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Abstract. We investigate the association between fertility and women's education in Italy, using data from the 2003 Household Multipurpose Survey *Family and Social Subjects*. We adopt a Bayesian event history approach to estimation and study the association between fertility and women's education in the presence of a time-varying unobserved component. It is shown that the usually made assumption of time-constant unobserved heterogeneity can lead to misleading results.

Keywords: Bayesian event history analysis; Italian fertility; Time-varying heterogeneity.

1. Introduction

The association between fertility and educational achievement is one of the strongest relationships ever recorded in social science. Education can be considered a marker of income, occupation or social status and it is often viewed as a surrogate of hard-to-measure concepts, such as opportunity costs (Castro Martín and Juárez, 1995). Also, for women, higher education often underlines the possibility to behave in autonomy of the male partner and of social norms (Hoem *et al.*, 2001).

In the socio-economic and demographic literature, there are two prominent theoretical perspectives on low fertility: the New Home Economics theory (Becker, 1981) and the

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Second Demographic Transition theory (Lesthaeghe, 1995). Both the theories predict lower fertility as women are gaining education. According to the first theory, the opportunity costs of children are heavier for highly educated women than for less educated ones. Therefore women with high education have fewer children and they enter into motherhood at a later age. According to the second theory, a modernized society, open to social and cultural changes, allows couples and individuals to develop a more personal lifestyle, so that having children becomes one of many possible options. Consistently, it is suggested that preference for children gets weaker as education level increases. In line with these two predictions, many studies have documented a negative association between women's educational attainment and fertility. Highly educated women delay the onset of childbearing and have overall fewer children compared with less educated women (e.g., Martín-García and Baizán, 2006; Brand and Davis, 2011).

Nevertheless, several studies have shown that transition rates to the second and third child do not decrease, but rather increase with women's education level. For instance, a positive association was found in many Western European countries, including West Germany and France (Koppen, 2006), Denmark (Gerster *et al.*, 2007), Sweden (Hoem and Hoem, 1989) and Austria (Hoem *et al.*, 2001). These studies aimed at evaluating whether the positive association between education and fertility could be attributed to the fact that higher education levels create better conditions for family formation. A key finding was that accounting for additional observed factors that might potentially be associated with fertility choices, generally downsizes the positive association between fertility and education, although it is neither nullified nor reversed.

Kravdal (2001) and Kreyenfeld (2002) strongly contributed to this debate suggesting that the positive association between education and second (and higher)-order fertility may be, at least partially, explained by the presence of a latent variable representing self-selection effects. Specifically, they argued that women with tertiary education who gave birth to the first child might have a marked and unobserved preference for children. Following the methodological framework proposed by Lillard and Panis (2000), Kravdal (2001) and Kreyenfeld (2002) assessed this hypothesis using a simultaneous-equations model. They jointly estimated the time-to-event for the first and the second child birth in the presence of a frailty component, shared by both the two possible events for each woman. Interpreting this subject-specific frailty in terms of woman's family orientation, their results suggested that the positive association between education and second births disappears once controlling for the unobserved *family proneness*.

A potentially relevant drawback of the approach proposed by Kravdal (2001) and Kreyenfeld (2002) is that it is based on the assumption of time-invariant unobserved-heterogeneity, which implies that family-orientation is constant over time. However, orientations towards work or family life may change over time: it may be amplified, reduced, or even reversed over individual's life courses.

In this paper we contribute to this vivid debate by investigating how a time-dependent frailty can relate to fertility dynamics in Italy. The Italian institutional context does not generally offer a family-friendly setting, so women who choose to set up a family are likely to be polarised between those with low career ambitions and those with a high family orientation (Matysiak and Vignoli, 2010). As a result, the self-selection hypothesis is expected to strongly apply in Italy. We will explore the role of educational attainment for fertility of Italian women by using three alternative approaches: an ordinary event history model without frailty component, which neglects self-selection effects; a time-independent frailty model, which describes a persistent family orientation over the life-course, and a time-dependent frailty model, which allows us to account for possible changes in family orientation during the life-course. Specifically, we use a piecewise exponential hazard model and adopt a Bayesian approach to estimation (e.g., Ibrahim *et al.*, 2001). The Bayesian paradigm has several advantages, including ease of computation via Monte Carlo Markov Chain (MCMC) methods, and the ability to incorporate prior information. From a Bayesian perspective, all unknown quantities, parameters as well as unobserved subject-specific frailties, are uncertain and they have a joint posterior distribution, conditional on the observed data. Therefore, inferences are based on posterior distributions, such as the posterior distributions of the subject-specific frailties, and the posterior hazard function.

The rest of the article is organized as follows. Section 2 introduces the fertility-education issue focusing on the case of Italy and briefly describes the data. Section 3 presents the notation and the methodology used in the application, whose results are discussed in Section 4. Section 5 concludes the paper, while Appendix A reports detailed tables on estimates generated by the adopted models.

2. The Italian fertility-education profile and data

The negative relationship between educational attainment and family formation has been suggested to be stronger in societies where the conflict between women's employment and family formation is larger (Blossfeld, 1995). In Italy, this conflict is still present. Although the country has experienced a strong increase in women's educational attainment and labour market participation since 1970s, it has not adjusted to the ongoing societal change: working hours, public services, family structures, and (generally limited) male participation in household chores, among others, indicate that the old-fashioned concept that women should be housewives is still alive. As a result, the traditional family-oriented welfare state and the women's increasing desire to invest in their human capital and participate in paid employment are being in conflict, leading to lower-than-desired fertility (McDonald, 2000). This argument partly explains the extremely low Italian fertility (1.4 children per woman in 2010).

Women's education has played a prominent role in shaping Italian fertility: the postponement of childbearing until older ages and the marked renunciation of marriage and children are widespread among highly educated women (Salvini, 2004). Moreover, the role of women's education has become more and more relevant in influencing overall tempo and quantum of fertility. In fact, the number of women holding a university degree is continuously increasing in succeeding cohorts, and currently there are more women than men in the age group 25 – 44, who have a university degree (Istat, 2009).

Recently, scientific research on Italian family demography has observed that couples with greater cultural and economic resources have a higher propensity to have children than their lower educated counterpart (Rosina and Testa, 2009; Régnier-Loilier and Vignoli, 2011). In this new state of affair, however, it is still not clear which role is played by the unobserved component usually interpreted as self-selection or family proneness. The only attempt to assess the potential influence of education on fertility accounting for the role of family proneness in Italy is due to Dalla Zuanna and Impicciatore (2008), who showed that the positive relationship between education and fertility significantly reverses once self-selection is taken into account. These authors, as Kravdal (2001) and Kreyenfeld (2002), considered a constant family proneness over women's life courses. Our contribution to this literature consists in further investigating the role of educational attainment for Italian fertility in the presence of unobserved heterogeneity, which can partially drive fertility and reasonably vary over time.

Our analyses are based on retrospective data, stemming from the Household Multipurpose Survey *Family and Social Subjects* (FSS). The FSS survey was conducted by the Italian National Statistical Office (Istat) in November 2003 on a sample of about 24 000 households and 49 451 individuals of all ages. The survey contains a wealth of information about individuals' and families' daily lives, including detailed fertility histories and educational attainment. The sample we use for our analyses consists of 9 029 women aged 20-45 at the time of the interview (i.e., cohorts 1958-1983). In the sample, 4 818 women have at least one child, while 3 025 women have at least two children. Education level is an ordinal variable with three levels: primary, secondary and tertiary education level. The first category comprises women who completed only compulsory education (eight years), as well as those who continued with basic vocational education, lasting three years in Italy. The secondary educated are those who completed at least four years of education at the upper-secondary level, as well as those who undertook post-secondary but non-tertiary education. Women who received a bachelor or a master's degree are classified as tertiary educated. For each woman, we consider the highest education level attained at the interview, that represents an exogenously fixed censoring time, neglecting possible dynamic dependences. There could be objections on the basis that it would have been more convenient to use education as a time-varying covariate. Nevertheless, the inclusion of the highest level of education ever reached is justified by the particular Italian pattern of family formation. People normally tend to form a family only after completing their education and training period (Salvini, 2004).

In the sample, only 12.5% of the women have a tertiary education level and most of them has no children (64%). Among these highly educated women, 16.3% has only one child, while 20.3% have at least two children. On the other hand, more than 70% of 3 243 (out of 9 029) women with a primary education level have at least one child, while about 49.2% have at least two children. In addition, as expected, the average age at the first childbirth for women with primary education is much lower than that for higher educated women (24 versus 31). An intermediate situation is recorded for women with a secondary education level, with 45.9% having at least one child and more than 25% with at least two children. Additional explanatory variables are also considered, including area of residence, cohort and parents' education level. Unfortunately, information about the partners were not included in the longitudinal FSS survey.

3. Event history models with time-dependent frailty

Event history models are an ideal framework for studying women's fertility process and for modelling the relationship between the risk of an event occurrence and selected predictors, such as, for example, women's education. In this section, we shortly describe event history models formulation as routinely adopted, to concentrate the attention on the less common formulation admitting a time-dependent frailty component, focusing on the application of interest.

Let us define the women's fertility process as a point process $X(t)$, with t representing the time-to-event, $t \in (0, T_c]$. The time origin, 0, corresponds to 14 years old age, while T_c is the duration till the interview. As fertility is here analysed limiting to the first and the second childbirth, $X(t)$ admits two kinds of event, and its state space is $\mathcal{S}_X = \{0, 1, 2\}$. State 0 represents the initial, transitional state of having no children, state 1 is for one child, and state 2 is an absorbing state for having the second child. Such multivariate process can be viewed as a marked point process $X(t, m)$ (Arjas, 1989), in which the mark $m \in \mathcal{M} = \{1, 2\}$ indicates the two kinds of event, $0 \to 1$ and $1 \to 2$. Notice that these kinds of event are not competing, but consecutive, as the second child cannot be born before the first one. Twins have been excluded from the analysis, as too few to be included with an adequate specification.

The complete description of the finite-dimensional distribution of this kind of process can be formulated in terms of its mark-specific hazard function $h_m(t)$, the instantaneous rate of having in t the m^{th} child. Similarly, the mark-specific survival function can be then specified as

$$S_m(t) = \exp\left\{-\int_0^t h_m(s)ds\right\}.$$

A set of explanatory variables can be included by defining a conditional version of the mark-specific hazard function. The likelihood function for the considered fertility process is then

$$\mathcal{L}(\boldsymbol{\theta}; \boldsymbol{y}) = \prod_{i=1}^{n} \prod_{m=1}^{2} h_m(t_{im} \mid Z_i, \mathcal{H}(t_{im}^-))^{\delta_{im}} \cdot S_m(t_{im} \mid Z_i, \mathcal{H}(t_{im}^-))^{\zeta_{im}},$$

in which t_{im} represents the event occurrence or censoring time for woman i for event m, Z_i is the vector of observed explanatory variables, $\mathcal{H}(t_{im}^-)$ represent the past history of the process and δ_{im} and ζ_{im} are adequately defined to deal with censured events, $\boldsymbol{\theta}$ represents the vector of parameters and $\boldsymbol{y}$ the matrix for all observed data (event times and explanatory variables).

In this work, we assume a model for the fertility process with a piecewise-constant specification. For $m = 1$, we divide the entire time axis $(0, T_c]$ into $K_1 = 6$ pre-specified sub-intervals $\mathcal{T}_{k_1} = (s_{k_1-1}, s_{k_1}]$ for $k_1 = 1, \ldots, K_1$, where $s_0 = 0$ and $s_{K_1} = T_c$. For women who experienced the birth of the first child and so are at risk of having a second child ($m = 2$), we also consider a second birth woman-specific time in terms of duration since the first-child occurrence time. The time axis is then $(0, T_c - t_{i1}]$, where t_{i1} corresponds to the woman i's age at the first childbirth. This time axis is divided into $K_2 = 4$ pre-specified sub-intervals $\mathcal{T}_{k_2} = (s_{k_2-1}, s_{k_2}]$ for $k_2 = 1, \ldots, K_2$, where $s_0 = 0$ and $s_{K_2} = T_c - t_{i1}$. The time at the first birth, t_{i1}, is also included as the past history of the process when $m = 2$.

The conditional mark-specific hazard function has the form

$$h_m(t_{im} \mid Z_i, \mathcal{H}(t_{im}^-)) = \sum_{k=1}^{K} (\lambda_{km} \cdot \mu_{im}) \cdot 1_{\{t_{k-1}<t\leq t_k\}}$$

where $\log(\mu_{im})$ is assumed as a linear function of the explanatory variables and past history of the process, not depending on k_m, $m = 1, 2$.

In fertility studies, it may be reasonable that the hazard function for each woman also depends on an unobserved woman-specific random frailty, which shall mean a part of the unexplained variation. Then, a time constant frailty component can be introduced as

$$\log(\mu_{im}) = \sum_{j=1}^{J} \beta_{jm} Z_{ij} \beta_{Hm} \mathcal{H}(t_{im}^-) + U_i$$

with, typically, $U_i \overset{iid}{\sim} N(0, \tau_U^2)$. This kind of model specification implies that the hazard function for each woman depends on the covariates as well as an unobserved woman-specific random frailty component, which may be interpreted in terms of family orientation (see Section 1).

However, family orientation, and so the woman-specific frailty, may reasonably change over time, leading to a time-varying random frailty model. A time-varying random component can be conceived in several ways. In certain situations, it can be modelled as a stochastic process moving over time. For example, in Yashin and Manton (1997) a diffusion process is assumed as frailty component, while Gjessing *et al.* (2003) adopt a Lévy-process. In this paper, in line with the functional form of the fertility process, the time-varying frailty component is assumed piecewise-constant, varying at fixed point in time, supposing three fixed time intervals,

$$\log(\mu_{im}) = \sum_{j=1}^{J} \beta_{jm} Z_{ij} + U_{1i}\mathcal{I}_1 + U_{2i}\mathcal{I}_2 + U_{3i}\mathcal{I}_3 \tag{1}$$

where each $\mathcal{I}_r$ equals 1 in the r^{th} time interval. Equivalently, this time-varying random effect specification can be viewed as correlated random slopes of time-varying dummy variables. Similar specifications can be found, for example, in Paik *et al.* (1994) and Wintrebert *et al.* (2004) for shared frailty survival models. Because of the particular application we have in mind, it seems sensible to assume the three random effects to be dependent, so that, for example,

$$U_{1i} = \varepsilon_{1i} \qquad U_{2i} = \delta_{12}\, U_{1i} + \varepsilon_{2i} \qquad U_{3i} = \delta_{13|2}\, U_{1i} + \delta_{23|1}\, U_{2i} + \varepsilon_{3i},$$

where the errors have zero mean Gaussian distribution. Equivalently, we can assume

$$\begin{pmatrix} U_{1i} \\ U_{2i} \\ U_{3i} \end{pmatrix} \overset{iid}{\sim} N\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \tau_1^2 & \tau_{12} & \tau_{13} \\ \tau_{12} & \tau_2^2 & \tau_{23} \\ \tau_{13} & \tau_{23} & \tau_3^2 \end{pmatrix}\right). \tag{2}$$

Whenever $\rho_{rs} = \tau_{rs}/\tau_r\tau_s$ is positive, the individual (unobserved) frailty levels will mainly be of the same sign when passing to period s. This implies, for instance, that women with a positive family proneness in period r will maintain such positive feeling towards family in period s. On the contrary, in the s^{th} period the frailty level will diminished, nullified or even reverse, when ρ_{rs} is negative. If $\rho_{rs} = 0$, the two frailty levels in periods r and s are marginally independent. Another interesting feature is the partial correlation coefficient between U_{1i} and U_{3i}, say $\rho_{13|2}$, which is a function of the elements of the inverse of the variance covariance matrix in (2). Whenever $\rho_{13|2} = 0$, then U_{3i} is independent of U_{1i} given U_{2i}, suggesting an AR(1) dependence model among the unobserved components.

To implement a Bayesian event history model (e.g., Ibrahim *et al.*, 2001), prior distributions for model parameters have to be specified. To reflect a vague prior knowledge, we opted for non informative, although proper, prior distributions. Particularly, denoting $\alpha_{k_m m} = \log \lambda_{k_m m}$, we assume

$$\alpha_{k_m m}|\alpha_{(k_m-1)m} \sim N\Big(\alpha_{(k_m-1)m}, \sigma_\alpha^2\Big) \qquad k_m = 1, \ldots K_m, \quad m = 1, 2$$

with $\alpha_{0m} = 0$ and $\sigma_\alpha^{-2} \sim$ Gamma $(0.01, 0.01)$. Note that we are assuming the variance parameter σ_α^2 of the normal prior distributions for $\alpha_{k_m m}|\alpha_{(k_m-1)m}$ does not depend on m, implying that the logarithm of the baseline hazard parameters for the first child and the second child have the same variance. The inverse of the variance-covariance matrix in (2) for the vector of random effects is assumed to have a prior Wishart distribution. Moreover, the prior distributions for the coefficients of the explanatory variables are assumed as $\beta_{jm} \sim N(0, 100)$ for each j and m.

We estimate three alternative models: an ordinary model without frailty component (Model A), a model with a time-constant frailty (Model B), and a third model with a time-varying frailty (Model C), with the frailty component changing, for each woman, at 28 and 35 years old. All the explanatory variables listed in Section 2 are included in each model. The posterior distributions of the parameters were obtained from Markov chain Monte Carlo (MCMC) methods, based on Gibbs sampler (Gelfand and Smith, 1990) using WinBugs (Lunn *et al.*, 2000). Two parallel MCMC chains were run with different starting values. In total, 90 000 iterations were run for models A and B, saving every 10^{th} iteration, after

Table 1. Summary statistics: Posterior distributions of the education-level parameters

	First Child ($j = 1$)				*Second Child* ($j = 2$)			
Parameter	*Mean*	*SD*	*2.5%*	*97.5%*	*Mean*	*SD*	*2.5%*	*97.5%*
				Model A (no frailty)				
Education Level (Primary)								
Secondary	-0.472	0.031	-0.533	-0.410	0.004	0.041	-0.077	0.084
Tertiary	-0.979	0.058	-1.094	-0.865	0.269	0.080	0.109	0.424
				Model B (time-constant frailty)				
Education Level (Primary)								
Secondary	0.077	0.055	-0.031	0.185	0.024	0.075	-0.120	0.173
Tertiary	0.030	0.086	-0.140	0.196	0.060	0.111	-0.163	0.275
				Model C (time-varying frailty)				
Education Level (Primary)								
Secondary	-0.515	0.033	-0.580	-0.450	-0.066	0.051	-0.167	0.034
Tertiary	-1.175	0.063	-1.301	-1.054	-0.166	0.117	-0.396	0.064

a burn-in stage of 10 000 iterations. For model C, the chains consist of 200 000 iterations, with the first 20 000 as burn-in and a thinning interval of 20 iterations. Various convergence checks were considered and no evidence against convergence was found. Inference is based on the remaining iterations, combining the two chains.

4. Results

In this section, we shall illustrate the results of the three models, A, B and C, aforementioned. In particular, posterior estimates for education and unobserved frailty components will be discussed in details. Such estimates are to be viewed conditionally on the other explanatory variables included (area of residence, cohort and parents' education level) whose posterior summaries are reported in the appendix.

Tables 1 and 2 present the posterior means, standard deviations, and 95% credible intervals for the education level parameters and the variances and covariances of the frailty components.

Our findings show that educational achievement is a key variable to explain fertility choices, but there is also a strong evidence that woman-specific random effects play a significant role, heavily influencing the association between educational achievement and fertility, especially when they are assumed to be time-dependent and focus is on the transition to

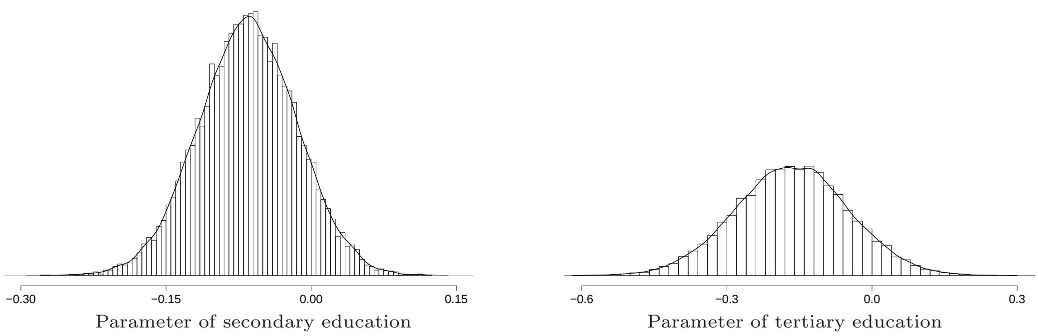

Figure 1. Histograms and densities of the posterior distributions of the education level parameters for Model C, second child birth ($m = 2$)

the second child.

Consider first the transition to the first birth. As it can be seen in Table 1, the model with a time-constant woman-specific frailty leads to posterior distributions of the education level parameters that are quite spread around zero with a large span, suggesting that the role of education on the transition to the first child is negligible. On another hand, when either no frailty or a time-varying frailty is used, a quite strong negative association between education and the transition to the first child arises, suggesting that lower educated women tend to have the first child at younger ages. For instance, Model C (with time-varying frailty) shows that, at the posterior mean, the first-child hazard for women with secondary and tertiary education reduces by about 40% ($1 - e^{-0.515} = 1 - 0.598$) and 70% ($1 - e^{-1.175} = 1 - 0.309$), respectively, compared to women with primary education.

In order to investigate whether the relationship between educational attainment and the transition to the second child differs by age at first birth, we also estimated the three alternative models including interaction terms between education level and woman's age at the first birth. Since higher educated women are generally older at their first birth, they have less time to give birth to their second child before reaching the biological limits of fertility. Therefore this type of women might be induced to anticipate the birth of the second child, as the result of a *time squeeze effect* (Kreyenfeld, 2002). Here we are focussing only on models without interactions between education level and age at the first childbirth, as the posterior distributions of the interaction coefficients provided no evidence that the relationship between educational attainment and second births changes as age at the first birth changes.

Table 2. Summary statistics: Posterior distributions of the variance parameters

Parameter	*Mean*	*SD*	*2.5%*	*97.5%*
		Model B (time-constant frailty)		
τ_U^2	0.063	0.033	0.021	0.150
		Model C (time-varying frailty)		
τ_1^2	0.021	0.008	0.009	0.041
τ_2^2	2.264	0.137	2.006	2.543
τ_3^2	4.612	0.503	3.659	5.689
τ_{12}	-0.125	0.045	-0.210	-0.037
τ_{13}	-0.163	0.063	-0.283	-0.040
τ_{23}	3.171	0.248	2.707	3.679

The role of unobserved heterogeneity seems to be even more dramatic when focus is on the second birth risk. Consistently with the literature, Model A, without any frailty component, leads to a quite strong positive association between education and second births. For instance, at the posterior mean, the estimated risk of experiencing a second birth for women with tertiary education is $e^{0.269} = 1.309$ times the risk for women with primary education. This positive association vanishes once controlling for woman-specific unobserved heterogeneity. Although the model with a time-constant frailty (Model B) provides positive posterior means for the education level parameters, their values are really small and the 95% posterior credible intervals cover 0, including both positive and negative values. Model C, with a time-dependent frailty, leads to negative posterior means for the education level parameters, suggesting that higher educated women tend to postpone the birth of a possible second child with respect to primary educated women. The 95% posterior credible intervals still cover 0, but they are skewed to the left, including more negative than positive values, and the posterior probabilities that the education level parameters are negative are greater than 90% (see also Figure 1).

In order to better understand the role of unobserved heterogeneity on the fertility process, and the potential benefits of using a time-varying frailty component, we now analyse in detail the posterior distributions of the variances of women's frailties.

As can be seen in Table 2, the posterior distributions of the variances of woman-specific frailties from the two alternative models B and C show a high degree of heterogeneity, especially when the assumption of time-constant frailty is relaxed.

Model B leads to a posterior distribution for the variance of the woman-specific frailty,

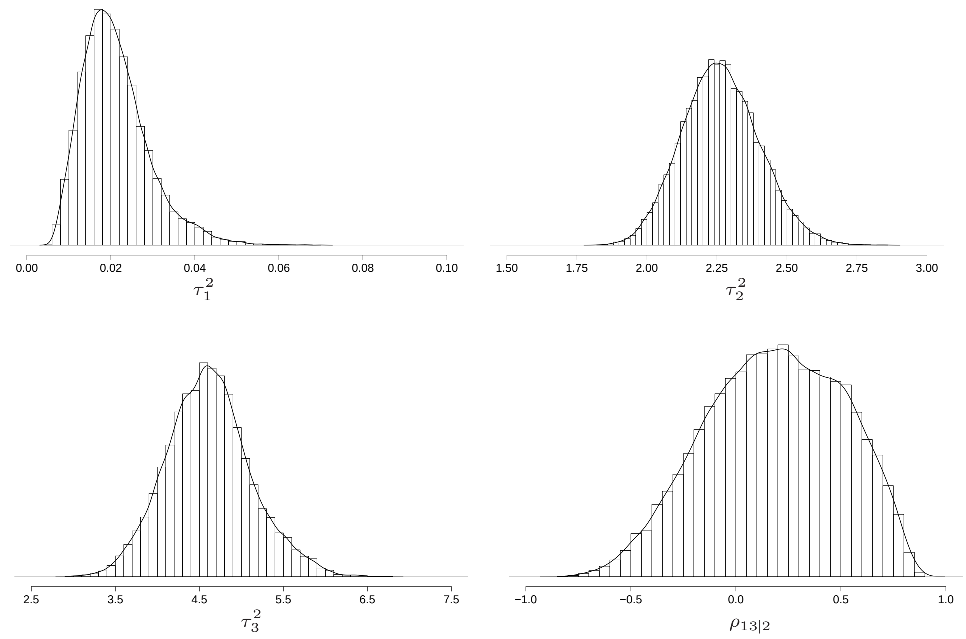

Figure 2. Model C: Histograms and densities of the posterior distributions of τ_1^2 (variance of U_{1i}), τ_2^2 (variance of U_{2i}), τ_3^2 (variance of U_{3i}), and $\rho_{13|2}$ (partial correlation between U_{1i} and U_{3i} given U_{2i})

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U_i, with mean of 0.063, implying that, at the posterior mean, a variation of $-\tau_U$ in U_i reduces the parity-specific fertility rates by 22.2% ($e^{-\sqrt{0.063}}$ = 0.778) and a rise of τ_U increases the parity-specific fertility rates by 28.5% ($e^{\sqrt{0.063}}$ = 1.285) irrespective of the woman's age.

When the assumption of time-constant frailty is relaxed (Model C), allowing individual's heterogeneity, and therefore family proneness, to depend on women's age, we find a strong posterior evidence that heterogeneity increases over time, implying that self-selection into family formation is expected to be very strong among older women. Specifically, among women between 14 and 28 years old, a positive (negative) variation of one standard deviation in the woman-specific random frailty has a rather small multiplicative effect on parity-specific fertility rates of $e^{\sqrt{0.021}}$ = 1.156 ($e^{-\sqrt{0.021}}$ = 0.865). This multiplicative effect goes up (down) to $e^{\sqrt{2.264}}$ = 4.503 ($e^{-\sqrt{2.264}}$ = 0.222) among women aged 28-35, and to $e^{\sqrt{4.612}}$ = 8.564 ($e^{-\sqrt{4.612}}$ = 0.117) among women older than 35 years. Therefore, for instance, a rise of one standard deviation in the frailty of a woman older than 35 years increases the fertility hazards by 8.564 times.

The first three graphs in Figure 2 show the histograms and densities of the posterior distributions of the three variance parameters: from the first period to the third period the posterior distributions of the variance parameters progressively move to the right, covering disjoint support intervals. Specifically, the posterior distribution of the variance of the frailty for the first period is skewed to the right with support in a very tight interval including values ranging from 0 to 0.07. The posterior distributions of the variances of the frailties for the second and third period − which are almost symmetric − have support in intervals including much higher values (see also Table 2). These results suggest that women's family proneness may not be a relevant factor in driving fertility choices at young ages, but it becomes a leading factor later. It is worth noting that the impact of the frailty components seems wider than the education effect. In fact, considering the posterior mean as a point estimate, we can see that the estimated marginal distribution for the third frailty component is $N(0, 4.612)$, so that about 40% of the higher educated women has a frailty strong enough to nullify or reverse the negative effect of the education ($u_{3i} \geq +1.175$, being -1.175 the posterior mean for the tertiary education parameter).

The posterior distributions of the variance and covariance parameters also provide information on the marginal and partial correlations between frailties over time. A negative and quite strong marginal correlation is found between the first-period frailty and the second- and third-period frailties, suggesting that a woman with a low proneness to family life at younger ages might develop a strong feeling towards family life at older ages and vice versa. Our results also suggest that there exists a strong positive marginal correlation between the second-period frailty and the third-period frailty: the posterior probability that this marginal correlation is greater than 0.90 is about 99.6%. Therefore, a woman who is very prone to family formation between 28 and 35 years old is expected to preserve this feeling in the last years of her fertility life.

The fourth graph in Figure 2 shows the posterior distribution of the partial correlation between U_{1i} and U_{3i} given U_{2i}, $\rho_{13|2}$. This posterior distribution is evenly spread around zero with a large span, and the 95% posterior credible interval, $(-0.455; 0.743)$, covers zero, suggesting that controlling for U_{2i} the association between U_{1i} and U_{3i} disappears. Although this result provides some evidence that there might exist an AR(1) dependence among the unobserved components, we did not further investigate this hypothesis, by focussing on our more general model, which does not impose any constrain on the correlation structure between frailties.

In order to further clarify the role of the time-varying unobserved heterogeneity on

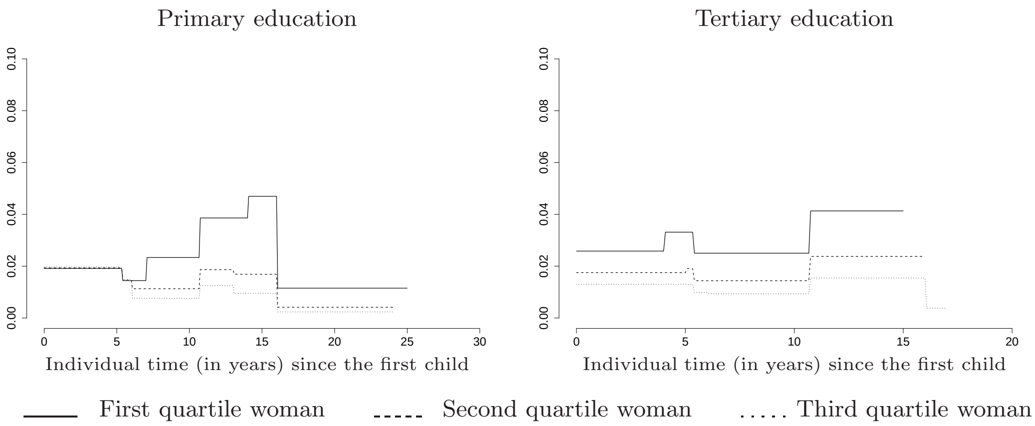

Figure 3. Model C: Estimated hazard function for primary and tertiary educated women on the first/second/third quartile of the unobserved first-period frailty distribution

fertility, we show in Figure 3 the hazard functions specific for the transition to the second child for three primary educated women (graph on the left) and three tertiary educated women (graph on the right). The chosen women have been selected from the sub-sample of reference women, living in North Italy, born between 1958 and 1965, and having both parents with education level lower than a bachelor's degree. The selected women have the frailties, U_{1i}, equal to the first, second and third quartiles of the posterior distributions of U_{1i} specific for the primary- and tertiary-educated reference women. Time is measured since the birth of the first child. The selected primary educated women gave birth to the first child at the age of 21 (first quartile woman) and 22 (second and third quartile woman) years. The selected tertiary educated women gave birth to the first child at the age of 31 (first quartile woman), 30 (second quartile woman) and 29 (third quartile woman) years.

As it can be seen in Figure 3, primary educated women have similar hazard values during the first years after the birth of their first child. In this period primary educated women are younger than 28 years old, so their family proneness is still described by U_{1i}, which has a small posterior variance (see Table 2). This implies that the quartiles of the distribution of U_{1i} and the corresponding hazards take on similar values. Due to the strong negative marginal correlations between U_{1i} and U_{2i}, and between U_{1i} and U_{3i}, the first quartile woman has a higher risk than the second and third quartile women after age 28. This difference is especially relevant between 11 and 16 years since the birth of the first child, where the risk ranges between 3.9% and 4.7% (first quartile woman), 1.7% and 1.9% (second quartile woman), and 0.9% and 1.3% (third quartile woman). A similar picture is drawn for the higher educated women. As expected, higher educated women experience the first birth later, implying that their second-child hazard is not directly related to the first-period frailty. The first quartile woman, who has a low orientation towards family life at younger ages, develops a high proneness to family life after 28 years, implying that her hazard function is translated upward with respect to the second and third quartile women.

5. Concluding remarks

In this paper, we propose a Bayesian event history model with time-dependent heterogeneity to analyse how women's education level is related to Italian fertility.

About the first child, our results corroborate the general view that higher educated women might have a stronger feeling towards the trade-off between work and family life than primary educated women. One reason for this is that better educated women may have more to lose in terms of foregone earnings. Timing of the first birth plays a key role, as the economic loss (the opportunity cost) of taking a break from the labour market constitutes a large part of the costs involved in having a child (e.g., Martín-García and Baizán, 2006).

As regards the transition to the second child, the model with a time-constant frailty provides positive posterior means of the education level parameters, but their values are really small and statistically negligible. Utilising a time-dependent frailty suggests that higher educated women tend to postpone the birth of a possible second child with respect to lower educated ones.

Overall, controlling for woman-specific family orientation changes the association between education and fertility dynamics, suggesting that some women might be very prone to family life compared to others with the same education level. However, we also showed that the usually made assumption of time-constant unobserved heterogeneity can lead to misleading results, by overestimating heterogeneity in the first part of women lives and underestimating it in older ages.

A. Appendix – Posterior distributions of the model parameters

Tables A1, A2, and A3 show summary statistics of the posterior distributions of the fixed effect parameters and the variance, σ^2_α, of the logarithm of the baseline hazard parameters for Models A, B, and C. Note that although summaries of the posterior distributions of the education level parameters are shown in Table 1 in the main text, for clarity they are also shown in Tables A1, A2, and A3.

Table A1. Summary statistics: Posterior distributions of the parameters of Model A (without frailty component)

	First Child (j = 1)				*Second Child* (j = 2)			
Parameter	*Mean*	*SD*	*2.5%*	*97.5%*	*Mean*	*SD*	*2.5%*	*97.5%*
$\alpha_1^{(j)}$	-6.448	0.054	-6.555	-6.345	-4.136	0.059	-4.253	-4.020
$\alpha_2^{(j)}$	-4.756	0.037	-4.827	-4.683	-4.475	0.067	-4.607	-4.345
$\alpha_3^{(j)}$	-3.913	0.038	-3.986	-3.840	-5.836	0.138	-6.108	-5.573
$\alpha_4^{(j)}$	-3.276	0.046	-3.367	-3.186	-7.267	0.384	-8.082	-6.583
$\alpha_5^{(j)}$	-2.907	0.093	-3.094	-2.726				
$\alpha_6^{(j)}$	-2.425	0.326	-3.112	-1.832				
Geographical area of residence (Nord)								
Center	-0.008	0.042	-0.091	0.073	0.046	0.055	-0.063	0.155
Sud-Islands	0.234	0.032	0.169	0.296	0.395	0.041	0.315	0.475
Coorte (1958-1965)								
1966-1973	0.051	0.032	-0.011	0.112	0.019	0.039	-0.058	0.096
1974-1983	0.777	0.051	0.676	0.877	-0.329	0.084	-0.498	-0.166
Education Level (Primary)								
Secondary	-0.472	0.031	-0.533	-0.410	0.004	0.041	-0.077	0.084
Tertiary	-0.979	0.058	-1.094	-0.865	0.269	0.080	0.109	0.424
Parents' education level								
At least one parent with BA	-0.023	0.043	-0.106	0.060	-0.027	0.057	-0.139	0.084
Age at the first child					-0.004	0.000	-0.004	-0.003
σ^2_α	11.014	10.502	3.125	34.796	9.981	22.656	1.850	39.669

Table A2. Summary statistics: Posterior distributions of the parameters of Model B (constant frailty)

	First Child (j = 1)				*Second Child* (j = 2)			
Parameter	*Mean*	*SD*	*2.5%*	*97.5%*	*Mean*	*SD*	*2.5%*	*97.5%*
$\alpha_1^{(j)}$	-6.102	0.065	-6.231	-5.976	-4.619	0.111	-4.835	-4.405
$\alpha_2^{(j)}$	-5.749	0.064	-5.876	-5.625	-5.189	0.139	-5.463	-4.921
$\alpha_3^{(j)}$	-5.579	0.166	-5.916	-5.266	-5.953	1.162	-8.614	-4.111
$\alpha_4^{(j)}$	-5.541	3.733	-12.900	2.040	-8.223	2.824	-15.080	-4.282
$\alpha_5^{(j)}$	-5.577	5.286	-16.220	5.027				
$\alpha_6^{(j)}$	-5.547	6.462	-18.410	7.616				
Geographical area of residence (Nord)								
Center	0.096	0.069	-0.039	0.231	-0.017	0.095	-0.204	0.168
Sud-Islands	0.127	0.053	0.023	0.231	0.108	0.072	-0.033	0.249
Coorte (1958-1965)								
1966-1973	0.025	0.059	-0.091	0.139	-0.010	0.080	-0.165	0.147
1974-1983	0.037	0.060	-0.081	0.154	-0.094	0.081	-0.253	0.066
Education Level (Primary)								
Secondary	0.077	0.055	-0.031	0.185	0.024	0.075	-0.120	0.173
Tertiary	0.030	0.086	-0.140	0.196	0.060	0.111	-0.163	0.275
Parents' education level								
At least one parent with BA	0.012	0.060	-0.106	0.129	-0.024	0.081	-0.184	0.133
Age at the first child					-0.002	0.001	-0.003	0.000

	First and Second Child			
Parameter	*Mean*	*SD*	*2.5%*	*97.5%*
σ_α^2	14.238	12.211	4.029	45.725

Table A3. Summary statistics: Posterior distributions of the parameters of Model C (time-dependent frailty)

	First Child ($j=1$)				*Second Child* ($j=2$)			
Parameter	*Mean*	*SD*	*2.5%*	*97.5%*	*Mean*	*SD*	*2.5%*	*97.5%*
$\alpha_1^{(j)}$	-5.715	0.041	-5.795	-5.636	-3.680	0.071	-3.823	-3.543
$\alpha_2^{(j)}$	-4.878	0.037	-4.950	-4.806	-3.962	0.091	-4.143	-3.787
$\alpha_3^{(j)}$	-4.403	0.042	-4.486	-4.322	-3.460	0.440	-4.386	-2.669
$\alpha_4^{(j)}$	-4.453	3.300	-11.110	2.078	-4.865	2.644	-11.070	-0.676
$\alpha_5^{(j)}$	-4.441	4.683	-13.680	4.962				
$\alpha_6^{(j)}$	-4.458	5.717	-15.870	6.897				
Geographical area of residence (Nord)								
Center	0.045	0.044	-0.043	0.131	0.117	0.069	-0.022	0.253
Sud-Islands	0.244	0.033	0.178	0.309	0.584	0.050	0.486	0.683
Coorte (1958-1965)								
1966-1973	-0.346	0.032	-0.410	-0.283	-0.055	0.048	-0.149	0.038
1974-1983	-1.156	0.051	-1.257	-1.058	-0.275	0.087	-0.448	-0.107
Education Level (Primary)								
Secondary	-0.515	0.033	-0.580	-0.450	-0.066	0.051	-0.167	0.034
Higher	-1.175	0.063	-1.301	-1.054	-0.166	0.117	-0.396	0.064
Parents' education level								
At least one parent with BA	-0.186	0.044	-0.274	-0.100	-0.041	0.073	-0.186	0.101
Age at the first child					-0.012	0.001	-0.013	-0.011

	First and Second Child			
Parameter	*Mean*	*SD*	*2.5%*	*97.5%*
σ_α^2	10.939	10.312	3.167	33.933

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