then the two scales realized volatility is defined as

$$rv_{(\mathsf{TS},\delta)\,t} \equiv \frac{1}{G} \sum_{g=1}^{G} rv^{g}_{(\mathsf{V},\delta)\,t} - \frac{n(\delta)_g}{n} rv_{(\mathsf{V},\delta_{\mathsf{f}})\,t}.$$

The expression "two scales" derives from the fact that this estimator combines the information from a slow ($\delta$) and fast ($\delta_{\mathsf{f}}$) time scale. Under appropriate assumptions ($\delta \to 0$ and $\delta^2/\delta_{\mathsf{f}} \to 0$), $rv_{(\mathsf{TS},\delta)\,t}$ converges to the latent volatility while realized volatility diverges to infinity.

The **Realized Kernel** (Barndorff-Nielsen et al. (2006)) is an estimator of the latent volatility analogous to the HAC estimator of the long-run variance of a stationary time series and is robust to dependent microstructure noise. It is defined as

$$rv_{(\mathsf{K},\delta)} \equiv \gamma_0(p_t) + \sum_{h=1}^{H} k\left(\frac{h-1}{H}\right)\{\gamma_h(p_t) + \gamma_{-h}(p_t)\},$$

where $\gamma_h(p_t)$ is equal to $\sum_{i=1}^{n(\delta)}(p_{i,t} - p_{i-1,t})(p_{i-h,t} - p_{i-h-1,t})$ and $k(\cdot)$ is some appropriate weight function. As the sampling frequency increases, and by choosing the $k(\cdot)$ weight function appropriately, the realized kernel is consistent and can attain the fastest convergence rate.

For comparison purposes we also consider the daily **Range** defined as

$$rv_{(\mathsf{R})\,t} \equiv \frac{1}{4\log(2)}(p_{\mathsf{high},t} - p_{\mathsf{low},t})^2,$$

where $p_{\mathsf{high},t}$ and $p_{\mathsf{low},t}$ are respectively the max and min log intra-daily prices of day $t$.

## 3.2 Data and Stylized Facts

Our empirical investigation is carried out on three NYSE stocks: Boeing (BA), General Electric (GE) and Johnson and Johnson (JNJ). The data is extracted from the NYSE-TAQ database. All the series analyzed in this study are derived from "cleaned" (c.f. Brownlees & Gallo (2006)) mid quotes from the NYSE between 9:30 and 16:05 for 12 intra–daily frequencies ranging from 30 seconds to 1 hour[3]. The realized volatility, bipower realized volatility and two scale realized volatility series follow CTS, while the realized kernel series follow TTS. The Two Scales Realized volatility is computed using the "high" fixed frequency equal to 15 seconds. The realized kernel is computed using the Modified Tukey-Hanning kernel (c.f. Barndorff-Nielsen et al. (2006)) and $H = 2$. The sample period is from February 2001 to December 2006 and contains 1465 daily observations. The analysis of these years is challenging in that this sample contains periods of very high volatility (early 2000s recession following the collapse of the Dot-com bubble, 9/11) followed by a period of very low volatility. Moreover, at the end of January 2001 the NYSE changed its ticksize and this event is likely to have had some impact on the empirical properties of the UHFD volatility measures established in studies

[3]The frequencies are: 30s, 1m, 2m, 3m, 4m, 5m, 6m, 10m, 15m, 20m, 30m and 1h.

on earlier samples.

Table 1, 2 and 3 about here.

Tables 1, 2 and 3 report some descriptive statistics on the series used in the analysis. It is worthwhile to pinpoint some features of the data that we use as guidance for the subsequent modeling effort:

- Upon visual inspection of the graphs (Figure 2), volatility clustering occurs around a changing level in average volatility (higher in the early part of the sample).
- The persistence and shape of the UHFD volatility measures appear to be frequency dependent. Serial correlation is higher at higher frequencies while the standard deviation decreases.
- Since the mean of realized volatility across sampling frequencies in excess of 30 seconds is substantially constant, it seems that the impact of *independent* microstructure noise for these series is less noticeable than in earlier/other datasets (c.f. Hansen & Lunde (2006), Barndorff-Nielsen et al. (2006)).
- There is evidence of *dependence* in microstructure noise as shown by the increase in the serial correlation and cross correlation between UHFD measures at higher frequencies, translating into the presence of biases (c.f. Hansen & Lunde (2006), Barndorff-Nielsen et al. (2006)).
- Almost all volatility measures systematically underestimate the variance of returns. This is due to the fact that the volatility measures are based only on *intra–daily* information while the daily return is made of an *intra–daily* and an *overnight* component.
- Daily returns standardized by the square root of the volatility measures do not exhibit ARCH effects but do not always appear to be normal.

# 4 Modeling Volatility Measures

The volatility measures exhibit different features according to the sampling frequency of the UHFD. They can be conveniently modelled and predicted referring to the MEM class imposing a reasonable

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| Meas. | Freq. | $rv_{(m,\delta)\,t}$ Mean | Std.Dev. | $\hat{\rho}_1$ | $\hat{\rho}_{m,\mathsf{V}}$ | $r_t/rv^{1/2}_{(m,\delta)\,t}$ Skew. | Kurt. | $Q^2_{10}$ |
|---|---|---|---|---|---|---|---|---|
| V | 30s | 2.61 | 2.63 | 0.81 | | 0.05 | 2.99 | 0.219 |
| | 1m | 2.80 | 2.91 | 0.82 | | 0.06 | 2.94 | 0.353 |
| | 2m | 2.92 | 3.15 | 0.81 | | 0.09 | 2.84 | 0.734 |
| | 3m | 2.95 | 3.29 | 0.76 | | 0.06 | 2.84 | 0.791 |
| | 4m | 2.92 | 3.31 | 0.77 | | 0.06 | 2.78* | 0.883 |
| | 5m | 2.87 | 3.32 | 0.77 | | 0.03 | 2.76** | 0.858 |
| | 6m | 2.83 | 3.38 | 0.71 | | 0.07 | 2.77** | 0.956 |
| | 10m | 2.69 | 3.35 | 0.71 | | 0.08 | 2.78* | 0.906 |
| | 15m | 2.61 | 3.27 | 0.59 | | 0.09 | 2.96 | 0.920 |
| | 20m | 2.54 | 3.48 | 0.58 | | 0.14 | 2.95 | 0.510 |
| | 30m | 2.47 | 3.84 | 0.40 | | 0.09 | 3.36** | 0.447 |
| | 1h | 2.04 | 3.20 | 0.42 | | 0.37*** | 5.38*** | 0.628 |
| B | 30s | 2.07 | 2.01 | 0.79 | 0.99 | 0.04 | 3.00 | 0.013 |
| | 1m | 2.48 | 2.57 | 0.80 | 0.99 | 0.05 | 2.97 | 0.143 |
| | 2m | 2.77 | 3.00 | 0.81 | 0.99 | 0.08 | 2.84 | 0.561 |
| | 3m | 2.82 | 3.14 | 0.78 | 0.99 | 0.03 | 2.91 | 0.460 |
| | 4m | 2.82 | 3.21 | 0.77 | 0.99 | 0.04 | 2.83 | 0.670 |
| | 5m | 2.80 | 3.26 | 0.77 | 0.99 | 0.03 | 2.80* | 0.565 |
| | 6m | 2.77 | 3.30 | 0.71 | 0.99 | 0.06 | 2.83 | 0.877 |
| | 10m | 2.64 | 3.43 | 0.66 | 0.99 | 0.11 | 2.83 | 0.942 |
| | 15m | 2.51 | 3.20 | 0.59 | 0.98 | 0.07 | 3.05 | 0.832 |
| | 20m | 2.44 | 3.32 | 0.53 | 0.98 | 0.17* | 2.99 | 0.330 |
| | 30m | 2.34 | 3.60 | 0.37 | 0.98 | -0.03 | 4.80*** | 0.881 |
| | 1h | 1.94 | 3.09 | 0.37 | 0.97 | 0.55*** | 6.65*** | 0.655 |
| TS | 30s | 4.83 | 4.69 | 0.82 | 0.99 | 0.04 | 3.05 | 0.063 |
| | 1m | 3.56 | 3.62 | 0.82 | 0.99 | 0.05 | 2.96 | 0.245 |
| | 2m | 3.26 | 3.46 | 0.80 | 0.98 | 0.05 | 2.87 | 0.572 |
| | 3m | 3.14 | 3.43 | 0.77 | 0.98 | 0.04 | 2.80 | 0.752 |
| | 4m | 3.06 | 3.40 | 0.76 | 0.98 | 0.04 | 2.78* | 0.826 |
| | 5m | 3.00 | 3.38 | 0.73 | 0.98 | 0.04 | 2.76** | 0.907 |
| | 6m | 2.96 | 3.41 | 0.73 | 0.97 | 0.04 | 2.71*** | 0.896 |
| | 10m | 2.86 | 3.44 | 0.69 | 0.96 | 0.04 | 2.67*** | 0.905 |
| | 15m | 2.79 | 3.46 | 0.63 | 0.94 | 0.03 | 2.71*** | 0.908 |
| | 20m | 2.73 | 3.48 | 0.60 | 0.92 | 0.04 | 2.80* | 0.853 |
| | 30m | 2.67 | 3.52 | 0.54 | 0.91 | 0.03 | 2.92 | 0.732 |
| | 1h | 2.45 | 3.63 | 0.43 | 0.77 | 0.08 | 3.12 | 0.608 |
| K | 30s | 3.32 | 3.75 | 0.79 | 0.95 | 0.07 | 2.83 | 0.755 |
| | 1m | 3.25 | 3.80 | 0.74 | 0.93 | 0.05 | 2.80* | 0.815 |
| | 2m | 3.14 | 3.78 | 0.67 | 0.92 | 0.05 | 2.82 | 0.781 |
| | 3m | 3.10 | 3.66 | 0.62 | 0.91 | 0.06 | 2.79* | 0.745 |
| | 4m | 3.07 | 3.63 | 0.59 | 0.90 | 0.10 | 2.77** | 0.500 |
| | 5m | 3.05 | 3.74 | 0.56 | 0.90 | 0.07 | 2.89 | 0.664 |
| | 6m | 3.08 | 3.83 | 0.59 | 0.90 | 0.07 | 2.81 | 0.846 |
| | 10m | 3.14 | 4.14 | 0.55 | 0.86 | 0.05 | 2.85 | 0.685 |
| | 15m | 3.23 | 4.43 | 0.47 | 0.82 | 0.09 | 3.10 | 0.845 |
| | 20m | 3.35 | 4.90 | 0.41 | 0.83 | 0.08 | 3.02 | 0.934 |
| | 30m | 3.37 | 4.84 | 0.42 | 0.75 | 0.05 | 3.86*** | 0.524 |
| | 1h | 3.58 | 5.50 | 0.36 | 0.75 | -0.10 | 9.39*** | 0.964 |
| R | | 2.80 | 3.82 | 0.51 | | 0.07 | 2.46*** | 0.572 |
| $r_t^2$ | | 3.74 | 12.3 | 0.17 | | | | |

Table 1: Descriptive statistics of the volatility measures. For each volatility measure and sampling frequency (when applicable) the table reports mean, standard deviation, skewness, kurtosis, first order autocorrelation coefficient, correlation with realized volatility $rv_{(\mathsf{V},\delta)}$ (computed at the same frequency). The second section pertains to returns standardized by the square root of the volatility measure reports with skewness and kurtosis, as well as the p–value of the Ljung-Box statistics ($Q^2_{10}$) computed on their squares. The last row of the table reports average, standard deviation and first order autocorrelation coefficient of the squared returns.

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| | | $rv_{(m,\delta)\,t}$ | | | | $r_t/rv^{1/2}_{(m,\delta)\,t}$ | | |
|---|---|---|---|---|---|---|---|---|
| Meas. | Freq. | Mean | Std.Dev. | $\hat{\rho}_1$ | $\hat{\rho}_{m,\mathsf{V}}$ | Skew. | Kurt. | $Q^2_{10}$ |
| V | 30s | 1.94 | 2.38 | 0.79 | | 0.21** | 3.44*** | 0.825 |
| | 1m | 2.13 | 2.72 | 0.78 | | 0.22** | 3.28** | 0.795 |
| | 2m | 2.27 | 3.08 | 0.74 | | 0.22** | 3.27** | 0.686 |
| | 3m | 2.29 | 3.25 | 0.72 | | 0.22** | 3.26* | 0.611 |
| | 4m | 2.30 | 3.40 | 0.68 | | 0.22** | 3.28** | 0.504 |
| | 5m | 2.29 | 3.44 | 0.65 | | 0.22** | 3.19 | 0.407 |
| | 6m | 2.27 | 3.42 | 0.65 | | 0.18* | 3.18 | 0.609 |
| | 10m | 2.17 | 3.27 | 0.59 | | 0.19* | 3.29** | 0.142 |
| | 15m | 2.13 | 3.67 | 0.46 | | 0.13 | 3.44*** | 0.114 |
| | 20m | 2.12 | 3.45 | 0.46 | | 0.15 | 3.45*** | 0.130 |
| | 30m | 2.05 | 3.62 | 0.43 | | 0.06 | 3.81*** | 0.659 |
| | 1h | 1.80 | 3.20 | 0.38 | | 0.04 | 4.79*** | 0.490 |
| B | 30s | 1.67 | 2.09 | 0.81 | 1.00 | 0.20** | 3.43*** | 0.860 |
| | 1m | 1.98 | 2.59 | 0.79 | 1.00 | 0.23** | 3.26* | 0.785 |
| | 2m | 2.18 | 2.99 | 0.75 | 1.00 | 0.21** | 3.25* | 0.590 |
| | 3m | 2.23 | 3.33 | 0.69 | 0.99 | 0.21** | 3.26* | 0.624 |
| | 4m | 2.25 | 3.47 | 0.65 | 0.99 | 0.22** | 3.25* | 0.536 |
| | 5m | 2.22 | 3.47 | 0.63 | 0.99 | 0.25** | 3.27** | 0.44 |
| | 6m | 2.21 | 3.50 | 0.63 | 0.99 | 0.19* | 3.23* | 0.524 |
| | 10m | 2.09 | 3.22 | 0.60 | 0.99 | 0.20** | 3.39*** | 0.152 |
| | 15m | 2.05 | 3.79 | 0.43 | 0.99 | 0.21** | 3.67*** | 0.162 |
| | 20m | 2.05 | 3.25 | 0.47 | 0.97 | 0.18* | 3.52*** | 0.097 |
| | 30m | 1.91 | 3.40 | 0.43 | 0.98 | -0.04 | 5.38*** | 0.051 |
| | 1h | 1.67 | 3.09 | 0.40 | 0.97 | 0.44*** | 9.91*** | 0.882 |
| TS | 30s | 3.71 | 4.63 | 0.76 | 1.00 | 0.22** | 3.41*** | 0.759 |
| | 1m | 2.78 | 3.64 | 0.75 | 0.99 | 0.23** | 3.28** | 0.760 |
| | 2m | 2.55 | 3.50 | 0.73 | 0.99 | 0.25** | 3.23* | 0.722 |
| | 3m | 2.48 | 3.48 | 0.71 | 0.99 | 0.24** | 3.23* | 0.637 |
| | 4m | 2.43 | 3.51 | 0.69 | 0.99 | 0.24** | 3.21 | 0.582 |
| | 5m | 2.40 | 3.56 | 0.66 | 0.99 | 0.23** | 3.19 | 0.522 |
| | 6m | 2.35 | 3.49 | 0.64 | 0.98 | 0.22** | 3.18 | 0.483 |
| | 10m | 2.27 | 3.45 | 0.59 | 0.97 | 0.19* | 3.19 | 0.267 |
| | 15m | 2.22 | 3.42 | 0.54 | 0.97 | 0.14 | 3.12 | 0.178 |
| | 20m | 2.19 | 3.41 | 0.53 | 0.96 | 0.12 | 3.13 | 0.164 |
| | 30m | 2.14 | 3.47 | 0.48 | 0.93 | 0.08 | 3.23* | 0.192 |
| | 1h | 1.41 | 2.61 | 0.35 | 0.74 | 0.06 | 3.68*** | 0.360 |
| K | 30s | 2.64 | 3.9 | 0.70 | 0.97 | 0.24** | 3.19 | 0.551 |
| | 1m | 2.54 | 3.89 | 0.67 | 0.94 | 0.25** | 3.33** | 0.403 |
| | 2m | 2.4 | 3.93 | 0.55 | 0.93 | 0.24** | 3.39*** | 0.334 |
| | 3m | 2.36 | 3.86 | 0.56 | 0.95 | 0.21** | 3.37** | 0.388 |
| | 4m | 2.37 | 3.94 | 0.49 | 0.93 | 0.20** | 3.29** | 0.418 |
| | 5m | 2.31 | 3.73 | 0.50 | 0.92 | 0.17* | 3.55*** | 0.243 |
| | 6m | 2.32 | 3.64 | 0.51 | 0.91 | 0.16* | 3.57*** | 0.263 |
| | 10m | 2.40 | 3.87 | 0.43 | 0.86 | 0.09 | 3.43*** | 0.141 |
| | 15m | 2.42 | 3.94 | 0.44 | 0.86 | 0.10 | 3.52*** | 0.372 |
| | 20m | 2.53 | 4.26 | 0.40 | 0.86 | 0.17* | 3.50*** | 0.194 |
| | 30m | 2.48 | 4.29 | 0.34 | 0.79 | -0.04 | 5.33*** | 0.789 |
| | 1h | 2.71 | 5.52 | 0.27 | 0.72 | 4.67*** | 87.76*** | 1.000 |
| R | | 2.43 | 4.64 | 0.41 | | 0.15 | 2.67*** | 0.045 |
| $r^2_t$ | | 3.03 | 8.65 | 0.17 | | | | |

Table 2: Descriptive statistics of the volatility measures. For each volatility measure and sampling frequency (when applicable) the table reports mean, standard deviation, skewness, kurtosis, first order autocorrelation coefficient, correlation with realized volatility $rv_{(\mathsf{V},\delta)}$ (computed at the same frequency). The second section pertains to returns standardized by the square root of the volatility measure reports with skewness and kurtosis, as well as the p–value of the Ljung-Box statistics ($Q^2_{10}$) computed on their squares. The last row of the table reports average, standard deviation and first order autocorrelation coefficient of the squared returns.

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| Meas. | Freq. | Mean | Std.Dev. | $\hat{\rho}_1$ | $\hat{\rho}_{m,\mathsf{V}}$ | Skew. | Kurt. | $Q^2_{10}$ |
|---|---|---|---|---|---|---|---|---|
| | | $rv_{(m,\delta)\,t}$ | | | | $r_t/rv^{1/2}_{(m,\delta)\,t}$ | | |
| V | 30s | 1.17 | 1.18 | 0.78 | | 0.11 | 4.20*** | 0.758 |
| | 1m | 1.33 | 1.51 | 0.76 | | 0.10 | 3.98*** | 0.864 |
| | 2m | 1.47 | 1.89 | 0.73 | | 0.11 | 3.72*** | 0.853 |
| | 3m | 1.50 | 2.08 | 0.71 | | 0.11 | 3.67*** | 0.765 |
| | 4m | 1.51 | 2.22 | 0.70 | | 0.10 | 3.49*** | 0.811 |
| | 5m | 1.51 | 2.33 | 0.72 | | 0.08 | 3.34** | 0.819 |
| | 6m | 1.50 | 2.59 | 0.62 | | 0.08 | 3.36** | 0.704 |
| | 10m | 1.41 | 2.43 | 0.55 | | 0.05 | 3.61*** | 0.504 |
| | 15m | 1.36 | 2.18 | 0.53 | | -0.01 | 3.70*** | 0.581 |
| | 20m | 1.33 | 2.26 | 0.48 | | 0.02 | 3.66*** | 0.631 |
| | 30m | 1.25 | 2.03 | 0.48 | | -0.08 | 4.26*** | 0.656 |
| | 1h | 0.99 | 2.05 | 0.29 | | -0.10 | 4.92*** | 0.742 |
| B | 30s | 0.95 | 0.93 | 0.77 | 0.99 | 0.09 | 4.20*** | 0.786 |
| | 1m | 1.2 | 1.38 | 0.77 | 0.99 | 0.12 | 3.97*** | 0.940 |
| | 2m | 1.41 | 1.87 | 0.75 | 0.99 | 0.14 | 3.83*** | 0.858 |
| | 3m | 1.46 | 2.08 | 0.70 | 0.99 | 0.12 | 3.67*** | 0.801 |
| | 4m | 1.47 | 2.24 | 0.68 | 0.99 | 0.10 | 3.40*** | 0.817 |
| | 5m | 1.46 | 2.37 | 0.66 | 0.99 | 0.06 | 3.34** | 0.825 |
| | 6m | 1.46 | 2.75 | 0.56 | 0.99 | 0.05 | 3.33** | 0.856 |
| | 10m | 1.36 | 2.47 | 0.52 | 0.99 | 0.08 | 3.55*** | 0.390 |
| | 15m | 1.32 | 2.23 | 0.49 | 0.98 | 0.06 | 3.93*** | 0.455 |
| | 20m | 1.29 | 2.26 | 0.45 | 0.98 | 0.01 | 3.58*** | 0.565 |
| | 30m | 1.19 | 2.16 | 0.43 | 0.98 | -0.09 | 4.48*** | 0.657 |
| | 1h | 0.91 | 1.79 | 0.29 | 0.97 | -0.58*** | 7.51*** | 0.869 |
| TS | 30s | 2.20 | 2.24 | 0.78 | 1.00 | 0.11 | 4.13*** | 0.773 |
| | 1m | 1.73 | 2.02 | 0.77 | 1.00 | 0.10 | 3.86*** | 0.873 |
| | 2m | 1.65 | 2.20 | 0.75 | 0.99 | 0.09 | 3.61*** | 0.843 |
| | 3m | 1.62 | 2.32 | 0.72 | 0.99 | 0.08 | 3.51*** | 0.815 |
| | 4m | 1.59 | 2.40 | 0.70 | 0.99 | 0.07 | 3.45*** | 0.815 |
| | 5m | 1.56 | 2.41 | 0.68 | 0.99 | 0.07 | 3.36** | 0.854 |
| | 6m | 1.53 | 2.43 | 0.66 | 0.98 | 0.07 | 3.32** | 0.878 |
| | 10m | 1.46 | 2.45 | 0.61 | 0.97 | 0.05 | 3.27** | 0.889 |
| | 15m | 1.41 | 2.28 | 0.59 | 0.94 | 0.05 | 3.39*** | 0.883 |
| | 20m | 1.37 | 2.16 | 0.55 | 0.95 | 0.04 | 3.32** | 0.862 |
| | 30m | 1.30 | 2.03 | 0.52 | 0.92 | -0.02 | 3.47*** | 0.831 |
| | 1h | 0.65 | 1.24 | 0.34 | 0.80 | -0.04 | 3.89*** | 0.352 |
| $K$ | 30s | 1.76 | 2.46 | 0.75 | 0.98 | 0.07 | 3.53*** | 0.689 |
| | 1m | 1.75 | 2.79 | 0.71 | 0.95 | 0.07 | 3.54*** | 0.700 |
| | 2m | 1.60 | 2.90 | 0.63 | 0.93 | 0.10 | 3.31** | 0.653 |
| | 3m | 1.57 | 2.88 | 0.59 | 0.93 | 0.03 | 3.50*** | 0.891 |
| | 4m | 1.56 | 2.81 | 0.55 | 0.94 | 0.06 | 3.44*** | 0.759 |
| | 5m | 1.57 | 2.84 | 0.57 | 0.95 | 0.08 | 3.38*** | 0.672 |
| | 6m | 1.54 | 2.65 | 0.56 | 0.94 | 0.06 | 3.67*** | 0.737 |
| | 10m | 1.54 | 2.54 | 0.54 | 0.89 | -0.01 | 3.46*** | 0.716 |
| | 15m | 1.63 | 3.25 | 0.45 | 0.89 | -0.09 | 3.75*** | 0.537 |
| | 20m | 1.70 | 3.49 | 0.35 | 0.85 | -0.03 | 3.46*** | 0.734 |
| | 30m | 1.61 | 4.06 | 0.25 | 0.75 | -0.20** | 3.97*** | 0.331 |
| | 1h | 1.70 | 4.47 | 0.16 | 0.73 | -0.24** | 5.34*** | 0.084 |
| R | | 1.36 | 2.23 | 0.47 | | 0.02 | 2.73** | 0.656 |
| $r_t^2$ | | 1.68 | 8.4 | 0.12 | | | | |

Table 3: Descriptive statistics of the volatility measures. For each volatility measure and sampling frequency (when applicable) the table reports mean, standard deviation, skewness, kurtosis, first order autocorrelation coefficient, correlation with realized volatility $rv_{(\mathsf{V},\delta)}$ (computed at the same frequency). The second section pertains to returns standardized by the square root of the volatility measure reports with skewness and kurtosis, as well as the p–value of the Ljung-Box statistics ($Q^2_{10}$) computed on their squares. The last row of the table reports average, standard deviation and first order autocorrelation coefficient of the squared returns.

amount of assumptions on the data.

## 4.1 A Family of Dynamic Models for Volatility Measures

Let the Multiplicative Error Model for the volatility measure $m$ sampled at frequency $\delta$, $rv_{(m,\delta)\ t}$, be defined as

$$rv_{(m,\delta)\,t} = \sigma^2_{(m,\delta)\,t}\,\varepsilon_{(m,\delta)\,t}, \tag{1}$$

where, conditional on the information set at $t-1$, $\mathcal{F}_{t-1}$, $\sigma^2_{(m,\delta)\ t}$ is a nonnegative predictable process function of a vector of parameters $\phi$,

$$\sigma^2_{(m,\delta)\ t} = \sigma^2_{(m,\delta)\ t}(\theta);$$

and $\varepsilon_{(m,\delta)\ t}$ is an iid innovation term with unit expected value

$$\varepsilon_{(m,\delta)\ t}|\mathcal{F}_{t-1} \sim Gamma(\phi, 1/\phi).$$

It then follows from standard properties of the gamma distribution that conditional on time $t$, the volatility measure is distributed as

$$rv_{(m,\delta)\,t+1}|\mathcal{F}_t \sim Gamma\left(\phi, \sigma^2_{(m,\delta)\,t+1}/\phi\right),$$

and its conditional expectation is

$$\mathsf{E}(rv_{(m,\delta)\,t+1}|\mathcal{F}_t) \equiv rv_{(m,\delta)\,t+1|t} = \sigma^2_{(m,\delta)\,t+1}.$$

Discussions and extensions on the properties of this model class can be found in Engle (2002), Engle & Gallo (2006), Cipollini et al. (2006).

The are a number of reasons why we argue that MEMs are a suitable specification for modeling volatility measures. The MEM is a nonnegative time series model and hence it always produces nonnegative predictions. Contrary to what happens when working with logs, it provides unbiased predictions without the need to transform forecasts. The Gamma distributional assumption is rather flexible depending on the shape parameter $\phi$, which does not affect the estimation of $\theta$ (cf. Engle & Gallo (2006)). Moreover, if the conditional expectation of the volatility measure is correctly specified, the expected value of the score of $\theta$ evaluated at the true parameters is zero irrespective of the Gamma assumption, that is, the ML estimator for $\theta$ is a QML estimator (White (1994)).

## 4.2 Base MEM

The challenge for successful forecasting lies in choosing an appropriate specification for $\sigma^2_{(m,\delta)\,t}$ in Equation (1). The Base MEM specification is

$$\sigma^2_{(m,\delta)\,t} = \omega + \alpha rv_{(m,\delta)\,t-1} + \beta\sigma^2_{(m,\delta)\,t-1} + \alpha^- rv^-_{(m,\delta)\,t-1} \tag{2}$$

with $rv^-_{(m,\delta)\,t-1} \equiv rv_{(m,\delta)\,t-1} 1_{\{r_{t-1}<0\}}$. It represents the analog of the GARCH(1,1) model with leverage effects (Glosten, Jagannanthan & Runkle (1993)) and it is estimated over the whole sample via maximum likelihood.

Tables 4, 5 and 6 about here.

Tables 4, 5 and 6 report the parameter estimates and residual diagnostics. The model is not always able to capture the dynamics of the series as the Ljung–Box test statistic is sometimes significant at standard levels. The GE residuals are quite dirty while the BA and JNJ residuals are much better behaved. Interestingly, evidence of autocorrelation in the GE residuals decreases as the sampling frequency decreases. The estimation results exhibit IGARCH type effects: the estimated persistence of shocks varies between 0.97 to 1.00. The shape of the innovation distribution changes with the sampling frequency: the higher the frequency, the more mound-shaped it is.

## 4.3 P-Spline MEM

The evidence of a unit root is consistent with long range dependence in the series; one can capture this empirical feature by specifying a trend component in the volatility dynamics. Early theoretical and

## BA

| | | Base | | | P-Spline | | | |
|---|---|---|---|---|---|---|---|---|
| Meas. | Freq. | Pers. | $\hat{\phi}$ | $Q_{10}$ | Pers. | $\hat{\phi}$ | $\hat{\lambda}$ | $Q_{10}$ |
| V | 30s | 0.99 | 6.20 | 0.082 | 0.82 | 7.88 | 9 | 0.601 |
| | 1m | 0.98 | 6.32 | 0.075 | 0.82 | 7.35 | 4 | 0.477 |
| | 2m | 0.98 | 5.29 | 0.065 | 0.82 | 6.34 | 2 | 0.330 |
| | 3m | 0.98 | 4.75 | 0.269 | 0.82 | 5.47 | 6 | 0.666 |
| | 4m | 0.98 | 4.34 | 0.098 | 0.82 | 5.04 | 3 | 0.291 |
| | 5m | 0.98 | 4.14 | 0.462 | 0.83 | 4.7 | 13 | 0.965 |
| | 6m | 0.98 | 3.83 | 0.076 | 0.84 | 4.17 | 12 | 0.260 |
| | 10m | 0.98 | 3.19 | 0.472 | 0.79 | 3.47 | 5 | 0.963 |
| | 15m | 0.99 | 2.58 | 0.391 | 0.78 | 2.71 | 6 | 0.845 |
| | 20m | 0.99 | 2.26 | 0.176 | 0.70 | 2.4 | 1 | 0.775 |
| | 30m | 0.99 | 1.79 | 0.990 | 0.77 | 1.89 | 5 | 0.998 |
| | 1h | 0.99 | 1.29 | 0.686 | 0.70 | 1.36 | 2 | 0.760 |
| B | 30s | 0.98 | 5.76 | 0.059 | 0.81 | 7.02 | 5 | 0.558 |
| | 1m | 0.98 | 5.74 | 0.078 | 0.82 | 6.58 | 4 | 0.444 |
| | 2m | 0.98 | 4.89 | 0.030 | 0.82 | 5.74 | 4 | 0.239 |
| | 3m | 0.98 | 4.10 | 0.246 | 0.83 | 4.99 | 11 | 0.652 |
| | 4m | 0.97 | 4.27 | 0.042 | 0.82 | 4.74 | 9 | 0.110 |
| | 5m | 0.98 | 3.87 | 0.305 | 0.84 | 4.39 | 17 | 0.881 |
| | 6m | 0.99 | 3.53 | 0.102 | 0.85 | 3.92 | 3 | 0.341 |
| | 10m | 0.98 | 2.90 | 0.366 | 0.79 | 3.18 | 4 | 0.823 |
| | 15m | 0.98 | 2.27 | 0.069 | 0.74 | 2.57 | 4 | 0.515 |
| | 20m | 0.99 | 2.03 | 0.128 | 0.67 | 2.29 | 3 | 0.726 |
| | 30m | 0.99 | 1.67 | 0.966 | 0.70 | 1.77 | 5 | 0.995 |
| | 1h | 0.99 | 1.15 | 0.657 | 0.61 | 1.25 | 1 | 0.557 |
| TS | 30s | 0.98 | 6.31 | 0.055 | 0.83 | 7.57 | 11 | 0.436 |
| | 1m | 0.98 | 5.61 | 0.034 | 0.82 | 7.04 | 3 | 0.288 |
| | 2m | 0.98 | 5.07 | 0.088 | 0.84 | 6.22 | 14 | 0.454 |
| | 3m | 0.98 | 4.65 | 0.193 | 0.83 | 5.57 | 4 | 0.626 |
| | 4m | 0.98 | 4.45 | 0.152 | 0.83 | 5.11 | 4 | 0.626 |
| | 5m | 0.98 | 4.13 | 0.158 | 0.82 | 4.76 | 3 | 0.704 |
| | 6m | 0.98 | 4.23 | 0.164 | 0.82 | 4.49 | 12 | 0.730 |
| | 10m | 0.98 | 3.31 | 0.149 | 0.79 | 3.68 | 7 | 0.928 |
| | 15m | 0.98 | 2.81 | 0.106 | 0.74 | 3.1 | 2 | 0.848 |
| | 20m | 0.98 | 2.50 | 0.083 | 0.71 | 2.77 | 5 | 0.729 |
| | 30m | 0.98 | 2.04 | 0.219 | 0.67 | 2.29 | 4 | 0.719 |
| | 1h | 0.98 | 1.44 | 0.787 | 0.61 | 1.65 | 3 | 0.789 |
| K | 30s | 0.99 | 4.52 | 0.019 | 0.82 | 5.25 | 4 | 0.314 |
| | 1m | 0.99 | 3.85 | 0.365 | 0.85 | 4.44 | 14 | 0.857 |
| | 2m | 0.98 | 3.30 | 0.275 | 0.84 | 3.61 | 17 | 0.575 |
| | 3m | 0.98 | 3.09 | 0.447 | 0.81 | 3.32 | 11 | 0.878 |
| | 4m | 0.98 | 2.69 | 0.532 | 0.79 | 3.02 | 7 | 0.980 |
| | 5m | 0.99 | 2.52 | 0.482 | 0.78 | 2.75 | 2 | 0.953 |
| | 6m | 0.98 | 2.47 | 0.302 | 0.77 | 2.71 | 8 | 0.966 |
| | 10m | 0.98 | 2.09 | 0.121 | 0.70 | 2.28 | 3 | 0.746 |
| | 15m | 0.98 | 1.84 | 0.246 | 0.79 | 1.89 | 5 | 0.482 |
| | 20m | 0.99 | 1.75 | 0.738 | 0.76 | 1.72 | 1 | 0.815 |
| | 30m | 0.99 | 1.37 | 0.521 | 0.49 | 1.52 | 2 | 0.643 |
| | 1h | 0.98 | 1.12 | 0.112 | 0.30 | 1.19 | 2 | 0.730 |
| R | | 0.97 | 1.84 | 0.514 | 0.69 | 1.96 | 5 | 0.927 |

Table 4: Estimation results for the volatility models. For each volatility measures, sampling frequency (when applicable) and volatility model (Base or P-Spline) the table reports the estimated persistence ($\hat{\alpha}+\hat{\beta}+\hat{\alpha}^{-}/2$), shape parameter $\hat{\phi}$ and the p–value of the Ljung–Box test on the residuals $rv_{(m,\delta)\,t}/\hat{\sigma}^2_{(m,\delta)\,t}$. Moreover, the table reports the selected shrinkage coefficients $\hat{\lambda}$ for the P-Spline MEM.

## GE

| Meas. | Freq. | Base Pers. | Base $\hat{\phi}$ | Base $Q_{10}$ | P-Spline Pers. | P-Spline $\hat{\phi}$ | P-Spline $\hat{\lambda}$ | P-Spline $Q_{10}$ |
|---|---|---|---|---|---|---|---|---|
| V | 30s | 1.00 | 6.02 | 0.022 | 0.90 | 8.15 | 2 | 0.187 |
| | 1m | 1.00 | 4.91 | 0.156 | 0.89 | 7.15 | 3 | 0.258 |
| | 2m | 0.99 | 4.54 | 0.010 | 0.86 | 5.99 | 2 | 0.171 |
| | 3m | 0.99 | 4.57 | 0.032 | 0.84 | 5.35 | 2 | 0.299 |
| | 4m | 0.99 | 4.17 | 0.028 | 0.85 | 4.80 | 3 | 0.282 |
| | 5m | 1.00 | 3.51 | 0.083 | 0.85 | 4.47 | 3 | 0.383 |
| | 6m | 1.00 | 3.25 | 0.051 | 0.85 | 4.18 | 3 | 0.237 |
| | 10m | 1.00 | 2.63 | 0.059 | 0.87 | 3.64 | 3 | 0.313 |
| | 15m | 1.00 | 2.47 | 0.139 | 0.86 | 2.96 | 4 | 0.468 |
| | 20m | 1.00 | 2.64 | 0.074 | 0.86 | 2.69 | 4 | 0.536 |
| | 30m | 0.99 | 1.94 | 0.119 | 0.86 | 2.15 | 2 | 0.190 |
| | 1h | 1.00 | 1.42 | 0.916 | 0.80 | 1.47 | 2 | 0.886 |
| B | 30s | 0.99 | 4.73 | 0.023 | 0.90 | 7.06 | 3 | 0.149 |
| | 1m | 0.99 | 4.88 | 0.023 | 0.88 | 6.34 | 2 | 0.196 |
| | 2m | 0.99 | 4.33 | 0.004 | 0.85 | 5.55 | 2 | 0.159 |
| | 3m | 0.99 | 4.23 | 0.020 | 0.84 | 4.97 | 2 | 0.351 |
| | 4m | 1.00 | 3.38 | 0.058 | 0.85 | 4.47 | 2 | 0.531 |
| | 5m | 0.99 | 3.23 | 0.047 | 0.85 | 4.15 | 3 | 0.339 |
| | 6m | 1.00 | 2.94 | 0.008 | 0.84 | 3.93 | 2 | 0.219 |
| | 10m | 0.99 | 2.84 | 0.030 | 0.87 | 3.41 | 3 | 0.321 |
| | 15m | 0.99 | 2.33 | 0.232 | 0.85 | 2.71 | 5 | 0.447 |
| | 20m | 1.00 | 2.05 | 0.210 | 0.86 | 2.46 | 5 | 0.636 |
| | 30m | 0.99 | 1.66 | 0.243 | 0.85 | 1.94 | 3 | 0.365 |
| | 1h | 0.99 | 1.26 | 0.743 | 0.81 | 1.33 | 3 | 0.957 |
| TS | 30s | 0.99 | 5.93 | 0.024 | 0.88 | 7.69 | 2 | 0.171 |
| | 1m | 1.00 | 4.85 | 0.027 | 0.87 | 6.68 | 2 | 0.197 |
| | 2m | 1.00 | 4.64 | 0.015 | 0.86 | 5.89 | 2 | 0.152 |
| | 3m | 0.99 | 4.11 | 0.035 | 0.85 | 5.38 | 1 | 0.252 |
| | 4m | 0.99 | 4.20 | 0.039 | 0.85 | 5.02 | 3 | 0.304 |
| | 5m | 0.99 | 4.18 | 0.035 | 0.84 | 4.76 | 2 | 0.309 |
| | 6m | 0.99 | 3.66 | 0.043 | 0.84 | 4.57 | 3 | 0.288 |
| | 10m | 1.00 | 3.28 | 0.017 | 0.84 | 3.99 | 3 | 0.166 |
| | 15m | 1.00 | 2.79 | 0.010 | 0.83 | 3.57 | 3 | 0.084 |
| | 20m | 1.00 | 2.95 | 0.006 | 0.82 | 3.28 | 4 | 0.065 |
| | 30m | 0.99 | 2.32 | 0.022 | 0.80 | 2.80 | 4 | 0.125 |
| | 1h | 0.99 | 1.81 | 0.608 | 0.74 | 2.03 | 3 | 0.570 |
| K | 30s | 0.99 | 3.93 | 0.105 | 0.85 | 4.6 | 2 | 0.436 |
| | 1m | 0.99 | 3.74 | 0.030 | 0.83 | 4.05 | 2 | 0.298 |
| | 2m | 1.00 | 2.97 | 0.069 | 0.81 | 3.49 | 3 | 0.435 |
| | 3m | 0.99 | 3.04 | 0.019 | 0.82 | 3.19 | 3 | 0.219 |
| | 4m | 0.99 | 2.77 | 0.080 | 0.81 | 3.18 | 4 | 0.479 |
| | 5m | 0.99 | 2.53 | 0.003 | 0.80 | 2.99 | 4 | 0.128 |
| | 6m | 0.99 | 2.46 | 0.018 | 0.81 | 2.91 | 4 | 0.156 |
| | 10m | 0.99 | 2.13 | 0.094 | 0.76 | 2.50 | 4 | 0.507 |
| | 15m | 0.99 | 2.03 | 0.341 | 0.70 | 2.19 | 3 | 0.832 |
| | 20m | 0.99 | 1.59 | 0.642 | 0.70 | 2.01 | 2 | 0.828 |
| | 30m | 1.00 | 1.52 | 0.859 | 0.69 | 1.70 | 2 | 0.071 |
| | 1h | 0.99 | 1.11 | 0.922 | 0.61 | 1.22 | 4 | 0.997 |
| R | | 1.00 | 1.76 | 0.681 | 0.75 | 1.9 | 3 | 0.939 |

Table 5: Estimation results for the volatility models. For each volatility measures, sampling frequency (when applicable) and volatility model (Base or P-Spline) the table reports the estimated persistence ($\hat{\alpha}+\hat{\beta}+\hat{\alpha}^-/2$), shape parameter $\hat{\phi}$ and the p–value of the Ljung–Box test on the residuals $rv_{(m,\delta)\,t}/\hat{\sigma}^2_{(m,\delta)\,t}$. Moreover, the table reports the selected shrinkage coefficients $\hat{\lambda}$ for the P-Spline MEM.

## JNJ

| Meas. | Freq. | Base Pers. | Base $\hat{\phi}$ | Base $Q_{10}$ | P-Spline Pers. | P-Spline $\hat{\phi}$ | P-Spline $\hat{\lambda}$ | P-Spline $Q_{10}$ |
|---|---|---|---|---|---|---|---|---|
| V | 30s | 0.99 | 5.31 | 0.261 | 0.84 | 7.54 | 5 | 0.861 |
| | 1m | 0.99 | 4.25 | 0.441 | 0.85 | 6.43 | 6 | 0.868 |
| | 2m | 1.00 | 3.58 | 0.103 | 0.85 | 5.33 | 10 | 0.581 |
| | 3m | 0.99 | 3.25 | 0.840 | 0.85 | 4.63 | 19 | 0.978 |
| | 4m | 1.00 | 3.66 | 0.473 | 0.84 | 4.18 | 3 | 0.983 |
| | 5m | 0.99 | 2.92 | 0.237 | 0.82 | 3.64 | 6 | 0.367 |
| | 6m | 1.00 | 3.09 | 0.695 | 0.83 | 3.50 | 3 | 0.994 |
| | 10m | 1.00 | 2.63 | 0.160 | 0.83 | 2.79 | 3 | 0.584 |
| | 15m | 1.00 | 2.01 | 0.385 | 0.75 | 2.27 | 1 | 0.793 |
| | 20m | 0.98 | 1.85 | 0.903 | 0.80 | 2.07 | 4 | 0.984 |
| | 30m | 0.99 | 1.55 | 0.165 | 0.72 | 1.71 | 4 | 0.848 |
| | 1h | 0.99 | 1.28 | 0.140 | 0.52 | 1.30 | 3 | 0.179 |
| B | 30s | 0.99 | 5.00 | 0.177 | 0.84 | 6.96 | 4 | 0.724 |
| | 1m | 0.99 | 4.48 | 0.512 | 0.86 | 6.08 | 6 | 0.834 |
| | 2m | 1.00 | 3.52 | 0.104 | 0.84 | 5.10 | 11 | 0.780 |
| | 3m | 1.00 | 3.54 | 0.822 | 0.86 | 4.51 | 3 | 0.960 |
| | 4m | 0.99 | 3.05 | 0.331 | 0.83 | 4.07 | 14 | 0.884 |
| | 5m | 1.00 | 2.28 | 0.380 | 0.83 | 3.46 | 3 | 0.712 |
| | 6m | 1.00 | 2.27 | 0.779 | 0.83 | 3.36 | 7 | 0.974 |
| | 10m | 1.00 | 2.56 | 0.385 | 0.81 | 2.68 | 6 | 0.729 |
| | 15m | 0.99 | 1.95 | 0.242 | 0.76 | 2.14 | 6 | 0.838 |
| | 20m | 0.99 | 1.65 | 0.879 | 0.79 | 1.94 | 3 | 0.965 |
| | 30m | 0.99 | 1.42 | 0.802 | 0.74 | 1.55 | 5 | 0.795 |
| | 1h | 1.00 | 1.18 | 0.073 | 0.85 | 1.15 | 4 | 0.112 |
| TS | 30s | 0.99 | 5.01 | 0.342 | 0.84 | 7.16 | 5 | 0.923 |
| | 1m | 0.99 | 4.08 | 0.608 | 0.84 | 6.14 | 8 | 0.919 |
| | 2m | 0.99 | 3.41 | 0.694 | 0.84 | 5.17 | 8 | 0.877 |
| | 3m | 0.99 | 3.22 | 0.449 | 0.84 | 4.61 | 7 | 0.867 |
| | 4m | 0.96 | 2.68 | 0.052 | 0.83 | 4.15 | 7 | 0.880 |
| | 5m | 1.00 | 3.22 | 0.305 | 0.83 | 3.85 | 3 | 0.913 |
| | 6m | 0.97 | 2.46 | 0.002 | 0.83 | 3.62 | 3 | 0.934 |
| | 10m | 1.00 | 2.57 | 0.481 | 0.83 | 3.11 | 9 | 0.973 |
| | 15m | 1.00 | 2.31 | 0.922 | 0.81 | 2.74 | 9 | 0.995 |
| | 20m | 1.00 | 1.89 | 0.714 | 0.81 | 2.45 | 8 | 0.995 |
| | 30m | 0.98 | 1.97 | 0.820 | 0.75 | 2.13 | 4 | 0.989 |
| | 1h | 1.00 | 1.49 | 0.693 | 0.72 | 1.56 | 2 | 0.987 |
| K | 30s | 1.00 | 2.80 | 0.356 | 0.84 | 4.33 | 6 | 0.727 |
| | 1m | 0.99 | 2.78 | 0.899 | 0.83 | 3.48 | 2 | 0.938 |
| | 2m | 1.00 | 2.79 | 0.497 | 0.83 | 2.99 | 4 | 0.979 |
| | 3m | 1.00 | 2.39 | 0.657 | 0.83 | 2.84 | 2 | 0.940 |
| | 4m | 1.00 | 2.34 | 0.743 | 0.83 | 2.63 | 2 | 0.972 |
| | 5m | 0.99 | 2.16 | 0.779 | 0.81 | 2.51 | 8 | 0.999 |
| | 6m | 1.00 | 2.38 | 0.248 | 0.81 | 2.47 | 7 | 0.968 |
| | 10m | 1.00 | 1.89 | 0.772 | 0.75 | 2.11 | 1 | 0.962 |
| | 15m | 1.00 | 1.56 | 0.593 | 0.76 | 1.81 | 5 | 0.973 |
| | 20m | 1.00 | 1.56 | 0.626 | 0.75 | 1.66 | 4 | 0.940 |
| | 30m | 1.00 | 1.29 | 0.939 | 0.81 | 1.38 | 1 | 0.938 |
| | 1h | 1.00 | 1.01 | 0.477 | 0.71 | 1.07 | 6 | 0.509 |
| R | | 0.99 | 1.7 | 0.485 | 0.81 | 1.76 | 18 | 0.870 |

Table 6: Estimation results for the volatility models. For each volatility measures, sampling frequency (when applicable) and volatility model (Base or P-Spline) the table reports the estimated persistence ($\hat{\alpha}+\hat{\beta}+\hat{\alpha}^-/2$), shape parameter $\hat{\phi}$ and the p–value of the Ljung–Box test on the residuals $rv_{(m,\delta)\,t}/\hat{\sigma}^2_{(m,\delta)\,t}$. Moreover, the table reports the selected shrinkage coefficients $\hat{\lambda}$ for the P-Spline MEM.

empirical justification of such an approach can be found in the work by Olsen & Associates research institute (e.g. Müller, Dacorogna, Davé, Olsen, Pictet & von Weizsäcker (1997)), who suggest the presence with a short and a long term component as a results of the interactions of different agents with different time–horizons in the financial markets: the long-term component is determined by "fundamentals" while the short-term component generates volatility clusters around the long-term component.

Following Engle & Rangel (2008), a flexible MEM specification for $\sigma^2_{(m,\delta)\ t}$ capable of capturing such long and short run dynamics is

$$\sigma^2_{(m,\delta)\ t} = \tau_{(m,\delta)\ t}\ g_{(m,\delta)\ t}, \tag{3}$$

where

$$\tau_{(m,\delta)\ t} \equiv \exp\left\{\sum \gamma_j B_j(t)\right\}, \tag{4}$$

captures the *long run trend* using some linear basis expansion of time $\{B_j(t)\}$, and

$$g_{(m,\delta)\ t} \equiv (1-\alpha-\beta-\alpha^-/2) + \alpha\frac{rv_{(m,\delta)\ t-1}}{\tau_{(m,\delta)\ t-1}} + \beta g_{(m,\delta)\ t-1} + \alpha^- \frac{rv^-_{(m,\delta)\ t-1}}{\tau_{(m,\delta)\ t-1}}, \tag{5}$$

captures the *short run persistence*.

To fully specify the model of Equations (3)–(5) some appropriate choice of the basis functions $B_j(\cdot)$ in Equation (4) has to be made. The spline volatility modeling approach *à la* Engle & Rangel (2008) fully specifies the spline model by using a quadratic spline basis, that is,

$$\{B_j(t)\} = \left\{1, t, t^2, \left[(t-\xi_1)_+\right]^2, ..., \left[(t-\xi_n)_+\right]^2\right\},$$

where $u_+ \equiv \max\{0, u\}$ and $\xi_1, ..., \xi_n$ are some (equally) spaced knots. The degrees of smoothness of the estimated trend will depend on the number of knots which Engle & Rangel (2008) determine on the basis of the BIC.

In practice, this modeling approach might have some drawbacks. Quadratic splines have very poor numerical properties that are expected to tangle nonlinear estimation. Choosing the knots via some model selection criterion is often not appealing in that it is usually not feasible to search over all the $2^n - 1$ knots combinations and some subjective ordering of possible combinations has to be chosen. Lastly, the BIC is an information criterion with very poor forecasting properties as the maximum asymptotic forecasting MSE implied by a BIC estimation strategy is infinite (Leeb & Pötscher (2005)).

In light of these considerations and building on the proposals of Eilers & Marx (1996), we propose a novel approach for the flexible modeling of volatility in the presence of trends that we name P-Spline MEM. The term P-Spline is short notation for Penalized B-splines. This modeling strategy consist of using a basis of B-splines with equidistant knots in Equation (4) and fitting the model by a penalized maximum likelihood estimation procedure depending on a shrinkage coefficient that controls the degree of smoothness of the estimated trend.

B-splines (Eilers & Marx (1996)) are a common basis of functions made up of polynomial pieces indexed by a set of knots, used for nonlinear approximation and smoothing in linear regressions (White (2006)). There are at least two properties of B-splines that turn out to be useful in this context. First, B-splines allow to simplify the numerical nonlinear estimation relative to Splines. Second, the derivatives of the log trend $\sum \gamma_j B_j(t)$ can be expressed as a linear combination of the finite differences of adjacent B-splines coefficients $\gamma_j$. It is hence possible to control the degree of smoothness of the trend by appropriately constraining the model parameters which suggests a penalized maximum likelihood (PML) estimation strategy.

Let $\psi \equiv (\gamma_1, ..., \gamma_K, \omega, \alpha, \alpha^-, \beta, \phi)'$ denote the model parameters and let $\gamma \equiv (\gamma_1, ..., \gamma_K)'$ denote the B-splines parameters. Then the penalized maximum likelihood estimator is defined as

$$\widehat{\psi}_\lambda \equiv \arg\max \{L_T(\psi) - \lambda \gamma' D_r' D_r \gamma\}$$

where $L_T(.)$ is the log–likelihood function, $D_r$ is the matrix representation of the difference operator of order $r$. The shrinkage coefficient $\lambda$ governs the bias/variance trade–off of the estimator: when $\lambda$ is 0 the PML estimator coincides with the ML estimator and as the shrinkage coefficient $\lambda$ grows to infinity, the estimated log trend collapses to a polynomial of degree $r - 1$.

We do not attempt to derive the large sample properties of the PML estimator: this can be done resorting a large sample framework under local alternatives for the biased parameters as in Knight & Fu (2000) and Hjort & Claeskens (2003)[4].

PML estimation techniques are not very common in the financial econometrics time series literature but have a long tradition in statistics since the seminal contribution of Hoerl & Kennard (1970). From a forecasting perspective an appealing feature of PML strategies is that the estimated trend tends not to be too sensitive to small changes in the data. In fact, shrinkage estimation strategies are called *stable* regularizing procedures as opposed to model selection strategies that are *unstable* (Breiman (1996)). This property is important in rolling or recursive prediction exercises in that the sequence of predicted values of the trend will not tend to change abruptly from one period to another.

In order to use the PML estimator in real applications, we need to determine some data–driven method to choose the amount of shrinkage $\lambda$ to impose on the estimates, We resort to a Corrected AIC type information criterion (Hurvich & Tsai (1989)). The $\mathsf{AIC_C}$ for the P–Spline MEM is defined as

$$\mathsf{AIC_C}(\lambda) = -2L_T(\widehat{\psi}_\lambda) + 2\widehat{\mathsf{d}}_\lambda + \frac{2\widehat{\mathsf{d}}_\lambda(\widehat{\mathsf{d}}_\lambda + 1)}{T - \widehat{\mathsf{d}}_\lambda - 1},$$

where

$$\widehat{\mathsf{d}}_\lambda = \mathsf{tr}\left\{ \left(\mathcal{I}(\widehat{\psi}_{ML}) + 2\lambda \mathbf{P}_r\right)^{-1} \mathcal{I}(\widehat{\psi}_{ML}) \right\},$$

with

$$\mathbf{P}_r = \begin{bmatrix} D_r' D_r & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix},$$

where $\mathcal{I}(\cdot)$ is the Fisher information matrix. This criterion uses as penalty for model complexity a function that is inversely proportional to the shrinkage coefficient $\lambda$ in analogy to the effective dimension of a linear smoother proposed in Hastie & Tibshirani (1990). We find this criterion appealing in that it leads to more parsimonious specifications in comparison to an AIC type criterion when the number of knots (hence parameters) is large with respect to the sample size.

[4] As with other parameter reduction techniques, there are problems with inference: as pointed out by Leeb & Pötscher (2006), the risk of the PML estimator cannot be uniformly consistently estimated (see also Hurvich & Tsai (1990)). This however does not have any consequences for our approach, since we are interested in point estimation and forecasting.

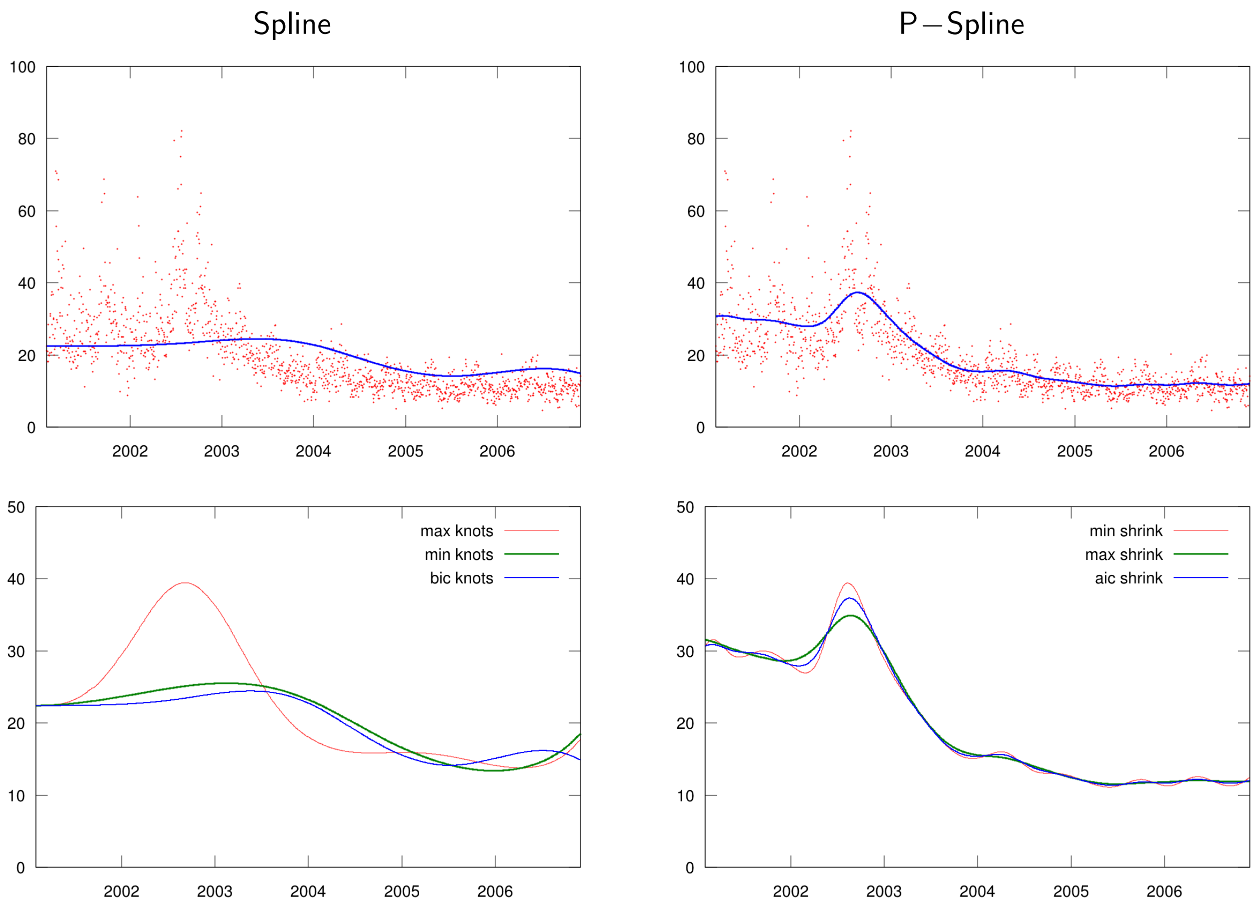


Figure 1: Cubic Splines with BIC and P–Splines with $\mathrm{AIC}_C$ fit comparison. The top graphs display the plot of the GE (annualized) realized volatility computed at a 5 min. frequency and the estimated volatility trend obtained with the Engle & Rangel approach and P-Splines. The bottom graphs show the differences in the estimated trends obtained with different estimation strategies.

Figure 1 about here.

Figure 1 shows the estimated trends obtained using the Engle & Rangel (2008) modeling approach and the P-Spline approach. Cubic splines with BIC driven knot selection and P-Splines are fitted to the 5 min. frequency realized volatility series of the GE stock (using the same number of maximum knots). The visual inspection of the graphs suggests that P-spline are able to capture the features of the data better. The condition number of the Hessian of the wide models (that is all knots or no shrinkage) drops from $10^{19}$ to $10^{6}$ using B-splines. As a result of the ill–conditioned Hessian, cubic splines tend to adapt very slowly to the data during the nonlinear estimation while B-splines are much better behaved. The BIC over penalizes (it only selects two knots) while our proposed AIC appears to behave satisfactorily.

Figure 2 about here.

The P-Spline MEM model is estimated over the full sample using 20 (equidistant) knots. The amount of smoothness $\hat{\lambda}$ imposed on the estimates is chosen by picking up the $\lambda$ which minimizes the $\mathsf{AIC_C}(\lambda)$ criterion over a grid of $\lambda$ values[5]. The right hand side of Tables 4, 5 and 6 report the parameter estimates and residual diagnostics of the model. Figure 2 reports the graphs of the annualized realized volatility series (5 minute frequency) together with their corresponding estimated trend. The specification captures the dynamics of the series satisfactorily as the Ljung–Box test statistic is always non significant at a 5% level. The persistence and shape of the innovation distribution depend on the sampling frequency in a similar way across measures and stocks. The persistence and shape parameter tend to be higher at higher frequencies, in accordance to the stylized facts.

[5]The grid of $\lambda$ values is $\{i\,\kappa : i = 1, ..., 20\}$ where $\kappa$ is a scale factor that is both sample size and stock dependent ($\kappa = cT$, with $c = 10^{-6}$ for BA, $c = 10^{-4}$ for GE and $c = 10^{-6}$ for JNJ).

# 5 Modeling Returns

## 5.1 A Conditional Heteroskedastic Model for Returns Based on Volatility Measures Predictions

Let $r_t$ denote the daily return, let $h_t$ be the conditional variance of the returns and let $rv_{(m,\delta)\ t|t-1}$ be the conditional expectation of the volatility measure at day $t$. We assume that the conditional variance $h_t$ is a linear function of the volatility measure conditional expectation

$$h_t = c + m\ rv_{(m,\delta)\ t|t-1}, \tag{6}$$

and we assume that the return standardized by its conditional standard deviation is well described by a standardized Student's $t$ distribution, that is

$$r_t = \sqrt{h_t}\, z_t, \qquad z_t \sim t_{1/\nu}, \tag{7}$$

where $t_{1/\nu}$ is a standardized (unit variance) $t$ distribution with $1/\nu$ dof (Fiorentini, Sentana & Calzolari (2003)). In other words the model for the returns of Equations (6) and (7) implies that the conditional heteroskedasticity of the returns series is captured by the conditional expectation of the volatility measures. The specification, however, does not require the volatility measures forecasts to be unbiased predictors of the returns' variance. The model allows us to test the Unbiased Volatility Predictor hypothesis $H_0 : c = 0 \quad m = 1$ (UVP test).

Tables 7, 8, 9 about here.

Equations 6 and 7 are estimated over the full sample using the series of 1 day ahead prediction of the volatility measures obtained by both the estimated Base and P-Spline specifications. Tables 7, 8, 9 report parameter estimates and diagnostics. The model and both series of volatility predictions are able to capture the squared returns dynamics satisfactorily as the Ljung–Box test statistic is always non significant at standard significance levels. However, the volatility predictions do not provide unbiased forecasts of the variance of returns in the great majority of cases as the UVP test statistic is

# BA

| Meas. | Freq. | Base $\hat{c}$ | Base $\hat{m}$ | Base $\hat{\nu}$ | Base UVP | Base $Q^2_{10}$ | P-Spline $\hat{c}$ | P-Spline $\hat{m}$ | P-Spline $\hat{\nu}$ | P-Spline UVP | P-Spline $Q^2_{10}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| V | 30s | 0.44 (0.21) | 1.2 (0.12) | 0.11 (0.02) | 0.000 | 0.231 | 0.36 (0.21) | 1.24 (0.12) | 0.1 (0.02) | 0.000 | 0.234 |
| | 1m | 0.48 (0.21) | 1.11 (0.12) | 0.11 (0.02) | 0.000 | 0.235 | 0.41 (0.2) | 1.14 (0.11) | 0.1 (0.02) | 0.000 | 0.231 |
| | 2m | 0.48 (0.2) | 1.06 (0.11) | 0.1 (0.02) | 0.000 | 0.222 | 0.43 (0.19) | 1.08 (0.11) | 0.1 (0.02) | 0.000 | 0.209 |
| | 3m | 0.48 (0.2) | 1.04 (0.11) | 0.1 (0.02) | 0.000 | 0.221 | 0.43 (0.19) | 1.07 (0.1) | 0.1 (0.02) | 0.000 | 0.213 |
| | 4m | 0.47 (0.2) | 1.05 (0.11) | 0.1 (0.02) | 0.000 | 0.187 | 0.41 (0.19) | 1.08 (0.1) | 0.1 (0.02) | 0.000 | 0.183 |
| | 5m | 0.47 (0.2) | 1.08 (0.11) | 0.1 (0.02) | 0.000 | 0.182 | 0.41 (0.19) | 1.1 (0.11) | 0.1 (0.02) | 0.000 | 0.181 |
| | 6m | 0.45 (0.2) | 1.09 (0.11) | 0.1 (0.02) | 0.000 | 0.188 | 0.38 (0.18) | 1.12 (0.11) | 0.1 (0.02) | 0.000 | 0.187 |
| | 10m | 0.39 (0.2) | 1.17 (0.11) | 0.09 (0.02) | 0.000 | 0.154 | 0.35 (0.18) | 1.2 (0.11) | 0.09 (0.02) | 0.000 | 0.142 |
| | 15m | 0.35 (0.2) | 1.21 (0.11) | 0.09 (0.02) | 0.000 | 0.178 | 0.34 (0.17) | 1.23 (0.11) | 0.09 (0.02) | 0.000 | 0.174 |
| | 20m | 0.35 (0.2) | 1.25 (0.12) | 0.09 (0.02) | 0.000 | 0.153 | 0.34 (0.18) | 1.26 (0.11) | 0.09 (0.02) | 0.000 | 0.161 |
| | 30m | 0.27 (0.21) | 1.33 (0.13) | 0.1 (0.02) | 0.000 | 0.166 | 0.31 (0.18) | 1.31 (0.12) | 0.09 (0.02) | 0.000 | 0.176 |
| | 1h | 0.11 (0.21) | 1.69 (0.16) | 0.09 (0.02) | 0.000 | 0.118 | 0.22 (0.18) | 1.64 (0.14) | 0.09 (0.02) | 0.000 | 0.125 |
| B | 30s | 0.37 (0.23) | 1.53 (0.16) | 0.11 (0.02) | 0.000 | 0.232 | 0.27 (0.22) | 1.59 (0.16) | 0.11 (0.02) | 0.000 | 0.235 |
| | 1m | 0.48 (0.22) | 1.24 (0.13) | 0.11 (0.02) | 0.000 | 0.242 | 0.37 (0.21) | 1.3 (0.13) | 0.11 (0.02) | 0.000 | 0.228 |
| | 2m | 0.51 (0.21) | 1.11 (0.12) | 0.11 (0.02) | 0.000 | 0.222 | 0.42 (0.2) | 1.15 (0.11) | 0.1 (0.02) | 0.000 | 0.209 |
| | 3m | 0.49 (0.2) | 1.09 (0.11) | 0.1 (0.02) | 0.000 | 0.239 | 0.42 (0.19) | 1.12 (0.11) | 0.1 (0.02) | 0.000 | 0.232 |
| | 4m | 0.46 (0.2) | 1.1 (0.11) | 0.1 (0.02) | 0.000 | 0.198 | 0.4 (0.19) | 1.13 (0.11) | 0.1 (0.02) | 0.000 | 0.195 |
| | 5m | 0.48 (0.2) | 1.1 (0.11) | 0.1 (0.02) | 0.000 | 0.186 | 0.4 (0.19) | 1.13 (0.11) | 0.1 (0.02) | 0.000 | 0.188 |
| | 6m | 0.49 (0.2) | 1.1 (0.11) | 0.1 (0.02) | 0.000 | 0.215 | 0.41 (0.19) | 1.14 (0.11) | 0.1 (0.02) | 0.000 | 0.215 |
| | 10m | 0.38 (0.2) | 1.21 (0.12) | 0.09 (0.02) | 0.000 | 0.158 | 0.35 (0.18) | 1.22 (0.11) | 0.09 (0.02) | 0.000 | 0.147 |
| | 15m | 0.38 (0.19) | 1.25 (0.12) | 0.09 (0.02) | 0.000 | 0.205 | 0.38 (0.17) | 1.25 (0.11) | 0.09 (0.02) | 0.000 | 0.198 |
| | 20m | 0.39 (0.2) | 1.27 (0.12) | 0.1 (0.02) | 0.000 | 0.183 | 0.37 (0.18) | 1.29 (0.12) | 0.09 (0.02) | 0.000 | 0.217 |
| | 30m | 0.35 (0.21) | 1.35 (0.14) | 0.1 (0.02) | 0.000 | 0.176 | 0.35 (0.18) | 1.36 (0.12) | 0.1 (0.02) | 0.000 | 0.204 |
| | 1h | 0.12 (0.21) | 1.76 (0.16) | 0.09 (0.02) | 0.000 | 0.110 | 0.24 (0.18) | 1.71 (0.15) | 0.09 (0.02) | 0.000 | 0.123 |
| TS | 30s | 0.39 (0.22) | 0.66 (0.07) | 0.11 (0.02) | 0.000 | 0.228 | 0.3 (0.21) | 0.68 (0.07) | 0.11 (0.02) | 0.000 | 0.227 |
| | 1m | 0.46 (0.21) | 0.88 (0.09) | 0.11 (0.02) | 0.052 | 0.219 | 0.39 (0.2) | 0.9 (0.09) | 0.1 (0.02) | 0.084 | 0.217 |
| | 2m | 0.48 (0.2) | 0.95 (0.1) | 0.1 (0.02) | 0.001 | 0.204 | 0.42 (0.19) | 0.97 (0.09) | 0.1 (0.02) | 0.001 | 0.199 |
| | 3m | 0.43 (0.2) | 1 (0.1) | 0.1 (0.02) | 0.000 | 0.202 | 0.39 (0.19) | 1.01 (0.1) | 0.1 (0.02) | 0.000 | 0.196 |
| | 4m | 0.42 (0.2) | 1.02 (0.1) | 0.1 (0.02) | 0.000 | 0.194 | 0.37 (0.19) | 1.04 (0.1) | 0.1 (0.02) | 0.000 | 0.188 |
| | 5m | 0.41 (0.2) | 1.04 (0.1) | 0.1 (0.02) | 0.000 | 0.195 | 0.35 (0.19) | 1.07 (0.1) | 0.1 (0.02) | 0.000 | 0.190 |
| | 6m | 0.39 (0.2) | 1.06 (0.1) | 0.1 (0.02) | 0.000 | 0.197 | 0.34 (0.18) | 1.08 (0.1) | 0.09 (0.02) | 0.000 | 0.189 |
| | 10m | 0.36 (0.2) | 1.11 (0.11) | 0.09 (0.02) | 0.000 | 0.187 | 0.3 (0.18) | 1.13 (0.1) | 0.09 (0.02) | 0.000 | 0.179 |
| | 15m | 0.33 (0.2) | 1.14 (0.11) | 0.09 (0.02) | 0.000 | 0.172 | 0.29 (0.18) | 1.16 (0.1) | 0.09 (0.02) | 0.000 | 0.164 |
| | 20m | 0.32 (0.2) | 1.17 (0.11) | 0.09 (0.02) | 0.000 | 0.169 | 0.29 (0.18) | 1.18 (0.11) | 0.09 (0.02) | 0.000 | 0.165 |
| | 30m | 0.26 (0.21) | 1.22 (0.12) | 0.09 (0.02) | 0.000 | 0.179 | 0.28 (0.18) | 1.21 (0.11) | 0.09 (0.02) | 0.000 | 0.183 |
| | 1h | 0.02 (0.23) | 7.84 (0.76) | 0.1 (0.02) | 0.000 | 0.176 | 0.14 (0.19) | 7.53 (0.66) | 0.1 (0.02) | 0.000 | 0.209 |
| K | 30s | 0.65 (0.2) | 0.88 (0.09) | 0.11 (0.02) | 0.000 | 0.220 | 0.55 (0.19) | 0.92 (0.09) | 0.1 (0.02) | 0.000 | 0.217 |
| | 1m | 0.57 (0.2) | 0.92 (0.09) | 0.1 (0.02) | 0.000 | 0.176 | 0.49 (0.18) | 0.95 (0.09) | 0.1 (0.02) | 0.000 | 0.177 |
| | 2m | 0.48 (0.2) | 0.97 (0.1) | 0.1 (0.02) | 0.000 | 0.191 | 0.44 (0.18) | 0.99 (0.09) | 0.1 (0.02) | 0.000 | 0.190 |
| | 3m | 0.46 (0.2) | 0.98 (0.1) | 0.1 (0.02) | 0.000 | 0.197 | 0.44 (0.18) | 0.99 (0.09) | 0.1 (0.02) | 0.000 | 0.187 |
| | 4m | 0.41 (0.2) | 1 (0.1) | 0.1 (0.02) | 0.000 | 0.201 | 0.38 (0.18) | 1.02 (0.09) | 0.1 (0.02) | 0.000 | 0.212 |
| | 5m | 0.36 (0.21) | 1.02 (0.1) | 0.1 (0.02) | 0.000 | 0.162 | 0.33 (0.18) | 1.04 (0.09) | 0.09 (0.02) | 0.000 | 0.181 |
| | 6m | 0.31 (0.21) | 1.04 (0.1) | 0.09 (0.02) | 0.001 | 0.175 | 0.31 (0.18) | 1.04 (0.09) | 0.09 (0.02) | 0.001 | 0.178 |
| | 10m | 0.24 (0.22) | 1.04 (0.1) | 0.09 (0.02) | 0.008 | 0.161 | 0.25 (0.19) | 1.04 (0.09) | 0.09 (0.02) | 0.005 | 0.160 |
| | 15m | 0.18 (0.22) | 1.03 (0.1) | 0.1 (0.02) | 0.056 | 0.187 | 0.17 (0.19) | 1.04 (0.09) | 0.09 (0.02) | 0.042 | 0.172 |
| | 20m | 0.18 (0.23) | 1 (0.1) | 0.1 (0.02) | 0.246 | 0.171 | 0.19 (0.19) | 0.99 (0.09) | 0.09 (0.02) | 0.197 | 0.189 |
| | 30m | 0.17 (0.23) | 0.99 (0.1) | 0.1 (0.02) | 0.360 | 0.252 | 0.18 (0.19) | 0.99 (0.09) | 0.09 (0.02) | 0.283 | 0.245 |
| | 1h | 0.02 (0.23) | 0.97 (0.09) | 0.1 (0.02) | 0.829 | 0.196 | 0.09 (0.19) | 0.96 (0.08) | 0.09 (0.02) | 0.865 | 0.226 |
| R | | 0.14 (0.22) | 1.22 (0.12) | 0.1 (0.02) | 0.000 | 0.273 | 0.16 (0.19) | 1.21 (0.11) | 0.09 (0.02) | 0.000 | 0.242 |

Table 7: Estimation results for the return model. For each measure, sampling frequencies (when applicable) and volatility model (Base or P-Spline) the table reports the estimates of the model parameters (standard errors in parenthesis), the p–value of the UVP test, and the p–value of the Ljung–Box test on the squared residuals $r_t^2/\hat{h}_t$.

# GE

| Meas. | Freq. | Base $\hat{c}$ | Base $\hat{m}$ | Base $\hat{\nu}$ | Base UVP | Base $Q^2_{10}$ | P-Spline $\hat{c}$ | P-Spline $\hat{m}$ | P-Spline $\hat{\nu}$ | P-Spline UVP | P-Spline $Q^2_{10}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| V | 30s | −0.23 (0.08) | 1.62 (0.11) | 0.1 (0.02) | 0.000 | 0.977 | −0.26 (0.08) | 1.67 (0.12) | 0.09 (0.02) | 0.000 | 0.976 |
| | 1m | −0.14 (0.08) | 1.47 (0.11) | 0.1 (0.02) | 0.000 | 0.980 | −0.18 (0.08) | 1.49 (0.11) | 0.09 (0.02) | 0.000 | 0.978 |
| | 2m | −0.11 (0.08) | 1.37 (0.1) | 0.1 (0.02) | 0.000 | 0.985 | −0.13 (0.08) | 1.37 (0.1) | 0.09 (0.02) | 0.000 | 0.983 |
| | 3m | −0.1 (0.07) | 1.33 (0.09) | 0.1 (0.02) | 0.000 | 0.991 | −0.13 (0.08) | 1.36 (0.09) | 0.09 (0.02) | 0.000 | 0.989 |
| | 4m | −0.09 (0.08) | 1.33 (0.09) | 0.1 (0.02) | 0.000 | 0.988 | −0.11 (0.07) | 1.33 (0.09) | 0.09 (0.02) | 0.000 | 0.985 |
| | 5m | −0.07 (0.08) | 1.31 (0.09) | 0.1 (0.02) | 0.000 | 0.988 | −0.11 (0.07) | 1.33 (0.09) | 0.09 (0.02) | 0.000 | 0.985 |
| | 6m | −0.07 (0.07) | 1.32 (0.09) | 0.1 (0.02) | 0.000 | 0.988 | −0.12 (0.07) | 1.35 (0.09) | 0.09 (0.02) | 0.000 | 0.985 |
| | 10m | −0.07 (0.07) | 1.37 (0.1) | 0.1 (0.02) | 0.000 | 0.987 | −0.12 (0.07) | 1.4 (0.1) | 0.09 (0.02) | 0.000 | 0.981 |
| | 15m | −0.08 (0.07) | 1.4 (0.1) | 0.1 (0.02) | 0.000 | 0.989 | −0.11 (0.07) | 1.43 (0.1) | 0.09 (0.02) | 0.000 | 0.985 |
| | 20m | −0.11 (0.08) | 1.43 (0.1) | 0.1 (0.02) | 0.000 | 0.988 | −0.12 (0.07) | 1.44 (0.1) | 0.08 (0.02) | 0.000 | 0.978 |
| | 30m | −0.05 (0.07) | 1.44 (0.1) | 0.1 (0.02) | 0.000 | 0.986 | −0.09 (0.07) | 1.47 (0.1) | 0.09 (0.02) | 0.000 | 0.976 |
| | 1h | −0.06 (0.08) | 1.61 (0.12) | 0.1 (0.02) | 0.000 | 0.990 | −0.1 (0.07) | 1.67 (0.11) | 0.08 (0.02) | 0.000 | 0.976 |
| B | 30s | −0.14 (0.08) | 1.86 (0.13) | 0.1 (0.02) | 0.000 | 0.975 | −0.16 (0.08) | 1.89 (0.13) | 0.09 (0.02) | 0.000 | 0.972 |
| | 1m | −0.1 (0.08) | 1.56 (0.11) | 0.1 (0.02) | 0.000 | 0.983 | −0.12 (0.08) | 1.58 (0.11) | 0.09 (0.02) | 0.000 | 0.981 |
| | 2m | −0.09 (0.08) | 1.42 (0.1) | 0.1 (0.02) | 0.000 | 0.986 | −0.11 (0.07) | 1.42 (0.1) | 0.09 (0.02) | 0.000 | 0.983 |
| | 3m | −0.09 (0.07) | 1.38 (0.1) | 0.1 (0.02) | 0.000 | 0.993 | −0.12 (0.07) | 1.4 (0.1) | 0.09 (0.02) | 0.000 | 0.990 |
| | 4m | −0.06 (0.08) | 1.32 (0.1) | 0.1 (0.02) | 0.000 | 0.986 | −0.09 (0.07) | 1.36 (0.1) | 0.09 (0.02) | 0.000 | 0.983 |
| | 5m | −0.06 (0.08) | 1.35 (0.1) | 0.1 (0.02) | 0.000 | 0.988 | −0.09 (0.07) | 1.37 (0.1) | 0.09 (0.02) | 0.000 | 0.984 |
| | 6m | −0.05 (0.07) | 1.34 (0.1) | 0.1 (0.02) | 0.000 | 0.986 | −0.09 (0.07) | 1.38 (0.1) | 0.09 (0.02) | 0.000 | 0.984 |
| | 10m | −0.07 (0.07) | 1.43 (0.1) | 0.1 (0.02) | 0.000 | 0.984 | −0.1 (0.07) | 1.45 (0.1) | 0.09 (0.02) | 0.000 | 0.981 |
| | 15m | −0.08 (0.08) | 1.49 (0.11) | 0.1 (0.02) | 0.000 | 0.985 | −0.1 (0.07) | 1.49 (0.1) | 0.08 (0.02) | 0.000 | 0.976 |
| | 20m | −0.09 (0.08) | 1.42 (0.1) | 0.1 (0.02) | 0.000 | 0.985 | −0.11 (0.07) | 1.48 (0.1) | 0.08 (0.02) | 0.000 | 0.967 |
| | 30m | −0.08 (0.08) | 1.58 (0.12) | 0.11 (0.02) | 0.000 | 0.979 | −0.09 (0.07) | 1.59 (0.11) | 0.09 (0.02) | 0.000 | 0.965 |
| | 1h | −0.03 (0.07) | 1.72 (0.12) | 0.11 (0.02) | 0.000 | 0.989 | −0.09 (0.07) | 1.81 (0.13) | 0.09 (0.02) | 0.000 | 0.979 |
| TS | 30s | −0.24 (0.08) | 0.87 (0.06) | 0.1 (0.02) | 0.000 | 0.977 | −0.27 (0.08) | 0.88 (0.06) | 0.09 (0.02) | 0.000 | 0.977 |
| | 1m | −0.14 (0.08) | 1.12 (0.08) | 0.1 (0.02) | 0.213 | 0.981 | −0.18 (0.08) | 1.14 (0.08) | 0.09 (0.02) | 0.083 | 0.981 |
| | 2m | −0.1 (0.08) | 1.2 (0.09) | 0.1 (0.02) | 0.040 | 0.985 | −0.14 (0.08) | 1.22 (0.09) | 0.09 (0.02) | 0.029 | 0.984 |
| | 3m | −0.1 (0.08) | 1.24 (0.09) | 0.1 (0.02) | 0.008 | 0.988 | −0.14 (0.08) | 1.25 (0.09) | 0.09 (0.02) | 0.008 | 0.987 |
| | 4m | −0.1 (0.08) | 1.25 (0.09) | 0.1 (0.02) | 0.003 | 0.988 | −0.13 (0.08) | 1.27 (0.09) | 0.09 (0.02) | 0.003 | 0.986 |
| | 5m | −0.1 (0.08) | 1.27 (0.09) | 0.1 (0.02) | 0.001 | 0.989 | −0.13 (0.07) | 1.28 (0.09) | 0.08 (0.02) | 0.002 | 0.987 |
| | 6m | −0.1 (0.08) | 1.29 (0.09) | 0.1 (0.02) | 0.000 | 0.990 | −0.13 (0.07) | 1.3 (0.09) | 0.08 (0.02) | 0.001 | 0.987 |
| | 10m | −0.09 (0.08) | 1.3 (0.09) | 0.1 (0.02) | 0.000 | 0.989 | −0.12 (0.07) | 1.33 (0.09) | 0.09 (0.02) | 0.000 | 0.983 |
| | 15m | −0.07 (0.07) | 1.33 (0.09) | 0.1 (0.02) | 0.000 | 0.987 | −0.11 (0.07) | 1.36 (0.09) | 0.08 (0.02) | 0.000 | 0.981 |
| | 20m | −0.07 (0.07) | 1.35 (0.1) | 0.1 (0.02) | 0.000 | 0.987 | −0.11 (0.07) | 1.38 (0.1) | 0.08 (0.02) | 0.000 | 0.979 |
| | 30m | −0.07 (0.07) | 1.4 (0.1) | 0.1 (0.02) | 0.000 | 0.988 | −0.1 (0.07) | 1.41 (0.1) | 0.08 (0.02) | 0.000 | 0.975 |
| | 1h | −0.07 (0.08) | 2.12 (0.16) | 0.11 (0.02) | 0.000 | 0.988 | −0.1 (0.07) | 2.17 (0.15) | 0.09 (0.02) | 0.000 | 0.966 |
| K | 30s | 0 (0.07) | 1.13 (0.08) | 0.1 (0.02) | 0.031 | 0.986 | −0.03 (0.07) | 1.14 (0.08) | 0.09 (0.02) | 0.052 | 0.986 |
| | 1m | −0.04 (0.07) | 1.18 (0.08) | 0.1 (0.02) | 0.013 | 0.991 | −0.08 (0.07) | 1.2 (0.08) | 0.08 (0.02) | 0.017 | 0.988 |
| | 2m | −0.07 (0.08) | 1.25 (0.09) | 0.1 (0.02) | 0.001 | 0.994 | −0.1 (0.07) | 1.27 (0.09) | 0.08 (0.02) | 0.001 | 0.988 |
| | 3m | −0.06 (0.08) | 1.26 (0.09) | 0.1 (0.02) | 0.000 | 0.993 | −0.1 (0.07) | 1.28 (0.09) | 0.08 (0.02) | 0.001 | 0.986 |
| | 4m | −0.06 (0.08) | 1.25 (0.09) | 0.1 (0.02) | 0.001 | 0.989 | −0.09 (0.07) | 1.27 (0.09) | 0.09 (0.02) | 0.001 | 0.983 |
| | 5m | −0.1 (0.08) | 1.29 (0.09) | 0.11 (0.02) | 0.000 | 0.986 | −0.11 (0.07) | 1.3 (0.09) | 0.08 (0.02) | 0.000 | 0.977 |
| | 6m | −0.07 (0.08) | 1.26 (0.09) | 0.1 (0.02) | 0.001 | 0.989 | −0.1 (0.07) | 1.28 (0.09) | 0.08 (0.02) | 0.001 | 0.982 |
| | 10m | −0.07 (0.08) | 1.22 (0.09) | 0.1 (0.02) | 0.004 | 0.989 | −0.09 (0.08) | 1.25 (0.09) | 0.08 (0.02) | 0.002 | 0.975 |
| | 15m | −0.07 (0.08) | 1.2 (0.09) | 0.1 (0.02) | 0.013 | 0.995 | −0.1 (0.08) | 1.24 (0.09) | 0.08 (0.02) | 0.005 | 0.978 |
| | 20m | −0.1 (0.08) | 1.18 (0.09) | 0.1 (0.02) | 0.078 | 0.993 | −0.11 (0.08) | 1.2 (0.08) | 0.08 (0.02) | 0.040 | 0.972 |
| | 30m | −0.06 (0.07) | 1.14 (0.08) | 0.11 (0.02) | 0.112 | 0.988 | −0.22 (0.09) | 1.3 (0.09) | 0.1 (0.02) | 0.004 | 0.940 |
| | 1h | −0.1 (0.08) | 1.09 (0.08) | 0.11 (0.02) | 0.495 | 0.985 | −0.15 (0.08) | 1.15 (0.08) | 0.09 (0.02) | 0.146 | 0.943 |
| R | | −0.03 (0.08) | 1.17 (0.08) | 0.1 (0.02) | 0.010 | 0.986 | −0.06 (0.07) | 1.22 (0.08) | 0.08 (0.02) | 0.003 | 0.975 |

Table 8: Estimation results for the return model. For each measure, sampling frequencies (when applicable) and volatility model (Base or P-Spline) the table reports the estimates of the model parameters (standard errors in parenthesis), the p–value of the UVP test, and the p–value of the Ljung–Box test on the squared residuals $r_t^2/\hat{h}_t$.

# JNJ

| Meas. | Freq. | Base $\hat{c}$ | Base $\hat{m}$ | Base $\hat{\nu}$ | Base UVP | Base $Q^2_{10}$ | P-Spline $\hat{c}$ | P-Spline $\hat{m}$ | P-Spline $\hat{\nu}$ | P-Spline UVP | P-Spline $Q^2_{10}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| V | 30s | −0.09 (0.07) | 1.4 (0.13) | 0.17 (0.02) | 0.000 | 0.199 | −0.08 (0.07) | 1.33 (0.11) | 0.15 (0.02) | 0.000 | 0.262 |
| | 1m | −0.05 (0.07) | 1.25 (0.11) | 0.17 (0.02) | 0.002 | 0.194 | −0.06 (0.06) | 1.26 (0.11) | 0.17 (0.02) | 0.003 | 0.266 |
| | 2m | 0.04 (0.06) | 1.07 (0.1) | 0.17 (0.03) | 0.062 | 0.202 | 0 (0.06) | 1.12 (0.1) | 0.17 (0.02) | 0.059 | 0.282 |
| | 3m | 0.03 (0.06) | 1.08 (0.1) | 0.17 (0.03) | 0.078 | 0.226 | 0.01 (0.06) | 1.1 (0.1) | 0.17 (0.02) | 0.099 | 0.301 |
| | 4m | 0.06 (0.06) | 1.03 (0.1) | 0.17 (0.03) | 0.078 | 0.232 | 0.02 (0.06) | 1.08 (0.1) | 0.16 (0.02) | 0.096 | 0.301 |
| | 5m | 0.07 (0.06) | 1.04 (0.1) | 0.17 (0.03) | 0.028 | 0.185 | 0.04 (0.06) | 1.07 (0.1) | 0.17 (0.02) | 0.047 | 0.297 |
| | 6m | 0.06 (0.06) | 1.06 (0.1) | 0.17 (0.03) | 0.018 | 0.210 | 0.04 (0.06) | 1.09 (0.1) | 0.16 (0.02) | 0.017 | 0.288 |
| | 10m | 0.13 (0.06) | 1.05 (0.1) | 0.18 (0.03) | 0.000 | 0.292 | 0.09 (0.06) | 1.1 (0.1) | 0.17 (0.02) | 0.000 | 0.352 |
| | 15m | 0.13 (0.06) | 1.09 (0.11) | 0.18 (0.03) | 0.000 | 0.262 | 0.09 (0.05) | 1.13 (0.1) | 0.16 (0.02) | 0.000 | 0.330 |
| | 20m | 0.13 (0.06) | 1.14 (0.12) | 0.18 (0.03) | 0.000 | 0.353 | 0.09 (0.06) | 1.15 (0.1) | 0.17 (0.02) | 0.000 | 0.384 |
| | 30m | 0.12 (0.06) | 1.18 (0.12) | 0.18 (0.03) | 0.000 | 0.282 | 0.07 (0.05) | 1.24 (0.11) | 0.16 (0.02) | 0.000 | 0.307 |
| | 1h | 0.09 (0.06) | 1.55 (0.15) | 0.18 (0.03) | 0.000 | 0.325 | 0.08 (0.06) | 1.54 (0.14) | 0.17 (0.02) | 0.000 | 0.608 |
| B | 30s | −0.08 (0.07) | 1.64 (0.14) | 0.16 (0.02) | 0.000 | 0.220 | −0.07 (0.07) | 1.59 (0.13) | 0.15 (0.02) | 0.000 | 0.296 |
| | 1m | −0.05 (0.07) | 1.37 (0.12) | 0.17 (0.02) | 0.000 | 0.190 | −0.05 (0.06) | 1.37 (0.12) | 0.16 (0.02) | 0.000 | 0.265 |
| | 2m | 0.03 (0.06) | 1.14 (0.11) | 0.17 (0.03) | 0.003 | 0.238 | 0.01 (0.06) | 1.17 (0.1) | 0.16 (0.02) | 0.004 | 0.291 |
| | 3m | 0.04 (0.06) | 1.11 (0.11) | 0.17 (0.03) | 0.007 | 0.228 | 0.01 (0.06) | 1.13 (0.1) | 0.17 (0.02) | 0.015 | 0.287 |
| | 4m | 0.05 (0.06) | 1.09 (0.1) | 0.17 (0.03) | 0.013 | 0.212 | 0.02 (0.06) | 1.12 (0.1) | 0.16 (0.02) | 0.022 | 0.295 |
| | 5m | 0.11 (0.06) | 1 (0.1) | 0.17 (0.03) | 0.009 | 0.169 | 0.04 (0.06) | 1.11 (0.1) | 0.16 (0.02) | 0.010 | 0.282 |
| | 6m | 0.07 (0.06) | 1.11 (0.11) | 0.17 (0.03) | 0.001 | 0.194 | 0.04 (0.06) | 1.12 (0.1) | 0.16 (0.02) | 0.002 | 0.267 |
| | 10m | 0.12 (0.06) | 1.1 (0.11) | 0.18 (0.03) | 0.000 | 0.280 | 0.08 (0.06) | 1.15 (0.11) | 0.17 (0.02) | 0.000 | 0.356 |
| | 15m | 0.13 (0.06) | 1.13 (0.11) | 0.18 (0.03) | 0.000 | 0.258 | 0.09 (0.05) | 1.17 (0.1) | 0.16 (0.02) | 0.000 | 0.312 |
| | 20m | 0.14 (0.06) | 1.15 (0.12) | 0.18 (0.03) | 0.000 | 0.267 | 0.1 (0.05) | 1.19 (0.11) | 0.17 (0.02) | 0.000 | 0.393 |
| | 30m | 0.13 (0.06) | 1.25 (0.12) | 0.18 (0.03) | 0.000 | 0.180 | 0.1 (0.05) | 1.27 (0.11) | 0.17 (0.02) | 0.000 | 0.292 |
| | 1h | 0.1 (0.06) | 1.62 (0.16) | 0.19 (0.03) | 0.000 | 0.275 | 0.06 (0.05) | 1.69 (0.15) | 0.17 (0.02) | 0.000 | 0.444 |
| TS | 30s | −0.09 (0.07) | 0.74 (0.07) | 0.17 (0.02) | 0.000 | 0.228 | −0.08 (0.07) | 0.72 (0.06) | 0.16 (0.02) | 0.000 | 0.283 |
| | 1m | −0.05 (0.07) | 0.97 (0.09) | 0.17 (0.03) | 0.120 | 0.232 | −0.06 (0.06) | 0.96 (0.08) | 0.16 (0.02) | 0.050 | 0.292 |
| | 2m | 0.01 (0.07) | 0.99 (0.09) | 0.17 (0.03) | 0.994 | 0.221 | −0.01 (0.06) | 1.01 (0.09) | 0.17 (0.02) | 0.992 | 0.303 |
| | 3m | 0.02 (0.07) | 1.02 (0.1) | 0.17 (0.03) | 0.655 | 0.218 | 0.01 (0.06) | 1.02 (0.09) | 0.17 (0.02) | 0.792 | 0.315 |
| | 4m | 0.03 (0.07) | 1.06 (0.1) | 0.18 (0.03) | 0.110 | 0.418 | 0.02 (0.06) | 1.04 (0.09) | 0.16 (0.02) | 0.497 | 0.329 |
| | 5m | 0.05 (0.06) | 1.02 (0.1) | 0.17 (0.03) | 0.187 | 0.291 | 0.02 (0.06) | 1.05 (0.09) | 0.16 (0.02) | 0.275 | 0.330 |
| | 6m | 0.06 (0.07) | 1.05 (0.1) | 0.18 (0.03) | 0.042 | 0.471 | 0.03 (0.06) | 1.07 (0.09) | 0.16 (0.02) | 0.134 | 0.341 |
| | 10m | 0.09 (0.06) | 1.05 (0.1) | 0.17 (0.03) | 0.003 | 0.301 | 0.05 (0.06) | 1.09 (0.1) | 0.16 (0.02) | 0.009 | 0.345 |
| | 15m | 0.09 (0.06) | 1.08 (0.1) | 0.17 (0.03) | 0.000 | 0.233 | 0.06 (0.05) | 1.12 (0.1) | 0.16 (0.02) | 0.001 | 0.328 |
| | 20m | 0.12 (0.06) | 1.08 (0.1) | 0.17 (0.03) | 0.000 | 0.221 | 0.07 (0.05) | 1.13 (0.1) | 0.16 (0.02) | 0.000 | 0.333 |
| | 30m | 0.1 (0.06) | 1.2 (0.12) | 0.18 (0.03) | 0.000 | 0.407 | 0.07 (0.05) | 1.17 (0.1) | 0.16 (0.02) | 0.000 | 0.362 |
| | 1h | 0.14 (0.06) | 3.92 (0.39) | 0.18 (0.03) | 0.000 | 0.474 | 0.07 (0.05) | 4.23 (0.36) | 0.16 (0.02) | 0.000 | 0.550 |
| K | 30s | 0.11 (0.06) | 0.86 (0.08) | 0.17 (0.03) | 0.147 | 0.244 | 0.08 (0.06) | 0.9 (0.08) | 0.17 (0.02) | 0.417 | 0.316 |
| | 1m | 0.1 (0.06) | 0.91 (0.09) | 0.17 (0.03) | 0.298 | 0.200 | 0.08 (0.06) | 0.92 (0.09) | 0.17 (0.02) | 0.372 | 0.303 |
| | 2m | 0.07 (0.06) | 1 (0.1) | 0.17 (0.03) | 0.133 | 0.256 | 0.04 (0.06) | 1.02 (0.09) | 0.16 (0.02) | 0.208 | 0.288 |
| | 3m | 0.08 (0.06) | 1 (0.1) | 0.18 (0.03) | 0.096 | 0.239 | 0.04 (0.06) | 1.04 (0.09) | 0.16 (0.02) | 0.164 | 0.301 |
| | 4m | 0.07 (0.06) | 1.01 (0.1) | 0.18 (0.03) | 0.091 | 0.236 | 0.04 (0.06) | 1.04 (0.09) | 0.16 (0.02) | 0.189 | 0.271 |
| | 5m | 0.09 (0.06) | 1 (0.1) | 0.18 (0.03) | 0.065 | 0.241 | 0.05 (0.06) | 1.02 (0.09) | 0.16 (0.02) | 0.155 | 0.324 |
| | 6m | 0.09 (0.06) | 1 (0.1) | 0.17 (0.03) | 0.054 | 0.284 | 0.05 (0.06) | 1.03 (0.09) | 0.16 (0.02) | 0.113 | 0.327 |
| | 10m | 0.07 (0.06) | 1 (0.1) | 0.18 (0.03) | 0.142 | 0.197 | 0.04 (0.05) | 1.02 (0.09) | 0.16 (0.02) | 0.247 | 0.314 |
| | 15m | 0.12 (0.06) | 0.9 (0.09) | 0.17 (0.03) | 0.094 | 0.193 | 0.07 (0.05) | 0.96 (0.08) | 0.16 (0.02) | 0.330 | 0.298 |
| | 20m | 0.12 (0.06) | 0.87 (0.08) | 0.18 (0.03) | 0.126 | 0.274 | 0.08 (0.05) | 0.91 (0.08) | 0.16 (0.02) | 0.324 | 0.332 |
| | 30m | 0.11 (0.06) | 0.93 (0.09) | 0.18 (0.03) | 0.161 | 0.182 | 0.08 (0.05) | 0.94 (0.08) | 0.16 (0.03) | 0.240 | 0.306 |
| | 1h | 0.11 (0.06) | 0.85 (0.08) | 0.18 (0.03) | 0.117 | 0.158 | 0.06 (0.05) | 0.88 (0.07) | 0.15 (0.03) | 0.211 | 0.301 |
| R | | 0.05 (0.06) | 1.13 (0.1) | 0.17 (0.03) | 0.001 | 0.281 | 0.02 (0.05) | 1.14 (0.09) | 0.15 (0.02) | 0.004 | 0.354 |

Table 9: Estimation results for the return model. For each measure, sampling frequencies (when applicable) and volatility model (Base or P-Spline) the table reports the estimates of the model parameters (standard errors in parenthesis), the p–value of the UVP test, and the p–value of the Ljung–Box test on the squared residuals $r_t^2/\hat{h}_t$.

almost always significant. The null of unbiasedness is convincingly not rejected only for two scales realized volatility sampled at frequencies around 1 minute and for the realized kernel sampled at low frequencies for the BA and GE stocks and almost all frequencies for JNJ. The JNJ stock exhibits less evidence against the UVP test null than the other stocks.

The UVP test may be too crude to evaluate the precision of the volatility measures predictions as they are expected to be downward biased. Straightforward calculations allow us to use the return specification to compute the MSE of the volatility measures forecasts. Consider

$$\mathsf{MSE}\big(rv_{(m,\delta)\ t|t-1}\big) \equiv \mathsf{E}\big(h_t - rv_{(m,\delta)\ t|t-1}\big)^2 ;$$

simple algebra leads to

$$\begin{aligned}\mathsf{E}\big(h_t - rv_{(m,\delta)\ t|t-1}\big)^2 &= \mathsf{E}\big(c + m\, rv_{(m,\delta)\ t|t-1} - rv_{(m,\delta)\ t|t-1}\big)^2 \\ &= \big(c + (m-1)\ \mathsf{E}(rv_{(m,\delta)\ t|t-1})\big)^2 + (m-1)^2\ \mathsf{Var}(rv_{(m,\delta)\ t|t-1}).\end{aligned}$$

Figure 3 about here.

For diagnostic purposes we estimate such a quantity by plugging in the sample counterparts of the population parameters and parameter estimates using the estimation results over the full sample. Figure 3 displays the plots of the estimated MSE as a function of the sampling frequency for each volatility measure, in the spirit of the volatility signature plot (Andersen, Bollerslev, Christoffersen & Diebold (2006)). Interestingly, the graphs are remarkably similar across stocks and forecasting methods with the only exception of the range whose relative position is different from stock to stock. The MSE of realized volatility initially decreases as the sampling frequency increases and then it steadily increases as the sampling frequency is higher than a few minutes. The MSE of bipower realized volatility follows exactly the same pattern but is systematically higher. The MSE of two scales realized volatility steeply decreases as the sampling frequency increases. At a 30 seconds frequency the MSE of two scales realized volatility does increase abruptly but this is probably a consequence of the two scales being to close to one another. The MSE of the realized kernel seems not to be too sensitive to the choice of the sampling frequency. The ranking between UHFD volatility measures is rather clear: the realized kernels achieves the best performance followed by two scales realized, realized volatility and bipower realized volatility. Importantly, it appears that the simple range benchmark is difficult to beat. The range can be convincingly beaten according to this metric only by the realized kernel and by the other UHFD measures at frequencies higher than 15 minutes.

# 6 Forecasting Value–at–Risk

We can evaluate the quality of the VaR forecasts via a two stage procedure, aimed at assessing their conditional coverage (*adequacy*), using a battery of tests on the binary indicator of VaR failure, and at measuring their precision (*accuracy*), using a goodness of fit loss function on the predicted returns' tails (cf. the methodology proposed by Sarma et al. (2003)).

The VaR forecasting exercise is performed by estimating the Base and P-Spline models using approximately 900 days of data and deriving the one–day ahead VaR prediction as

$$\widehat{\mathsf{VaR}}^{p}_{t+1|t} = -\mathsf{F}^{-1}_{t_{1/\hat{\nu}}}(p)\,\sqrt{\hat{c} + \hat{m}\;\widehat{rv}_{(m,\delta)\;t+1|t}},$$

where $\widehat{rv}_{(m,\delta)\;t+1|t}$ is the one–step ahead volatility measure prediction obtained by the Base and P-Spline methods. The latter is estimated using 10 knots and the choice of the shrinkage coefficient $\lambda$ is performed via the AIC on the first rolling sample and then kept fixed for the rest of the prediction exercise. For comparison purposes, we also estimate a GARCH(1,1) model with leverage effects and Student's $t$ innovations. We then move ahead the sample by one day and repeat the procedure until we gather the series of 1 day ahead predictions (spanning about 3 years).

## 6.1 VaR Forecasting Adequacy

Let the failure process $\{H_{t+1}\}$ for the VaR be defined as

$$\left\{H_{t+1} \equiv \left(r_{t+1} < -\widehat{\mathsf{VaR}}^{p}_{t+1|t}\right)\right\}$$

If the sequence of VaR prediction is adequate, then the VaR conditional coverage should be equal to $p$ for any $t$, that is

$$\mathsf{E}(H_{t+1}|\mathcal{F}_t) = p. \tag{8}$$

Many of the VaR evaluation tests proposed in the literature attempt at assessing the adequacy of VaR predictions by testing against different types of departures from Equation (8).

**Unconditional Coverage test** (Christoffersen (1998)) Assuming that $\{H_{t+1}\}$ is an independently distributed failure process, the null hypothesis of the unconditional coverage test is that the failure probability is equal to $p$, and it is tested against the alternative of a failure rate different from $p$. Under the null, the test statistic is

$$LR_{\mathsf{uc}} = -2\log\frac{p^{n_1}(1-p)^{n_0}}{\hat{\pi}^{n_1}(1-\hat{\pi})^{n_0}} \sim \chi^2_{(1)},$$

where $n_0$ and $n_1$ are, respectively, the number of 0's and 1's in the series and $\hat{\pi} = n_1/(n_0+n_1)$.

**Independence test** (Christoffersen (1998)) The null hypothesis of the independence test is that the failure process $\{H_{t+1}\}$ is independently distributed, and it is tested against the alternative of a first order Markov process. Under the null, the test statistic is

$$LR_{\mathsf{ind}} = -2\log\frac{(1-\hat{\pi}_2)^{(n_{00}+n_{10})}\hat{\pi}_2^{(n_{01}+n_{11})}}{(1-\hat{\pi}_{01})^{n_{00}}\hat{\pi}_{01}^{n_{01}}(1-\hat{\pi}_{11})^{n_{10}}\hat{\pi}_{11}^{n_{11}}} \sim \chi^2_{(1)},$$

where $n_{ij}$ is the number of $i$ values followed by a $j$ in the $H_{t+1}$ series, $\hat{\pi}_{01} = n_{01}/(n_{00}+n_{01})$, $\hat{\pi}_{11} = n_{11}/(n_{10}+n_{11})$ and $\hat{\pi}_2 = (n_{01}+n_{11})/(n_{00}+n_{01}+n_{10}+n_{11})$.

**Conditional Coverage test** (Christoffersen (1998)) The null hypothesis of the conditional coverage test is that the failure process $\{H_{t+1}\}$ is an independent failure process with failure probability $p$, and it is tested against the alternative of a first–order Markov failure process with a different transition probability matrix. Under the null, the test statistic is

$$LR_{\mathsf{cc}} = -2\log\frac{p^{n_1}(1-p)^{n_0}}{(1-\hat{\pi}_{01})^{n_{00}}\hat{\pi}_{01}^{n_{01}}(1-\hat{\pi}_{11})^{n_{10}}\hat{\pi}_{11}^{n_{11}}} \sim \chi^2_{(2)}.$$

Note that, conditionally on the first observation, $LR_{\text{cc}} = LR_{\text{uc}} + LR_{\text{ind}}$.

**Dynamic Quantile test.** (Engle & Manganelli (2004)[6]) The version of the Dynamic Quantile test employed here aims at detecting no correlation between the sequence of $H_{t+1}$ (arranged in a vector $H$) and its past (here represented by four lagged values $(H_t, H_{t-1}, H_{t-2}, H_{t-3})$, gathered in a matrix $X$ which contains a vector of ones as well). Regressing $H - p\iota$ on $X$, we derive the LS estimator

$$\hat{\beta}_{\mathsf{LS}} = (X'X)^{-1}X'(H - p\iota);$$

from which we derive the Dynamic Quantile Hit test statistic for the null hypothesis $H_0 : \beta = 0$

$$DQ_{\mathsf{hit}} = \frac{\hat{\beta}'_{\mathsf{LS}} X'X \hat{\beta}_{\mathsf{LS}}}{p(1-p)} \sim \chi^2_q.$$

Tables 10, 11, 12 about here.

Tables 10, 11, 12 report the average number of failures at a nominal 99% coverage, the average VaR and the p–values of the adequacy tests[7]. and show that at a 1% significance level all the nulls of VaR adequacy are not rejected. In the BA and GE stock there is some mild evidence of over coverage that is stronger in the BA case using the Base forecasts and becomes weaker using the P-Spline forecasts at lower frequencies. In the JNJ stock there is some evidence of dependence in the VaR failures using the P-Spline forecasts. The volatility measure systematically lead to smaller average VaR than a GARCH and the P-Spline predictions systematically lead to smaller average VaR than the corresponding Base predictions. Overall, the adequacy of the VaR forecasts appears to be quite similar across all forecasting methods and it is difficult to find evidence that UHFD volatility measure provide significantly more adequate VaR forecasts than the forecasts based on the range or GARCH.

[6]Berkowitz, Christoffersen & Pelletier (2006) contains a Monte Carlo comparison of several VaR adequacy tests. The Dynamic Quantile tests appears to have the best finite sample properties.

[7]The 95% VaR adequacy results provide similar evidence and are not reported in the paper.

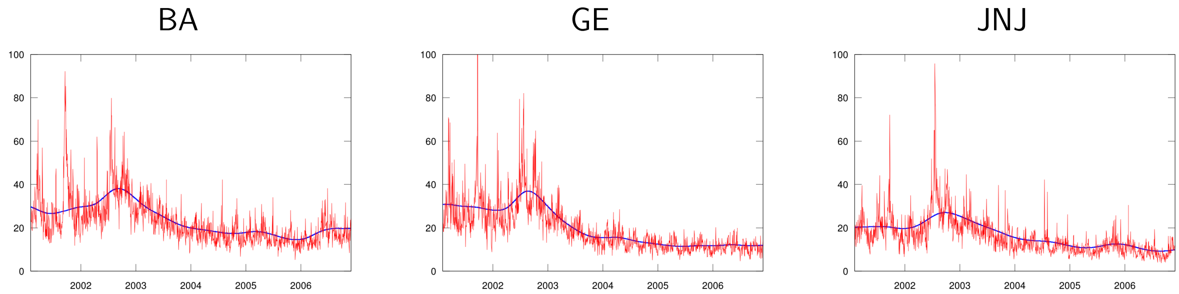


Figure 2: Annualized volatility Feb. 2001 – Dec 2006. The graphs display the plot of (annualized) realized volatility computed at a 5 min. frequency and the estimated volatility trend of the series

# BA

| | | Base | | | | | | P-Spline | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Meas. | Freq. | $\overline{H}$ | $\overline{\text{VaR}}$ | $\text{LR}_{\text{uc}}$ | $\text{LR}_{\text{ind}}$ | $\text{LR}_{\text{cc}}$ | $\text{DQ}_{\text{Hit}}$ | $\overline{H}$ | $\overline{\text{VaR}}$ | $\text{LR}_{\text{uc}}$ | $\text{LR}_{\text{ind}}$ | $\text{LR}_{\text{cc}}$ | $\text{DQ}_{\text{Hit}}$ |
| V | 30s | 0.36 | 334 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 326.83 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 1m | 0.36 | 334.94 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 328.02 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 2m | 0.18 | 333.31 | 0.017 | 0.952 | 0.057 | 0.589 | 0.54 | 325.99 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 3m | 0.18 | 332.52 | 0.017 | 0.952 | 0.057 | 0.589 | 0.36 | 325.62 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 4m | 0.18 | 330.41 | 0.017 | 0.952 | 0.057 | 0.589 | 0.54 | 322.56 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 5m | 0.18 | 330.57 | 0.017 | 0.952 | 0.057 | 0.589 | 0.36 | 320.92 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 6m | 0.36 | 326.75 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 315.98 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 10m | 0.18 | 327.41 | 0.017 | 0.952 | 0.057 | 0.589 | 0.54 | 316.59 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 15m | 0.18 | 324.84 | 0.017 | 0.952 | 0.057 | 0.589 | 0.36 | 313.44 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 20m | 0.36 | 326.87 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 317.32 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 30m | 0.36 | 323.72 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 318.23 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 1h | 0.37 | 328.26 | 0.090 | 0.903 | 0.236 | 0.827 | 0.55 | 315.33 | 0.254 | 0.855 | 0.513 | 0.954 |
| B | 30s | 0.36 | 342.74 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 334.19 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 1m | 0.18 | 340.49 | 0.017 | 0.952 | 0.057 | 0.589 | 0.36 | 332.64 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 2m | 0.18 | 336.76 | 0.017 | 0.952 | 0.057 | 0.589 | 0.36 | 329.13 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 3m | 0.18 | 333.67 | 0.017 | 0.952 | 0.057 | 0.589 | 0.54 | 327.36 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 4m | 0.18 | 331.68 | 0.017 | 0.952 | 0.057 | 0.589 | 0.36 | 323.38 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 5m | 0.18 | 331.96 | 0.017 | 0.952 | 0.057 | 0.589 | 0.36 | 322.24 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 6m | 0.18 | 328.43 | 0.017 | 0.952 | 0.057 | 0.589 | 0.36 | 318.78 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 10m | 0.18 | 328.26 | 0.017 | 0.952 | 0.057 | 0.589 | 0.36 | 318.21 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 15m | 0.18 | 325.27 | 0.017 | 0.952 | 0.057 | 0.589 | 0.36 | 313.48 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 20m | 0.36 | 327.31 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 314.30 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 30m | 0.36 | 326.1 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 313.34 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 1h | 0.37 | 328.64 | 0.090 | 0.903 | 0.236 | 0.827 | 0.54 | 315.47 | 0.233 | 0.857 | 0.484 | 0.946 |
| TS | 30s | 0.36 | 336.77 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 329.48 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 1m | 0.36 | 336.18 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 328.45 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 2m | 0.18 | 334.3 | 0.017 | 0.952 | 0.057 | 0.589 | 0.36 | 327.15 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 3m | 0.18 | 332.43 | 0.017 | 0.952 | 0.057 | 0.589 | 0.36 | 324.19 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 4m | 0.18 | 330.41 | 0.017 | 0.952 | 0.057 | 0.589 | 0.54 | 321.76 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 5m | 0.18 | 328.94 | 0.017 | 0.952 | 0.057 | 0.589 | 0.54 | 320.17 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 6m | 0.18 | 327.69 | 0.017 | 0.952 | 0.057 | 0.589 | 0.54 | 318.92 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 10m | 0.18 | 326.82 | 0.017 | 0.952 | 0.057 | 0.589 | 0.54 | 315.63 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 15m | 0.18 | 327.86 | 0.017 | 0.952 | 0.057 | 0.589 | 0.54 | 315.88 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 20m | 0.36 | 328.17 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 316.02 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 30m | 0.36 | 329.6 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 316.68 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 1h | 0.37 | 332.67 | 0.090 | 0.903 | 0.236 | 0.827 | 0.37 | 315.39 | 0.090 | 0.903 | 0.236 | 0.827 |
| K | 30s | 0.36 | 326.86 | 0.081 | 0.904 | 0.217 | 0.811 | 0.72 | 318.4 | 0.486 | 0.809 | 0.762 | 0.992 |
| | 1m | 0.36 | 326.22 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 316.81 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 2m | 0.36 | 324.93 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 316.8 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 3m | 0.36 | 334.37 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 322.58 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 4m | 0.18 | 338.87 | 0.017 | 0.952 | 0.057 | 0.589 | 0.36 | 324.21 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 5m | 0.18 | 336.61 | 0.017 | 0.952 | 0.057 | 0.589 | 0.54 | 322.13 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 6m | 0.36 | 333.46 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 320.92 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 10m | 0.36 | 323.65 | 0.081 | 0.904 | 0.217 | 0.811 | 0.72 | 311.76 | 0.486 | 0.809 | 0.762 | 0.992 |
| | 15m | 0.36 | 332.95 | 0.081 | 0.904 | 0.217 | 0.811 | 0.72 | 316.02 | 0.486 | 0.809 | 0.762 | 0.992 |
| | 20m | 0.36 | 342.47 | 0.081 | 0.904 | 0.217 | 0.811 | 0.72 | 314.23 | 0.486 | 0.809 | 0.762 | 0.992 |
| | 30m | 0.36 | 350.45 | 0.081 | 0.904 | 0.217 | 0.811 | 0.72 | 315.18 | 0.486 | 0.809 | 0.762 | 0.992 |
| | 1h | 0.37 | 355.57 | 0.090 | 0.903 | 0.236 | 0.827 | 0.72 | 319.89 | 0.486 | 0.809 | 0.762 | 0.992 |
| R | | 0.36 | 340.44 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 319.69 | 0.233 | 0.857 | 0.484 | 0.946 |
| GARCH | | 0.9 | 380.1 | 0.233 | 0.857 | 0.484 | 0.946 | | | | | | |

Table 10: 99% VaR forecasting adequacy results. For each measure, sampling frequencies (when applicable) and volatility model (Base or P-Spline) the table reports the average number of VaR failures, the average VaR and the p–values of the adequacy tests.

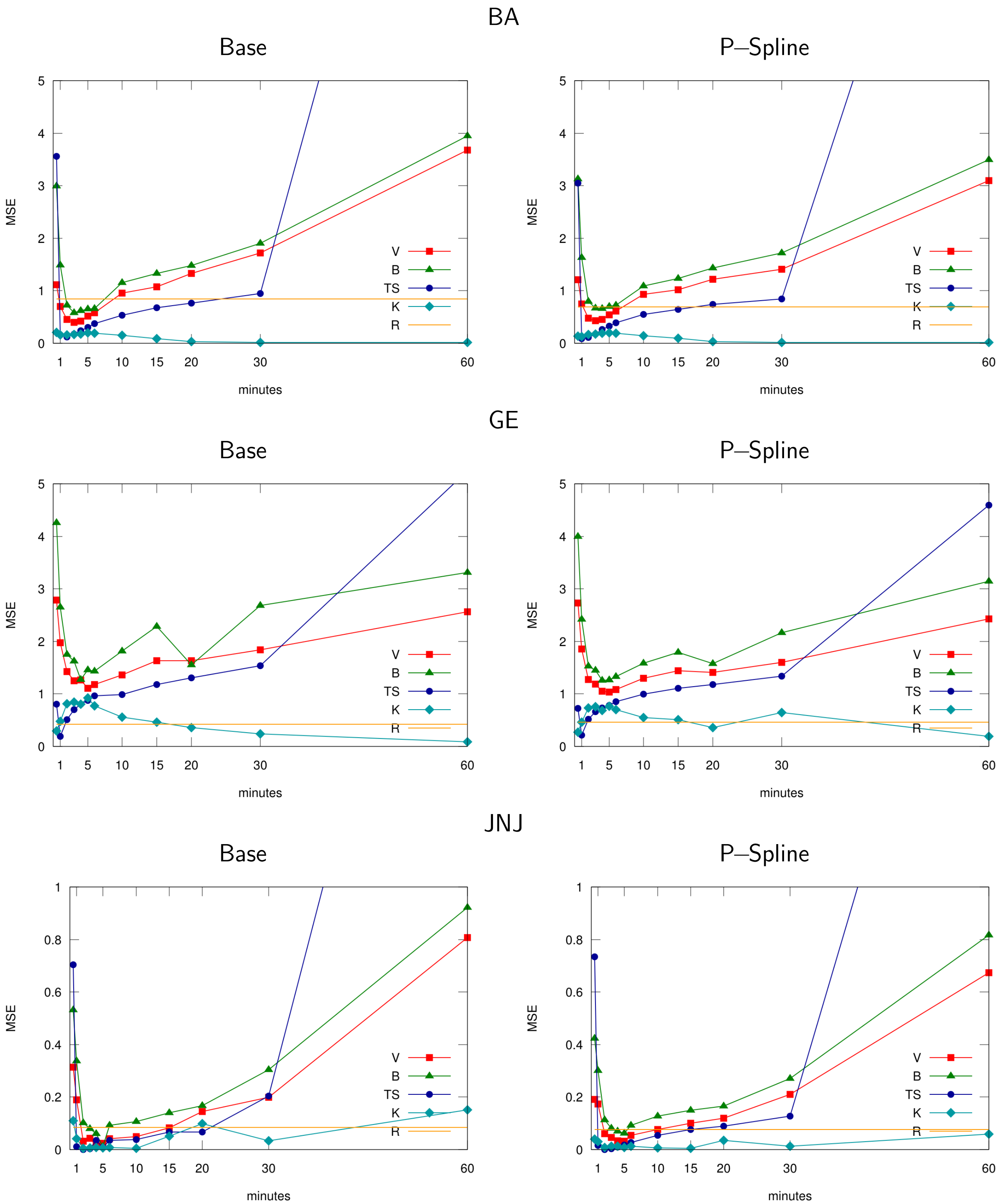


Figure 3: In–sample volatility MSE of the volatility measures. The graphs display the estimated MSEs of the volatility measures as a function of the sampling frequency.

# GE

| | | Base | | | | | | P-Spline | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Meas. | Freq. | $\overline{H}$ | $\overline{\text{VaR}}$ | $\text{LR}_{\text{uc}}$ | $\text{LR}_{\text{ind}}$ | $\text{LR}_{\text{cc}}$ | $\text{DQ}_{\text{Hit}}$ | $\overline{H}$ | $\overline{\text{VaR}}$ | $\text{LR}_{\text{uc}}$ | $\text{LR}_{\text{ind}}$ | $\text{LR}_{\text{cc}}$ | $\text{DQ}_{\text{Hit}}$ |
| V | 30s | 0.36 | 231.91 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 222.86 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 1m | 0.36 | 230.39 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 220.63 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 2m | 0.36 | 228.77 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 217.61 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 3m | 0.36 | 229.74 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 216.54 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 4m | 0.36 | 229.28 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 217.77 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 5m | 0.36 | 229.09 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 215.58 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 6m | 0.36 | 228.28 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 214.53 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 10m | 0.36 | 229.55 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 215.4 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 15m | 0.36 | 226.4 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 211.11 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 20m | 0.36 | 224.93 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 210.39 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 30m | 0.36 | 232.53 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 215.26 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 1h | 0.36 | 232.93 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 213.25 | 0.081 | 0.904 | 0.217 | 0.811 |
| B | 30s | 0.36 | 232.03 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 223.08 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 1m | 0.36 | 229.96 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 220.04 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 2m | 0.36 | 228.25 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 216.64 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 3m | 0.36 | 229 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 214.92 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 4m | 0.36 | 230.08 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 217.82 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 5m | 0.36 | 231.35 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 217.7 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 6m | 0.36 | 229.12 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 215.03 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 10m | 0.36 | 229.84 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 215.82 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 15m | 0.36 | 226.81 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 211.11 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 20m | 0.36 | 221.16 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 209.17 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 30m | 0.36 | 230.77 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 213.46 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 1h | 0.37 | 237.83 | 0.092 | 0.903 | 0.240 | 0.831 | 0.37 | 218.25 | 0.092 | 0.903 | 0.240 | 0.831 |
| TS | 30s | 0.36 | 232.65 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 223.5 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 1m | 0.36 | 231.64 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 221.52 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 2m | 0.36 | 230.28 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 219.75 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 3m | 0.36 | 229.69 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 218.03 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 4m | 0.36 | 229.55 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 217.17 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 5m | 0.36 | 229.56 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 216.55 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 6m | 0.36 | 229.44 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 216.88 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 10m | 0.36 | 230.08 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 216.94 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 15m | 0.36 | 228.59 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 215.13 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 20m | 0.36 | 226.74 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 212.07 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 30m | 0.36 | 226.81 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 213.21 | 0.233 | 0.857 | 0.484 | 0.946 |
| | 1h | 0.36 | 232.7 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 212.39 | 0.081 | 0.904 | 0.217 | 0.811 |
| K | 30s | 0.36 | 220.86 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 207.83 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 1m | 0.36 | 223.71 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 208.24 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 2m | 0.36 | 223.69 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 206.1 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 3m | 0.36 | 224.58 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 208.2 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 4m | 0.36 | 222.23 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 204.68 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 5m | 0.36 | 220.04 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 205.43 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 6m | 0.36 | 222.9 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 208.7 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 10m | 0.36 | 223.15 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 204.57 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 15m | 0.36 | 225.65 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 205.83 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 20m | 0.36 | 227.63 | 0.081 | 0.904 | 0.217 | 0.811 | 0.36 | 205.84 | 0.081 | 0.904 | 0.217 | 0.811 |
| | 30m | 0.36 | 232.57 | 0.081 | 0.904 | 0.217 | 0.811 | 0.54 | 206.43 | 0.233 | 0.008 | 0.015 | 0.000 |
| | 1h | 0.18 | 236.42 | 0.017 | 0.952 | 0.057 | 0.589 | 0.54 | 204.86 | 0.233 | 0.008 | 0.015 | 0.000 |
| R | | 0.36 | 224.53 | 0.081 | 0.904 | 0.217 | 0.811 | 0.72 | 204.32 | 0.486 | 0.809 | 0.762 | 0.992 |
| GARCH | | 0.36 | 240.8 | 0.081 | 0.904 | 0.217 | 0.811 | | | | | | |

Table 11: 99% VaR forecasting adequacy results. For each measure, sampling frequencies (when applicable) and volatility model (Base or P-Spline) the table reports the average number of VaR failures, the average VaR and the p–values of the adequacy tests.

# JNJ

| | | Base | | | | | | P-Spline | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Meas. | Freq. | $\overline{H}$ | $\overline{\text{VaR}}$ | $\text{LR}_{\text{uc}}$ | $\text{LR}_{\text{ind}}$ | $\text{LR}_{\text{cc}}$ | $\text{DQ}_{\text{Hit}}$ | $\overline{H}$ | $\overline{\text{VaR}}$ | $\text{LR}_{\text{uc}}$ | $\text{LR}_{\text{ind}}$ | $\text{LR}_{\text{cc}}$ | $\text{DQ}_{\text{Hit}}$ |
| V | 30s | 0.54 | 218.02 | 0.233 | 0.857 | 0.484 | 0.946 | 0.9 | 207.28 | 0.811 | 0.763 | 0.929 | 0.999 |
| | 1m | 0.9 | 213.54 | 0.811 | 0.763 | 0.929 | 0.999 | 0.9 | 203.24 | 0.811 | 0.763 | 0.929 | 0.999 |
| | 2m | 0.9 | 211.44 | 0.811 | 0.763 | 0.929 | 0.999 | 1.08 | 200.52 | 0.850 | 0.717 | 0.920 | 0.997 |
| | 3m | 1.08 | 212.08 | 0.850 | 0.717 | 0.920 | 0.997 | 1.08 | 201.32 | 0.850 | 0.717 | 0.920 | 0.997 |
| | 4m | 1.08 | 211.53 | 0.850 | 0.717 | 0.920 | 0.997 | 1.08 | 199.75 | 0.850 | 0.717 | 0.920 | 0.997 |
| | 5m | 0.9 | 213.37 | 0.811 | 0.763 | 0.929 | 0.999 | 1.26 | 200.63 | 0.552 | 0.672 | 0.766 | 0.020 |
| | 6m | 1.08 | 209.39 | 0.850 | 0.717 | 0.920 | 0.997 | 1.26 | 196.7 | 0.552 | 0.672 | 0.766 | 0.020 |
| | 10m | 0.9 | 215.36 | 0.811 | 0.763 | 0.929 | 0.999 | 1.26 | 199.18 | 0.552 | 0.672 | 0.766 | 0.020 |
| | 15m | 1.08 | 218.07 | 0.850 | 0.717 | 0.920 | 0.997 | 1.26 | 198.77 | 0.552 | 0.672 | 0.766 | 0.020 |
| | 20m | 0.9 | 220.65 | 0.811 | 0.763 | 0.929 | 0.999 | 1.26 | 202.1 | 0.552 | 0.672 | 0.766 | 0.020 |
| | 30m | 0.9 | 215.6 | 0.811 | 0.763 | 0.929 | 0.999 | 0.9 | 198.3 | 0.811 | 0.763 | 0.929 | 0.001 |
| | 1h | 0.75 | 213.26 | 0.547 | 0.805 | 0.810 | 0.995 | 0.94 | 199.38 | 0.888 | 0.758 | 0.944 | 0.999 |
| B | 30s | 0.54 | 224.33 | 0.233 | 0.857 | 0.484 | 0.946 | 0.72 | 211.79 | 0.486 | 0.809 | 0.762 | 0.992 |
| | 1m | 0.54 | 215.2 | 0.233 | 0.857 | 0.484 | 0.946 | 0.9 | 204.39 | 0.811 | 0.763 | 0.929 | 0.999 |
| | 2m | 0.9 | 212.67 | 0.811 | 0.763 | 0.929 | 0.999 | 1.08 | 200.71 | 0.850 | 0.717 | 0.920 | 0.997 |
| | 3m | 1.08 | 211.61 | 0.850 | 0.717 | 0.920 | 0.997 | 1.08 | 200.92 | 0.850 | 0.717 | 0.920 | 0.997 |
| | 4m | 1.08 | 211.36 | 0.850 | 0.717 | 0.920 | 0.997 | 1.08 | 199.8 | 0.850 | 0.717 | 0.920 | 0.997 |
| | 5m | 0.9 | 213.07 | 0.811 | 0.763 | 0.929 | 0.999 | 1.26 | 201.12 | 0.552 | 0.672 | 0.766 | 0.020 |
| | 6m | 0.9 | 210.83 | 0.811 | 0.763 | 0.929 | 0.999 | 1.26 | 197.92 | 0.552 | 0.672 | 0.766 | 0.020 |
| | 10m | 0.9 | 215.99 | 0.811 | 0.763 | 0.929 | 0.999 | 1.26 | 199.85 | 0.552 | 0.672 | 0.766 | 0.020 |
| | 15m | 1.08 | 218.11 | 0.850 | 0.717 | 0.920 | 0.997 | 1.26 | 197.08 | 0.552 | 0.672 | 0.766 | 0.020 |
| | 20m | 0.9 | 220.22 | 0.811 | 0.763 | 0.929 | 0.999 | 1.26 | 202.16 | 0.552 | 0.672 | 0.766 | 0.020 |
| | 30m | 1.08 | 218.5 | 0.850 | 0.717 | 0.920 | 0.997 | 0.9 | 200.96 | 0.811 | 0.763 | 0.929 | 0.001 |
| | 1h | 0.75 | 215.63 | 0.547 | 0.805 | 0.810 | 0.995 | 0.94 | 201.24 | 0.888 | 0.758 | 0.944 | 0.999 |
| TS | 30s | 0.54 | 219.39 | 0.233 | 0.857 | 0.484 | 0.946 | 0.72 | 208.76 | 0.486 | 0.809 | 0.762 | 0.992 |
| | 1m | 0.72 | 213.67 | 0.486 | 0.809 | 0.762 | 0.992 | 1.08 | 204.2 | 0.850 | 0.717 | 0.920 | 0.007 |
| | 2m | 0.9 | 212.45 | 0.811 | 0.763 | 0.929 | 0.999 | 1.08 | 201.63 | 0.850 | 0.717 | 0.920 | 0.997 |
| | 3m | 1.08 | 212.93 | 0.850 | 0.717 | 0.920 | 0.997 | 1.08 | 201.26 | 0.850 | 0.717 | 0.920 | 0.997 |
| | 4m | 1.08 | 213.78 | 0.850 | 0.717 | 0.920 | 0.997 | 1.08 | 201.08 | 0.850 | 0.717 | 0.920 | 0.997 |
| | 5m | 1.08 | 213.89 | 0.850 | 0.717 | 0.920 | 0.997 | 1.26 | 201.13 | 0.552 | 0.672 | 0.766 | 0.020 |
| | 6m | 1.08 | 214.18 | 0.850 | 0.717 | 0.920 | 0.997 | 1.26 | 200.29 | 0.552 | 0.672 | 0.766 | 0.020 |
| | 10m | 1.08 | 215.56 | 0.850 | 0.717 | 0.920 | 0.997 | 1.26 | 200.52 | 0.552 | 0.672 | 0.766 | 0.020 |
| | 15m | 1.08 | 215.89 | 0.850 | 0.717 | 0.920 | 0.997 | 1.26 | 198.93 | 0.552 | 0.672 | 0.766 | 0.020 |
| | 20m | 1.08 | 216.36 | 0.850 | 0.717 | 0.920 | 0.997 | 1.26 | 199.17 | 0.552 | 0.672 | 0.766 | 0.020 |
| | 30m | 0.9 | 218.5 | 0.811 | 0.763 | 0.929 | 0.999 | 1.08 | 199.51 | 0.850 | 0.717 | 0.920 | 0.007 |
| | 1h | 0.75 | 221.8 | 0.547 | 0.805 | 0.810 | 0.995 | 0.94 | 199.87 | 0.888 | 0.758 | 0.944 | 0.001 |
| K | 30s | 0.72 | 219.92 | 0.486 | 0.809 | 0.762 | 0.992 | 1.08 | 205.29 | 0.850 | 0.717 | 0.920 | 0.997 |
| | 1m | 0.9 | 217.01 | 0.811 | 0.763 | 0.929 | 0.999 | 1.08 | 205.76 | 0.850 | 0.717 | 0.920 | 0.997 |
| | 2m | 0.9 | 213.46 | 0.811 | 0.763 | 0.929 | 0.999 | 1.08 | 200.53 | 0.850 | 0.717 | 0.920 | 0.007 |
| | 3m | 0.72 | 215.4 | 0.486 | 0.809 | 0.762 | 0.992 | 0.9 | 201.45 | 0.811 | 0.763 | 0.929 | 0.001 |
| | 4m | 0.72 | 216.31 | 0.486 | 0.809 | 0.762 | 0.992 | 1.08 | 199.72 | 0.850 | 0.717 | 0.920 | 0.007 |
| | 5m | 0.9 | 213.95 | 0.811 | 0.763 | 0.929 | 0.999 | 1.08 | 199.25 | 0.850 | 0.717 | 0.920 | 0.997 |
| | 6m | 0.9 | 214.24 | 0.811 | 0.763 | 0.929 | 0.999 | 1.08 | 197.00 | 0.850 | 0.717 | 0.920 | 0.997 |
| | 10m | 0.54 | 221.73 | 0.233 | 0.857 | 0.484 | 0.946 | 1.26 | 203.31 | 0.552 | 0.672 | 0.766 | 0.020 |
| | 15m | 0.9 | 219.88 | 0.811 | 0.763 | 0.929 | 0.999 | 1.80 | 199.89 | 0.088 | 0.544 | 0.194 | 0.000 |
| | 20m | 0.9 | 217.99 | 0.811 | 0.763 | 0.929 | 0.999 | 1.26 | 200.54 | 0.552 | 0.672 | 0.766 | 0.020 |
| | 30m | 0.9 | 227.56 | 0.811 | 0.763 | 0.929 | 0.999 | 1.80 | 201.23 | 0.088 | 0.544 | 0.194 | 0.000 |
| | 1h | 0.56 | 239.78 | 0.271 | 0.854 | 0.536 | 0.960 | 1.13 | 210.11 | 0.772 | 0.711 | 0.895 | 0.007 |
| R | | 0.9 | 226.82 | 0.811 | 0.763 | 0.929 | 0.999 | 1.08 | 207.39 | 0.850 | 0.717 | 0.920 | 0.007 |
| GARCH | | 0.9 | 254.03 | 0.811 | 0.763 | 0.929 | 0.999 | | | | | | |

Table 12: 99% VaR forecasting adequacy results. For each measure, sampling frequencies (when applicable) and volatility model (Base or P-Spline) the table reports the average number of VaR failures, the average VaR and the p–values of the adequacy tests.

## 6.2 VaR Forecasting Accuracy

We evaluate the out–of–sample accuracy of the VaR forecast using the probability deviation loss functions proposed by Kuester et al. (2006), The loss function is computed using the series of probability integral transformations of the returns using their estimated one day ahead cdf, i.e. $\hat{u}_{t+1} = \widehat{\mathsf{F}}_{t+1|t}(r_{t+1})$. For each of such $\hat{u}_{t+1}$ in $(0, 0.10]$, the probability deviations $\hat{d}_u$ are defined as the difference between the empirical cdf of the $\hat{u}$'s and a uniform cdf. We can then construct goodness of fit measures of the models on the left tail of the return distribution as the mean of squared and of absolute probability deviations, that is

$$\mathsf{MSE} \equiv \sum_{\hat{u} \in (0,0.10]} \hat{d}_u^{\,2} \qquad \mathsf{MAE} \equiv \sum_{\hat{u} \in (0,0.10]} |\hat{d}_u|.$$

Such loss functions have interesting prequential appeal (Dawid (1984)) and are also reminiscent of previous work on density forecast evaluation like Diebold, Gunther & Tay (1998).

Tables 13 about here.

Figure 4 about here.

Tables 13 report the MSE and MAE and Figure 4 displays the graphs of the volatility measures MSE as functions of the sampling frequency. The Base out of sample performance of the UHFD volatility measures behaves rather similarly across stocks. The P-Spline method results tell a slightly different story. First, P-Spline forecasts systematically increase the out–of–sample accuracy of the VaR forecasts over the Base counterparts. While realized volatility, bipower realized volatility and two scale realized volatility still have a very similar out–of–sample performance, realized kernel performs systematically better at higher frequencies. In most cases performance improves as the sampling frequency decreases and the best out of sample performance is obtained around 20–30 minutes. Coherently with the in–sample results, realized kernel in conjunction with the P-spline method has a very promising out–of–sample performance that is fairly invariant to the choice of the sampling frequency. The UHFD measures always produce more accurate forecasts than the GARCH benchmark, with the exception of the Base forecasts for the GE stock. However, the range appears

| | | BA | | | | GE | | | | JNJ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Base | | P-Spline | | Base | | P-Spline | | Base | | P-Spline | |
| Meas. | Freq. | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE | MSE | MAE |
| V | 30s | 3.147 | 1.683 | 2.229 | 1.409 | 10.492 | 3.131 | 7.334 | 2.628 | 2.374 | 1.385 | 0.503 | 0.609 |
| | 1m | 3.343 | 1.73 | 2.492 | 1.469 | 9.366 | 2.966 | 6.037 | 2.382 | 2.008 | 1.263 | 0.338 | 0.487 |
| | 2m | 3.222 | 1.704 | 2.066 | 1.342 | 7.994 | 2.741 | 4.433 | 2.032 | 2.133 | 1.288 | 0.47 | 0.544 |
| | 3m | 3.024 | 1.656 | 1.968 | 1.306 | 8.511 | 2.829 | 3.634 | 1.842 | 2.4 | 1.378 | 0.445 | 0.581 |
| | 4m | 2.477 | 1.497 | 1.436 | 1.075 | 8.213 | 2.777 | 3.423 | 1.795 | 2.307 | 1.352 | 0.341 | 0.488 |
| | 5m | 2.707 | 1.551 | 1.454 | 1.084 | 7.918 | 2.724 | 2.884 | 1.644 | 2.519 | 1.402 | 0.108 | 0.244 |
| | 6m | 2.497 | 1.492 | 0.889 | 0.786 | 7.877 | 2.721 | 2.062 | 1.388 | 1.768 | 1.145 | 0.18 | 0.314 |
| | 10m | 2.057 | 1.379 | 0.551 | 0.68 | 8.035 | 2.733 | 2.758 | 1.588 | 2.288 | 1.323 | 0.191 | 0.333 |
| | 15m | 2.191 | 1.403 | 0.32 | 0.513 | 7.596 | 2.666 | 1.617 | 1.199 | 3.082 | 1.57 | 0.07 | 0.222 |
| | 20m | 2.282 | 1.459 | 0.298 | 0.478 | 6.697 | 2.502 | 1.977 | 1.312 | 3.885 | 1.726 | 0.09 | 0.242 |
| | 30m | 1.639 | 1.202 | 0.132 | 0.32 | 8.602 | 2.835 | 1.35 | 1.109 | 1.993 | 1.248 | 0.09 | 0.242 |
| | 1h | 1.933 | 1.288 | 0.03 | 0.143 | 8.145 | 2.758 | 0.483 | 0.641 | 2.85 | 1.446 | 0.404 | 0.508 |
| B | 30s | 4.391 | 2.004 | 3.059 | 1.661 | 10.537 | 3.147 | 7.471 | 2.654 | 3.563 | 1.697 | 0.712 | 0.763 |
| | 1m | 4.51 | 2.012 | 3.308 | 1.705 | 9.305 | 2.96 | 5.811 | 2.335 | 2.183 | 1.32 | 0.361 | 0.478 |
| | 2m | 3.689 | 1.824 | 2.279 | 1.395 | 7.801 | 2.707 | 4.138 | 1.96 | 2.405 | 1.371 | 0.449 | 0.529 |
| | 3m | 3.417 | 1.739 | 2.135 | 1.348 | 7.53 | 2.661 | 3.315 | 1.76 | 2.228 | 1.333 | 0.417 | 0.532 |
| | 4m | 2.694 | 1.561 | 1.374 | 1.045 | 8.355 | 2.801 | 3.03 | 1.687 | 2.493 | 1.396 | 0.382 | 0.52 |
| | 5m | 2.921 | 1.602 | 1.498 | 1.075 | 8.376 | 2.799 | 2.991 | 1.676 | 2.326 | 1.35 | 0.077 | 0.211 |
| | 6m | 2.424 | 1.462 | 0.989 | 0.858 | 7.721 | 2.691 | 1.993 | 1.345 | 1.939 | 1.205 | 0.189 | 0.327 |
| | 10m | 2.156 | 1.416 | 0.652 | 0.771 | 7.727 | 2.68 | 2.898 | 1.632 | 2.601 | 1.416 | 0.236 | 0.427 |
| | 15m | 2.303 | 1.44 | 0.464 | 0.591 | 8.11 | 2.749 | 2.352 | 1.447 | 3.175 | 1.584 | 0.07 | 0.224 |
| | 20m | 2.275 | 1.463 | 0.295 | 0.496 | 6.274 | 2.43 | 2.101 | 1.377 | 3.63 | 1.663 | 0.15 | 0.32 |
| | 30m | 1.536 | 1.157 | 0.441 | 0.727 | 7.523 | 2.649 | 1.087 | 0.963 | 2.261 | 1.357 | 0.102 | 0.26 |
| | 1h | 1.571 | 1.188 | 0.792 | 0.85 | 10.208 | 3.088 | 1.775 | 1.271 | 3.516 | 1.578 | 0.628 | 0.633 |
| TS | 30s | 3.725 | 1.829 | 2.547 | 1.504 | 10.492 | 3.13 | 6.688 | 2.511 | 2.912 | 1.531 | 0.438 | 0.579 |
| | 1m | 3.647 | 1.808 | 2.478 | 1.471 | 10.108 | 3.08 | 5.711 | 2.314 | 2.26 | 1.332 | 0.343 | 0.501 |
| | 2m | 3.461 | 1.761 | 2.309 | 1.377 | 9.056 | 2.913 | 4.301 | 1.993 | 2.47 | 1.399 | 0.443 | 0.57 |
| | 3m | 3.252 | 1.709 | 1.867 | 1.213 | 8.3 | 2.785 | 3.619 | 1.839 | 2.644 | 1.449 | 0.419 | 0.553 |
| | 4m | 2.753 | 1.581 | 1.287 | 0.98 | 8.027 | 2.74 | 3.103 | 1.705 | 2.823 | 1.5 | 0.238 | 0.39 |
| | 5m | 2.485 | 1.496 | 1.041 | 0.866 | 7.909 | 2.721 | 3.035 | 1.687 | 2.773 | 1.48 | 0.144 | 0.295 |
| | 6m | 2.376 | 1.459 | 0.995 | 0.854 | 7.813 | 2.704 | 2.585 | 1.555 | 2.772 | 1.47 | 0.121 | 0.268 |
| | 10m | 2.267 | 1.426 | 0.47 | 0.564 | 7.902 | 2.716 | 2.518 | 1.532 | 3.136 | 1.54 | 0.153 | 0.332 |
| | 15m | 2.23 | 1.426 | 0.356 | 0.468 | 7.353 | 2.621 | 2.094 | 1.381 | 2.965 | 1.491 | 0.097 | 0.241 |
| | 20m | 2.307 | 1.456 | 0.341 | 0.472 | 7.405 | 2.63 | 2.1 | 1.393 | 2.881 | 1.481 | 0.076 | 0.226 |
| | 30m | 2.477 | 1.504 | 0.197 | 0.395 | 7.574 | 2.665 | 2.123 | 1.395 | 3.204 | 1.563 | 0.077 | 0.238 |
| | 1h | 2.578 | 1.467 | 0.276 | 0.443 | 9.977 | 3.055 | 3.068 | 1.646 | 5.658 | 2.143 | 0.426 | 0.568 |
| K | 30s | 4.068 | 1.905 | 2.83 | 1.545 | 9.315 | 2.947 | 4.431 | 2.017 | 2.223 | 1.323 | 0.231 | 0.390 |
| | 1m | 3.499 | 1.767 | 1.863 | 1.193 | 8.45 | 2.809 | 2.682 | 1.565 | 2.862 | 1.503 | 0.177 | 0.332 |
| | 2m | 2.801 | 1.553 | 0.979 | 0.876 | 7.98 | 2.735 | 1.91 | 1.302 | 3.142 | 1.578 | 0.168 | 0.320 |
| | 3m | 2.37 | 1.454 | 0.734 | 0.712 | 9.332 | 2.96 | 2.455 | 1.51 | 3.424 | 1.636 | 0.183 | 0.343 |
| | 4m | 2.019 | 1.349 | 0.326 | 0.471 | 7.844 | 2.703 | 2.24 | 1.41 | 4.321 | 1.837 | 0.310 | 0.410 |
| | 5m | 2.436 | 1.485 | 0.208 | 0.378 | 8.026 | 2.738 | 1.986 | 1.326 | 2.098 | 1.305 | 0.050 | 0.186 |
| | 6m | 2.798 | 1.59 | 0.484 | 0.606 | 7.9 | 2.719 | 1.816 | 1.296 | 2.657 | 1.425 | 0.091 | 0.225 |
| | 10m | 2.635 | 1.554 | 0.173 | 0.35 | 9.053 | 2.9 | 2.233 | 1.442 | 3.267 | 1.615 | 0.117 | 0.306 |
| | 15m | 2.795 | 1.569 | 0.14 | 0.288 | 8.744 | 2.86 | 2.177 | 1.416 | 1.733 | 1.178 | 0.610 | 0.706 |
| | 20m | 2.416 | 1.475 | 0.132 | 0.203 | 7.957 | 2.739 | 1.585 | 1.175 | 1.969 | 1.275 | 0.317 | 0.480 |
| | 30m | 2.686 | 1.507 | 0.343 | 0.393 | 8.568 | 2.838 | 1.234 | 1.133 | 4.364 | 1.852 | 0.215 | 0.422 |
| | 1h | 3.424 | 1.724 | 0.36 | 0.523 | 8.937 | 2.906 | 0.142 | 0.316 | 9.129 | 2.610 | 1.182 | 0.884 |
| R | | 3.225 | 1.661 | 0.266 | 0.462 | 5.367 | 2.257 | 0.314 | 0.428 | 5.188 | 1.954 | 0.431 | 0.54 |
| GARCH | | 10.858 | 2.863 | | | 7.791 | 2.7 | | | 9.36 | 2.783 | | |

Table 13: VaR forecasting accuracy results. For each stock, measure, sampling frequencies (when applicable) and volatility model (Base or P-Spline) the table reports the MSE and MAE.

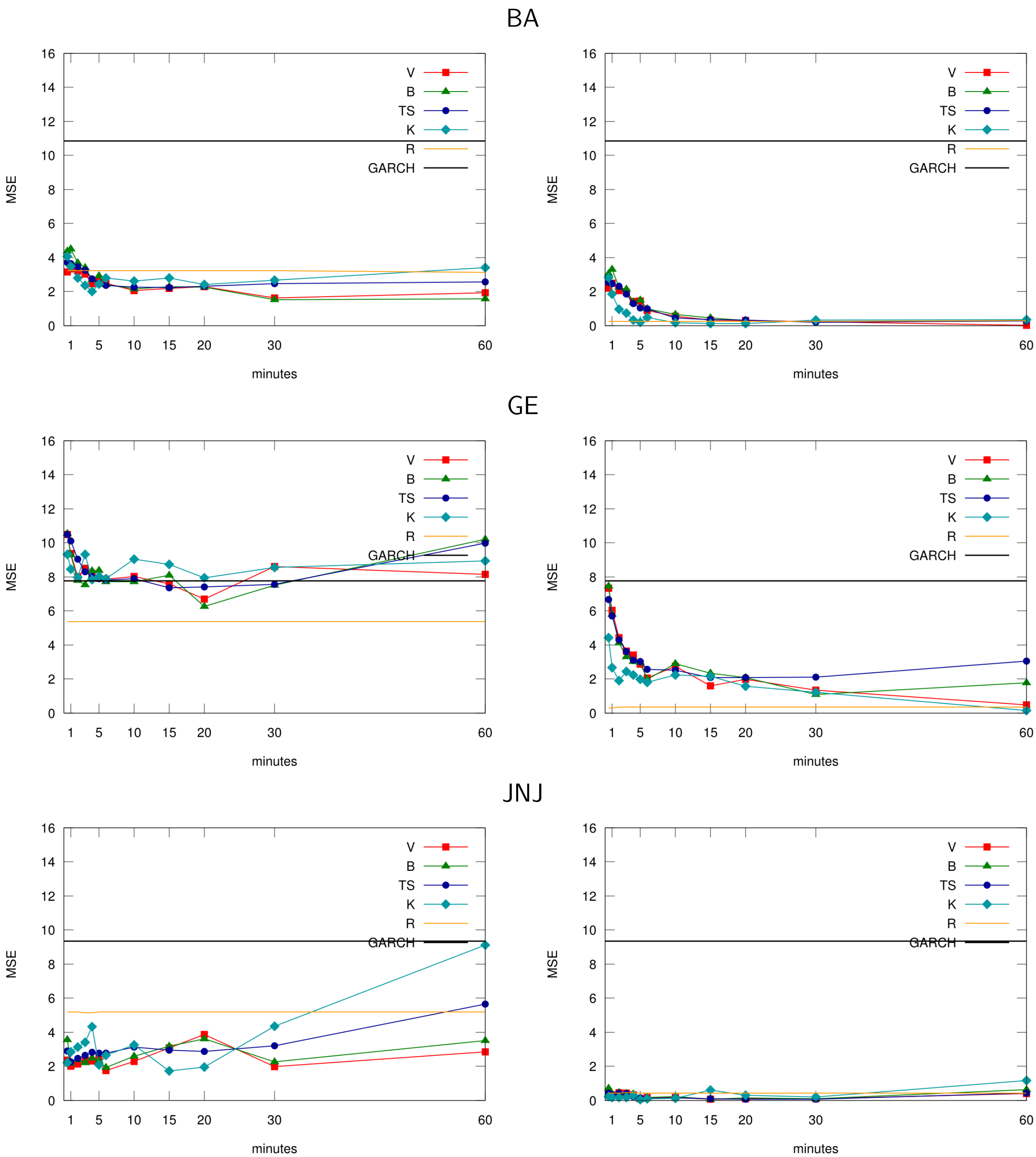


Figure 4: Out–of–sample VaR MSE of the volatility measures. The graphs display the estimated MSE of the volatility measures as a function of the sampling frequency.

hard to beat. In the GE stock the range forecasts systematically perform better than all the other measures. In the BA and JNJ stock the Base range forecasts are beaten by the UHFD measures at most sampling frequency but the P-Spline range forecasts have a substantially close performance.

# 7 Conclusions

In this paper we have engaged in a VaR forecasting comparison of prediction methods based on different volatility measures.

We find that UHFD volatility measures perform similarly in terms of VaR forecasting and obtain the best forecasting results at "low" frequencies (20/30min). Modeling volatility trends using our novel P-Spline MEM systematically improves forecasting ability, and realised kernel and P-spline have a forecasting ability less dependent on the choice of the sampling frequency. However, models for realized volatility measures produce VaR forecasts which are more *accurate* than a standard GARCH but yet as *adequate* and do not appear to outperform the range. The empirical evidence suggests that the range has a very good cost–to–quality ratio for VaR prediction.

The empirical evidence of this paper can be somehow counterintuitive. The UHFD volatility measures literature argues that by using *all* the data it is possible to construct arbitrarily precise estimates of volatility and it is not uncommon to find papers claiming that using UHFD volatility measures corresponds to "observe" volatility.

We believe that there are some straightforward arguments that explain our findings. A contribution of Granger (Granger (1998)) on the advent of UHFD points out that asymptotic theory assumes that the amount of information increases with the amount of data, but there are many situations in which this will just not hold, e.g. *"by observing earth movements more carefully we do not observe more large earthquakes"* (Granger (1998)). The empirical findings suggest that microstructure dynamics seem to bias volatility dynamics at very high frequencies and this compromises the benefits of sampling at increasingly higher frequencies. In fact, the realized kernel that is more robust to these type of microstructure noise dynamics seems to provide slightly better forecasting performance provided that is used with an appropriate model for forecasting.

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