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Abstract

Empirical evidence shows that the dynamics of high frequency–based measures of volatility exhibit persistence and occasional abrupt changes in the average level. By looking at volatility measures for major indices, we notice similar patterns (including jumps at about the same time), with stronger similarities, the higher the degree of company capitalization represented in the indices. We adopt the recent Markov Switching Asymmetric Multiplicative Error Model to model the dynamics of the conditional expectation of realized volatility. This allows us to address the issues of a slow moving average level of volatility and of different dynamics across regimes. An extension sees a more flexible model combining the characteristics of Markov Switching and smooth transition dynamics.

Keywords: MEM models, regime switching, smooth transition, realized volatility

1 Introduction

Direct measures of financial volatility were made possible by the availability of ultra high frequency data: several estimators were developed (for a review, see Andersen et al., 2010) under a number of assumptions on the underlying continuous time process driving prices. In what follows, we will use the version called realized kernel volatility, proposed by Barndoff-Nielsen et al. (2008), shown to filter out the presence of market microstructure noise and jumps.

When put next to one another, financial market volatilities generally exhibit similar behavior, being also subject to sudden, seemingly common, changes. Whether patterns of spillover can be

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detected from one market to another is the object of a large debate in the literature (see, for example, Gallo and Otranto, 2008; Engle et al., 2012, and references therein) which extends to the consequences of capital markets integration for portfolio diversification. In this paper we want to model volatilities in a univariate context with the aim to identify which indices present common features in the dynamics, given that each of them represents a different degree of market capitalization. The econometric approach is an extension of the Multiplicative Error Model pioneered by Engle (2002) in the direction of identifying regimes of volatility with a Markov Switching behavior. Sudden changes typically occur when large shocks hit the markets and possibly showing up in the series as common to several indices. This impression is confirmed by the graphs in Fig. 1 where volatilities of six US indices are plotted on the period between January 3, 1996 and February 27, 2009: Standard & Poor's 500 (S&P500, 3263 obs.), Dow Jones 30 (DJ30, 3261 obs.), the S&P400 Midcap (S&P400, 3258 obs.), Russell1000 (RU1, 3262 obs.), Russell2000 (RU2, 3264 obs.) and Russell3000 (RU3, 3262 obs.).[1]

The visual inspection of these time series reveals a high degree of persistence and several abrupt changes, particularly clear in the most recent period, with turbulence leading to the burst of the tech bubble, the 2001 recession, the low level of volatility in mid decade and then the explosion of uncertainty following the subprime mortgage crisis. On the other hand, these peaks seem less marked, especially in the first part of the series, for S&P400 and RU2, which are indices representing companies with a lower degree of capitalization.

Recently, Gallo and Otranto (2012) have conjectured the presence of changing levels of the prevailing average volatility by subperiods: the series show in fact alternating regimes which visually involve changes in the level but may also correspond to differences in the dynamics in the series. They propose to extend the class of Multiplicative Error Models (MEMs), developed by Engle (2002) and expanded by Engle and Gallo (2006), including a Markov Switching dynamics in the parameters to capture the presence of regimes. Being a MEM, this class of models applies to non–negative valued processes, therefore capturing dynamics without resorting to logs and producing forecasts of volatility (and not of log–volatility); moreover, considering the presence of regimes, these models capture the different phases of volatility, characterized by quiet periods, turmoil phases and accommodating brief abnormal peaks, leading to more realistic interpretations. In particular, applying their model to the same S&P500 volatility series analyzed here, Gallo and Otranto (2012) show that it is possible to obtain a better fit relative to the standard MEM, to avoid the high persistence in the estimated series (which contrasts with the empirical evidence) and to eliminate the residual autocorrelation which affect many realized volatility models. In this paper we propose a further extension of this class of models, allowing also for the possibility that the parameters relative to the error distribution can follow a different change in regime than the parameters of the conditional expected volatility. This is achieved by considering, as in Otranto (2011), smooth transition dynamics for the error coefficients, along the lines of Chan and Tong (1986); for some series this extension improves the model performance. We select the best model

[1] Data are expressed as percentage annualized volatility, i.e. the square root of the realized variance series taken from the *Oxford-Man Institute's realised library* version 0.1 (Heber et al., 2009), and multiplied by $\sqrt{252} * 100$.

in this class judging upon its statistical properties and drawing some considerations about the similarities in the changes in regimes across the six series.

The paper is organized as follows: in the next section we introduce the new class of Markov Switching models within the MEM framework. In section 3 we show the empirical results, keeping the standard MEM model as a benchmark. Some final remarks will conclude the paper.

2 A Class of Markov Switching AMEM

The basic MEM idea is introduced in Engle (2002) and successively developed in Engle and Gallo (2006): for what is of interest here, the volatility x_t of a certain financial time series is modeled as the product of a time varying scale factor μ_t (the conditional mean of x_t) which follows a GARCH-type dynamics, and a nonnegative valued error ε_t:

$$x_t = \mu_t \varepsilon_t, \qquad \varepsilon_t | \Psi_{t-1} \sim Gamma(a, 1/a) \quad \forall t$$
$$\mu_t = \omega + \alpha x_{t-1} + \beta \mu_{t-1} + \gamma D_{t-1} x_{t-1}, \qquad \text{where } D_t = \begin{cases} 1 & if \quad r_t < 0 \\ 0 & if \quad r_t \geq 0 \end{cases} \tag{2.1}$$

where Ψ_t represents the information available at time t. This *base* specification takes the presence of asymmetric responses of volatility to the sign of the returns (Engle and Gallo, 2006), where the coefficient γ captures a stronger reaction to past volatility when accompanied by negative returns. We call this model *Asymmetric MEM* (AMEM); setting γ to zero gives us the standard MEM. Constraints can be imposed to ensure the positiveness of μ_t ($\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$, $\gamma \geq 0$) and the stationarity of the process (persistence $(\alpha + \beta + \gamma/2)$ less than 1). The Gamma distribution depends only on a single parameter a, providing a mean and a variance of the conditional error equal to 1 and $1/a$ respectively. Correspondingly, the conditional mean and variance of x_t are μ_t and μ_t^2/a respectively. Further lags could be added.

In order to extend the capabilities of the model to capture extreme events which change market characteristics, such as sudden and persistent changes in the level of the series, Gallo and Otranto (2012) introduce the Markov–Switching AMEM (MS–AMEM):

$$x_t = \mu_{t,s_t} \varepsilon_t, \qquad \varepsilon_t | \Psi_{t-1} \sim Gamma(a_{s_t}, 1/a_{s_t}) \quad \forall t$$
$$\mu_{t,s_t} = \omega + \sum_{i=1}^{n} k_i I_{s_t} + \alpha_{s_t} x_{t-1} + \beta_{s_t} \mu_{t-1,s_{t-1}} + \gamma_{s_t} D_{t-1} x_{t-1} \tag{2.2}$$

where s_t is a discrete latent variable which ranges in $[1, \ldots, n]$, representing the regime at time t. I_{s_t} is an indicator equal to 1 when $s_t \leq i$ and 0 otherwise; $k_i \geq 0$ and $k_1 = 0$. Accordingly, the constant in regime j is given by $(\omega + \sum_{i=1}^{j} k_i)$. The changes in regime are driven by a Markov chain, such that:

$$Pr(s_t = j | s_{t-1} = i, s_{t-2}, \ldots) = Pr(s_t = j | s_{t-1} = i) = p_{ij} \tag{2.3}$$

Also in (2.2) the positiveness and stationary constraints given for (2.1) hold within each regime. Gallo and Otranto (2012) identify three regimes for the S&P500 realized kernel volatility, that can be interpreted as the low, medium-high and very high volatility states. Dealing with the same S&P500 series and in order to compare the changes in regimes of this series with other five series with similar dynamics, we also fix $n = 3$. When $\gamma_{s_t} = 0$, no asymmetric effects are present (MS–MEM).[2]

Accordingly, the unconditional mean of the volatility within each regime is:

$$m_{s_t} = \frac{\omega + \sum_{i=1}^{n} k_i I_{s_t}}{1 - \alpha_{s_t} - \beta_{s_t} - \gamma_{s_t}/2}, \tag{2.4}$$

The hypothesis that all the coefficients follow the same Markovian dynamics could be quite restrictive; for example, it would be plausible to think that the coefficient of the Gamma distribution follows its own dynamics not subject to the same regime changes as the coefficients of the conditional mean μ_t. We propose an alternative model, to be used when the MS–AMEM does not fit the data adequately: we add another equation to (2.2) for the time-varying parameter of the Gamma distribution which changes more or less abruptly, depending on the value of the returns:

$$a_t = b_0 + b_1\{1 + exp[-\delta(r_{t-1} - c)]\}^{-1} \tag{2.5}$$

where $b_0 > 0$, $b_1 \geq 0$, $\delta > 0$ and c are unknown parameters. In practice, we are adding a time-varying smooth transition variance (see Teräsvirta, 2009), not dependent on regimes, but with a suitable dynamic behavior. We call the model (2.2)-(2.3)-(2.5), the MS–AMEM with Smooth Transition Variance (MS–AMEM–STV). This specification would provide more flexibility to the Markov Switching model, in particular to capture the sizeable jumps, such as the highest peaks in 2008 (see Fig. 1). When δ approaches ∞, equation (2.5) is equivalent to a threshold model (Tong, 1990), and equation (2.5) is substituted by:

$$a_t = \begin{cases} b_0 & if \quad r_{t-1} \leq c \\ b_0 + b_1 & if \quad r_{t-1} > 0 \end{cases} \tag{2.6}$$

In this case we obtain different regimes for the conditional mean equation and for the Gamma coefficient, which will follow proper dynamics with two regimes. We call the model represented by equations (2.2)-(2.3)-(2.6), the MS–AMEM with Threshold Variance (MS–AMEM–TV).

[2] Details about the reparameterization of β_{s_t} to guarantee a certain coherence between the regime and the level of volatility, and about the solution of possible estimation problems, are in Gallo and Otranto (2012). In the same work another specification of the MS–AMEM is given, in which the asymmetry deriving from the sign of the returns may affect also the transition probabilities (the so called *Asymmetry in Probability MS–AMEM*).

3 Empirical Results

As a complement to the profile of the six volatility series shown in Fig. 1 above, we have calculated the usual descriptive statistics (not shown here to save space). They confirm the compatibility with the presence of regimes, especially a very large range with a thick right tail (high kurtosis). Time dependence is reflected in the autocorrelation functions, which are characterized by slowly declining high values, a fact typically seen as evidence of the presence of regimes.

We estimate the MS–AMEMs for all the series and verify if they have a good performance in terms of fitting and statistical tests; in particular we adopt the autocorrelation pattern of the residuals as a guideline, in the sense that, if they are correlated, we estimate the alternative MS–AMEM–STV (the MS–AMEM–TV if δ diverges) choosing the one with better properties in terms of results of the Ljung-Box statistics. This procedure selects the MS-AMEM only for the S&P500 volatility (as shown in detail in Gallo and Otranto, 2012), the MS–MEM for the DJ30 series, the MS–AMEM–STV for the S&P400 volatility and the MS–AMEM–TV for the three Russell indices.

We have also estimated the original AMEM, shown in equation (2.1), for all the series and compared its statistical performance with respect to the selected MS models. For this purpose we calculated the AIC[3] and some loss functions of interest, namely, the Root Mean Squared Error (RMSE), the Mean Absolute Error (MAE) and Theil's U (the latter calculated using the first differences of observed and forecasted data to detect the capability of the model to capture the turning points). On all accounts Markov Switching behavior is detected (see Table 1), with a strong improvement in the residual diagnostics.

One of the motivations to adopt a MS volatility model is the presence of autocorrelated residuals in the AMEM. In the same Table 1 we show the p-values of the Ljung-Box test statistics, in correspondence of lags 1, 5 and 10, for the AMEM and the selected MS models to check how uncorrelated the residuals are.[4] What we observe is that the models with three regimes are able to capture a large portion of the strong residual dependence structure still present in the AMEM.

The estimation results for the MS models are reported in Tables 2 and 3. From Table 2, we notice a strong difference in model dynamics when the assumption of common dependence of the coefficients on the regimes is relaxed. Starting from the intercepts, the MS models show a significant increase in these coefficients when regimes change, with an increase of more than 5 points in the high volatility third regime; the models with MS and STV or TV do not show similar differences in the intercepts. In this case the more flexible variance is able to capture also abrupt jumps in the series maintaining small intercepts. As a consequence, volatility dynamics

[3]Tests based on the likelihood function cannot be used to compare the AMEM with respect to the corresponding MS models because of the presence of nuisance parameters present only under the alternative hypothesis; in this case, with the proper caution, a classical information criterion could provide some information (see Psaradakis and Spagnolo, 2003); in particular the AIC seems to choose the correct state dimension more successfully than the BIC, provided that the parameter changes are not too small and the hidden Markov chain is fairly persistent.

[4]For MS models we have used the generalized residuals, introduced by Gourieroux et al. (1987) for latent variable models, defined as $E(\hat{\varepsilon}_t|\Psi_{t-1}) = \sum_{i=1}^{3} \hat{\varepsilon}_{s_t,t} t Pr(s_t = i|\Psi_{t-1})$, where $\hat{\varepsilon}_{s_t,t}$ are the residuals at time t derived from the parameters of the model in state s_t.

is represented by different coefficient behavior; in the case of models type (2.2), the α and γ coefficients increase with the regime whereas the β coefficients show an opposite behavior; this involves a strong dependence on the most recent observation and on the sign of returns for the regimes of high volatility and a lower persistence. The models containing equation (2.6) show that the third regime depends only on the values corresponding to negative returns and an increasing persistence in the third regime. It is interesting to note also that the estimated coefficients of the RU1 and RU3 volatility are very similar (pointing to a common DGP), whereas they differ from the one of RU2. We can argue that the companies with larger capitalization present in both RU1 and RU3 dominate the behavior of the volatility, while the smaller caps in RU2 behave differently.

In terms of transition probabilities (Table 3), it is evident that there is a strong permanence in the same regime for all the indices, in particular in regime 1 and 2. Regime 3 is less persistent for all Russell's and for S&P400. Some further insights are gained by looking at the off-diagonal elements of the transition probability matrix, with similar considerations for all the indices. Being in regime 1 there is a very low probability to switch to either of the other two regimes. From the regime of intermediate volatility there is a higher probability to move to the high volatility regime than to revert to a low volatility regime. By the same token, we note that the downward transition from the high volatility states occurs preferably with a move to the intermediate state: joint with the considerations above, there seems to be a strong interaction between regimes 2 and 3 while the period of low volatility is a sort of self standing regime.

The different behavior of the previous coefficients could be misleading in the interpretation of regimes; the level of the volatility within each regime is represented by the unconditional mean (2.4) by regime, which is shown in Table 4 for each series and signals the interpretability of regimes as increasing volatility. Moreover, S&P500, DJ30, RU1 and RU3 present similar levels of volatility in regime 1 and 2, which are higher with respect to the corresponding levels of S&P400 and RU2. The third regime is the one presenting the main differences among the six series; S&P500 and DJ30 are again similar, whereas S&P400, RU1 and RU2 show very high levels of average volatility, with RU3 in an intermediate position. Using smoothed probabilities, we can superimpose the average volatility levels to the observed series as in Figure 2. Bursts of volatilities, as well as sudden reductions in their values, correspond to a discrete change in the average value around which volatility follows its dynamics. More erratic behavior is apparent in the less frequently inspected indices. It is clear that the MS–AMEM with STV or TV consider the third regime as a state which absorbs the highest peaks, whereas the MS–AMEM corresponds to a higher duration in the regime.

In practice, it seems that there is a certain consistency in the behavior of the high capitalization indices, whereas S&P400 and RU2 show a sort of definitive permanent level shift at the end of 1998, with consistently higher levels of volatility from there on. The coherence among the indices can be evaluated in Table 5, where we show the percentage of cases in which the indices fall in the same regime. The high capitalization indices are in the same regime in more than 82% of cases, with a maximum in correspondence of RU1 and RU3 (96%). The coherence between high and low capitalization indices is low (between 44% and 65%), whereas there is a high coherence between S&P400 and RU2 (88.5%).

Figure 3 addresses the issue of whether regimes are coherent across indices or whether the non–homogeneity is relative to a specific state: we build a bar graph where the frequency in each regime for one index is broken down by the frequency across its own regimes for another index. The S&P500 is the reference index in five panels while the last one reports results between S&P400 and RU2. If regimes agreed perfectly we would have each side bar of the color of the same regime. Variety signals different regime partition. The most striking result is the large coherence between S&P500 and DJ30 at one hand (top left panel) and between S&P400 and RU2 on the other (bottom right panel). The most striking contrast is between the latter two each with the S&P500 (right column, top and middle). This suggests that the small cap companies have a more similar behavior as the mid caps, while the largest of the large caps (well represented by the S&P500) dominate volatility behavior when inserted within an index.

4 Concluding Remarks

With direct volatility measurement, many interesting questions can be addressed about its dynamics. We have investigated the possibility that abrupt changes seen in the time series of realized kernel volatility may signal the presence of regimes corresponding to different average levels of turbulence. With our Markov Switching specification of a Multiplicative Error Model we have allowed for the possibility that the shape parameter of the Gamma distribution ruling the tails of the error term may be made dependent on the value of the lagged returns. This significantly adds to the catalog of available volatility models for forecasting. While run on individual series, the analysis allows to compare results and establish commonalities. Company size shows up in different behavior in the volatility of indices for large caps on the one side and for mid and small caps on the other, an issue which is seldom investigated.

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Tables and Figures

Table 1: Likelihood-based criteria, in sample forecasting performance and autocorrelation tests[a] for AMEM and MS models. Sample: January 3, 1996 to February 27, 2009.

	Log-lik	AIC	RMSE	MAE	Theil U	$p(Q_1)$	$p(Q_5)$	$p(Q_{10})$
				S&P500				
AMEM	-8389.66	5.145	4.490	2.811	0.381	0.002	0.000	0.002
MS–AMEM	-8328.77	5.118	4.428	2.632	0.367	0.140	0.027	0.103
				Dow Jones				
AMEM	-8056.77	4.944	4.059	2.478	0.370	0.002	0.000	0.000
MS–MEM	-8025.82	4.933	4.074	2.330	0.361	0.809	0.008	0.091
				S&P400				
AMEM	-7516.42	4.617	3.528	2.197	0.339	0.001	0.000	0.005
MS–AMEM–STV	-7467.62	4.598	3.312	1.929	0.295	0.140	0.014	0.011
				Russell 1000				
AMEM	-8158.18	5.005	4.194	2.620	0.378	0.002	0.000	0.002
MS–AMEM–TV	-8100.88	4.980	3.716	2.303	0.334	0.270	0.017	0.023
				Russell 2000				
AMEM	-7462.25	4.576	3.717	2.245	0.376	0.003	0.000	0.003
MS–AMEM–TV	-7423.91	4.562	3.515	1.970	0.325	0.169	0.005	0.001
				Russell 3000				
AMEM	-8054.88	4.942	4.081	2.542	0.376	0.001	0.000	0.001
MS–AMEM–TV	-8013.71	4.927	3.668	2.260	0.335	0.146	0.036	0.052

[a] In the table, $p(Q_j)$ $(j = 1, 5, 10)$ indicates the p-values of the Ljung-Box Statistics at lag j.

Table 2: Coefficient estimates of the main equation of Markov Switching AMEM specifications with three regimes (standard errors in parentheses). Sample: January 3, 1996 to February 27, 2009.[a]

	S&P500	Dow Jones		S&P400	Russell 1000	Russell 2000	Russell 3000
ω	1.872	2.367		0.911	1.652	0.843	1.637
	(0.229)	(0.927)		(0.018)	(0.166)	(0.121)	(0.151)
k_2	0.685	1.048		0.000	0.001	0.000	0.000
	(0.228)	(0.242)		(0.001)	(0.006)	(0.005)	(0.001)
k_3	5.188	5.990		0.989	0.827	0.901	0.717
	(1.326)	(2.775)		(0.002)	(0.117)	(0.082)	(0.334)
α_1	0.199	0.311		0.160	0.180	0.206	0.182
	(0.028)	(0.112)		(0.007)	(0.022)	(0.021)	(0.021)
α_2	0.161	0.270		0.073	0.098	0.055	0.101
	(0.029)	(0.096)		(0.003)	(0.011)	(0.010)	(0.012)
α_3	0.257	0.385		0.000	0.000	0.000	0.000
	(0.058)	(0.086)		(0.000)	(0.000)	(0.000)	(0.000)
β_1	0.525	0.377		0.637	0.563	0.554	0.559
	(0.056)	(0.224)		(0.008)	(0.043)	(0.051)	(0.041)
β_2	0.594	0.460		0.789	0.714	0.807	0.707
	(0.058)	(0.191)		(0.004)	(0.024)	(0.020)	(0.023)
β_3	0.343	0.230		0.932	0.896	0.928	0.876
	(0.108)	(0.171)		(0.009)	(0.063)	(0.013)	(0.031)
γ_1	0.076			0.042	0.077	0.035	0.077
	(0.007)			(0.010)	(0.010)	(0.023)	(0.010)
γ_2	0.083			0.067	0.091	0.083	0.091
	(0.010)			(0.014)	(0.008)	(0.006)	(0.009)
γ_3	0.143			0.068	0.104	0.072	0.124
	(0.018)			(0.009)	(0.062)	(0.013)	(0.031)
a_1	15.808	21.062	b_0	4.314	5.226	3.945	5.012
	(0.632)	(1.249)		(1.554)	(1.947)	(1.259)	(2.247)
a_2	18.742	20.677	b_1	14.537	11.249	11.568	11.256
	(1.452)	(1.675)		(1.557)	(2.068)	(1.518)	(2.280)
a_3	10.946	11.127	δ	0.936			
	(0.751)	(1.194)		(0.021)			
			c	-3.531	-3.636	-4.310	-3.630
				(0.734)	(0.022)	(0.021)	(0.008)

[a] The model selected are: a MS–AMEM for S&P500, a MS–MEM for Dow Jones, a MS–AMEM–STV for S&P400, a MS–AMEM–TV for Russell1000, Russell2000 and Russell3000.

Table 3: Coefficient estimates of the transition probabilities for Markov Switching AMEM specifications with three regimes (standard errors in parentheses). Sample: January 3, 1996 to February 27, 2009.[a]

	S&P500	Dow Jones	S&P400	Russell 1000	Russell 2000	Russell 3000
p_{11}	0.989	0.977	0.994	0.992	0.985	0.993
	(0.002)	(0.009)	(0.002)	(0.001)	(0.004)	(0.001)
p_{12}	0.007	0.018	0.000	0.004	0.010	0.004
	(0.001)	(0.004)	(0.000)	(0.001)	(0.006)	(0.001)
p_{13}	0.004	0.005	0.006	0.004	0.005	0.003
p_{21}	0.007	0.013	0.000	0.003	0.003	0.004
	(0.001)	(0.005)	(0.000)	(0.002)	(0.002)	(0.001)
p_{22}	0.977	0.975	0.951	0.945	0.948	0.950
	(0.002)	(0.003)	(0.002)	(0.016)	(0.009)	(0.009)
p_{23}	0.016	0.012	0.049	0.052	0.049	0.046
p_{31}	0.006	0.007	0.007	0.014	0.013	0.011
	(0.002)	(0.012)	(0.001)	(0.009)	(0.008)	(0.004)
p_{32}	0.042	0.049	0.247	0.210	0.203	0.170
	(0.001)	(0.008)	(0.013)	(0.097)	(0.019)	(0.042)
p_{33}	0.952	0.944	0.746	0.776	0.784	0.819

[a] The model selected are: a MS–AMEM for S&P500, a MS–MEM for Dow Jones, a MS–AMEM–STV for S&P400, a MS–AMEM–TV for Russell1000, Russell2000 and Russell3000. The coefficients p_{i3} ($i = 1, 2, 3$) are not directly estimated, but they are obtained as $p_{i3} = 1 - p_{i1} - p_{i2}$.

Table 4: Unconditional mean of the volatility within each regime of US financial indices in the period from January 3, 1996 to February 27, 2009.

	S&P500	Dow Jones	S&P400	Russell 1000	Russell 2000	Russell 3000
m_1	7.891	7.599	5.011	7.561	3.780	7.424
m_2	12.609	12.648	8.551	11.557	8.719	11.184
m_3	23.570	24.442	55.937	47.675	48.513	37.986

Table 5: Percentage of common regimes between pairs of US financial indices in the period from January 3, 1996 to February 27, 2009.

	Dow Jones	S&P400	Russell 1000	Russell 2000	Russell 3000
S&P500	89.26	52.71	86.45	44.47	85.74
Dow Jones		52.30	83.15	45.23	82.20
S&P400			62.54	88.49	64.68
Russell 1000				54.23	95.92
Russell 2000					56.74

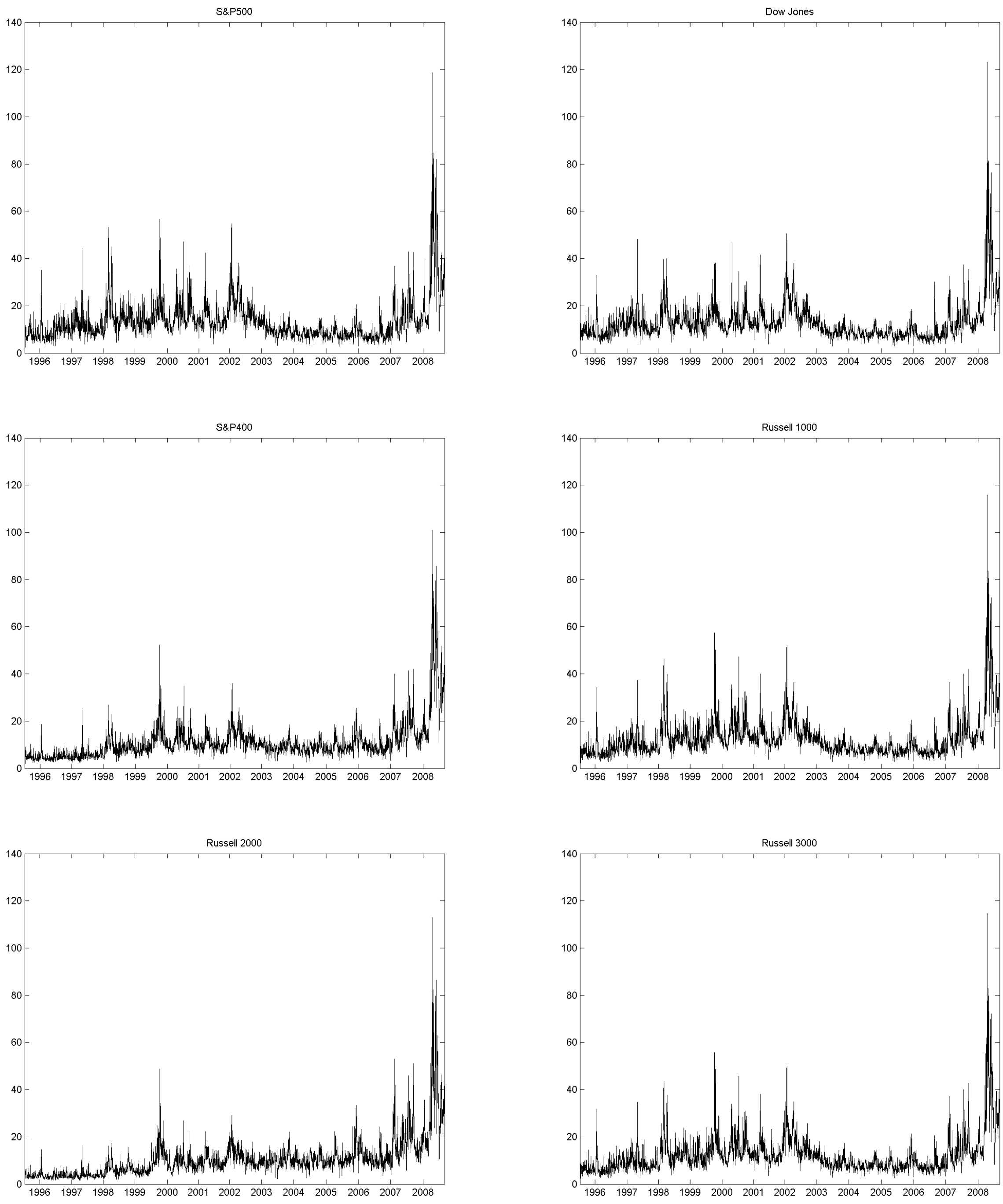

Figure 1: Realized kernel volatility of six US indices.

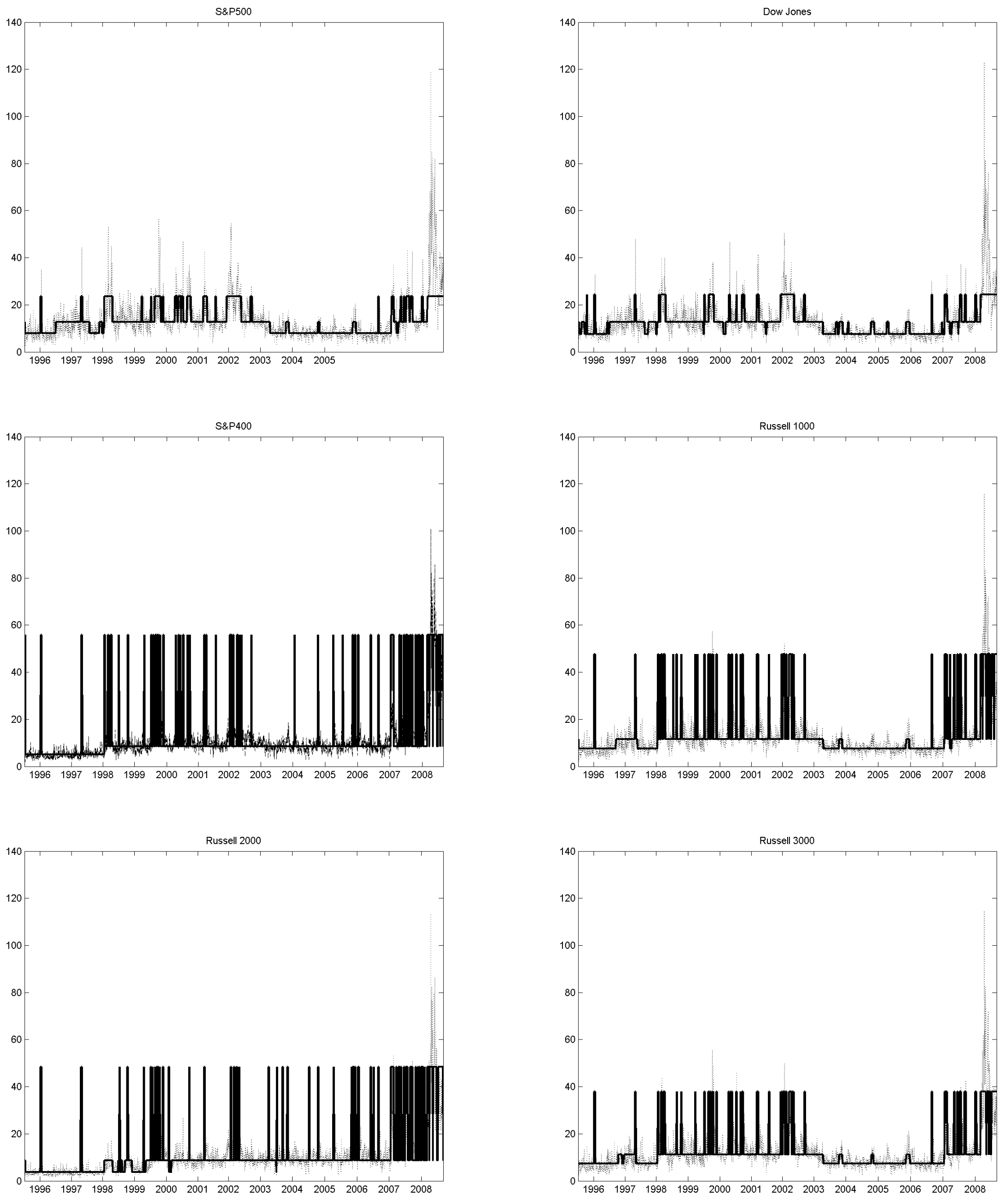

Figure 2: Original series (dotted lines) with regime-specific average volatilities (bold lines) in the sample period January 3, 1996 to February 27, 2009.

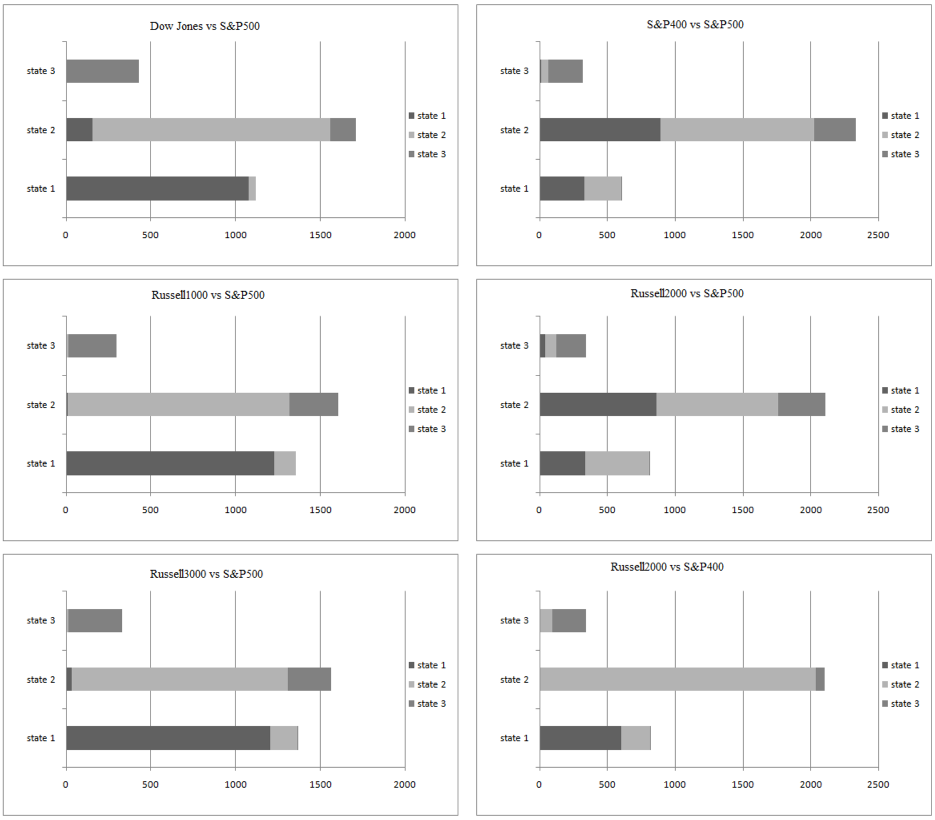

Figure 3: Distribution of the regimes between pairs of financial US indices in the sample period January 3, 1996 to February 27, 2009. A vs B indicates that the frequency in each regime for index A is broken down by the frequency across its own regimes for index B.