# Evaluation of student performance through a multidimensional latent class IRT model with nonignorable missingness

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## Aim

- Analysis of the performance of university students, with reference to 6 compulsory courses of the first year
- The main goal is to classify the students into ability classes on the basis of their performance on the 6 exams; for each exam, the performance is measured by two pieces of information:
  1. enrollment to the exam: whether the student decides to *take the exam* in the observation period (one year)
  2. exam result: conditional on enrollment the student obtain a result (*failed* or *passed with a grade*).
- If the student does not enroll to a given exam, the result is missing: this is informative about the student ability, thus the missingness cannot be ignored





## Methods

- We adopt a multidimensional latent class IRT model for the analysis of item responses affected by non-ignorable missingness
- We extend the proposal of Bacci and Bartolucci (2015) to allow for
  - ordinal responses (instead of binary)
  - two kinds of missingness:
    1. structural missing data: the result is missing because the exam is not due (structural missing data are not considered in the model of Bacci and Bartolucci, 2015)
    2. genuine (potentially informative) missing data: the result is missing even if the exam is due because the student did not take it

Bacci S., Bartolucci F. (2015) A multidimensional finite mixture SEM for non-ignorable missing responses to test items, *Structural Equation Modeling*





## Data

- We consider the freshmen of A.Y. 2013/2014 in two degree programs (*Economics*; *Business*) of the University of Florence
- Outcome: performance on the compulsory first-year exams in year 2014
- We analyze data about 861 active freshmen (those who enrolled for at least one exam in year 2014 - 89% of the total)
- Exams can be taken in any order (and freely repeated), enrolling via web
  - Courses of semester I: exams in any of the 6 sessions from Jan to Dec
  - Courses of semester II: exams in any of the 4 sessions from June to Dec

Table: *Enrollment rates and exam results for first-year exams (year 2014)*

| Course | (sem.) | Enroll. rate (%) | Exam grade (%) fail | 18-21 | 22-24 | 25-27 | $\geq 28$ | Passing rate (%) enroll. | overall |
|---|---|---|---|---|---|---|---|---|---|
| Accounting | (I) | 93.5 | 42.5 | 15.9 | 17.3 | 17.0 | 7.3 | 57.5 | 53.8 |
| Math | (I) | 67.8 | 65.8 | 16.2 | 7.3 | 6.8 | 4.0 | 34.2 | 21.1 |
| Law | (I) | 48.3 | 47.1 | 14.2 | 16.1 | 14.4 | 8.2 | 52.9 | 25.6 |
| Management | (II) | 72.5 | 30.6 | 8.2 | 16.7 | 23.2 | 21.3 | 69.4 | 50.3 |
| MicroEcon | (II) | 41.8 | 41.9 | 10.6 | 11.4 | 18.1 | 18.1 | 58.1 | 24.3 |
| Statistics | (II) | 67.0 | 39.7 | 16.8 | 13.5 | 11.4 | 18.5 | 60.3 | 40.4 |

# Covariates + the course group indicator

| | | $N$ | Average number of exams enrolled to | Average number of exams passed |
|---|---|---|---|---|
| **All freshmen** | | **861** | **3.8** | **2.2** |
| *Gender* | Male | 502 | 3.8 | 2.1 |
| | Female | 359 | 3.9 | 2.2 |
| *High school type* | Technical | 765 | 3.8 | 2.1 |
| | Humanities | 201 | 3.7 | 1.9 |
| | Scientific | 321 | 4.1 | 2.4 |
| | Other | 284 | 3.6 | 1.8 |
| *High school grade* | $< 80$ | 596 | 3.6 | 1.7 |
| | $\geq 80$ | 265 | 4.4 | 3.3 |
| *Late matriculation* | No | 759 | 4.0 | 2.3 |
| | Yes | 102 | 3.0 | 1.3 |
| *Degree program* | Business | 588 | 3.8 | 2.1 |
| | Economics | 273 | 3.9 | 2.3 |
| *Course group* | A-C | 257 | 3.7 | 2.2 |
| | D-L | 240 | 3.8 | 2.2 |
| | M-P | 204 | 4.0 | 2.1 |
| | Q-Z | 160 | 3.9 | 2.3 |

Each course has 4 groups (classes), based on the first letter of the student's surname: e.g. Mr. Rossi is assigned to "Statistics Q-Z", therefore the exam "Statistics Q-Z" is due (we can observe a result), while "Statistics A-C" etc. are not due (we cannot observe a result - structural missingness)





# Model: basic notation

Remark: in our application 'item' ⇔ 'first-year exam'

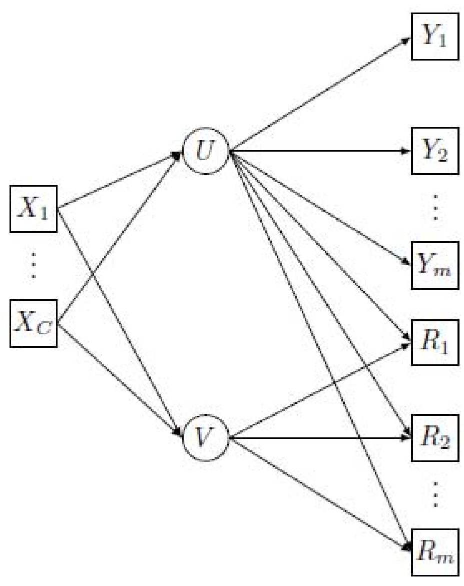
- $Y_j = y_j$: response provided by the subject to ordinal item $j$, $j = 1, ..., m$
  $1, \ldots, L$ if item $j$ is observed
  "NA" if item $j$ is skipped
- $R_j = r_j$: item indicator of response
  1 if $Y_j$ is observed
  0 if $Y_j$ is skipped (informative missing)
  "NA" if $Y_j$ is not due (structural missing)
- $X_1, \ldots, X_C$: exogenous individual covariates
- $U$: latent variable denoting the latent trait (ability) measured by the test items
- $V$: latent variable denoting an individual preference in choosing the test items to answer (determining if an item is observed or missing)





# Model: path diagram

Multidimensional LC-IRT model: item binary indicators of answer $R_j$, item ordinal responses $Y_j$, latent ability $U$, latent preference in choosing the test items to answer $V$








# Model: our specification

- The model allows for multidimensional latent variables of each type, but for simplicity we assume
  - a single latent ability $U$
  - a single latent preference in choosing the exams $V$
- The exam result is an element of the following set: $\{\text{failed}, 18, 19, \ldots, 29, 30, 30 \text{ with honors}\}$ - this implies 15 ordinal categories, but for simplicity we reduce the categories to 5 as follows:

$$\begin{cases} Y_j = NA & \text{if} \quad R_j = 0 \\ Y_j = 0 & \text{if} \quad R_j = 1 \text{ and } grade = NA \\ Y_j = 1 & \text{if} \quad R_j = 1 \text{ and } 18 \leq grade \leq 21 \\ Y_j = 2 & \text{if} \quad R_j = 1 \text{ and } 22 \leq grade \leq 24 \\ Y_j = 3 & \text{if} \quad R_j = 1 \text{ and } 25 \leq grade \geq 27 \\ Y_j = 4 & \text{if} \quad R_j = 1 \text{ and } grade \geq 28 \end{cases}$$

- Each of the **6 courses** has **4 groups** (i.e. classes with different teachers)
  - any item $j$ refers to a course for a given group ⇒ there are $m$=6×4=24 items ('Accounting A-C', 'Accounting D-L', ..., 'Statistics M-Z')
  - each student is assigned to one group, thus the outcomes for the other groups are missing by construction: $R_j = NA$ (structural missing)

## Model: distribution of the latent variables

- We assume that the latent variables $U$ and $V$ have discrete distributions
  - $U$ has support points (latent classes) $u_{h_U}$
    the number of support points $k_U$ has to be estimated
  - $V$ has support points (latent classes) $v_{h_V}$
    the number of support points $k_V$ has to be estimated
- Discrete latent variables $\rightarrow$ clustering the individuals into latent classes that are homogeneous with respect to the latent traits
- In the spirit of concomitant variable LC models (Dayton and Macready, 1988), we allow the membership probabilities of the latent classes to depend on observed covariates through a multinomial logit model (Bacci and Bartolucci, 2015):

$$\begin{aligned}\log \frac{\lambda_{h_U}(\boldsymbol{x})}{\lambda_1(\boldsymbol{x})} &= \boldsymbol{x}'\boldsymbol{\phi}_{h_U}, \quad h_U = 2,\dots,k_U \\ \log \frac{\pi_{h_V}(\boldsymbol{x})}{\pi_1(\boldsymbol{x})} &= \boldsymbol{x}'\boldsymbol{\psi}_{h_V}, \quad h_V = 2,\dots,k_V\end{aligned}$$





## Model: measurement component

The relationships between the latent variables $U$ and $V$ and the manifest variables (item responses $Y_1 \dots Y_j \dots Y_m$ and response indicators $R_1 \dots R_j \dots R_m$) are described by the measurement part of the model:

- $R_j$ given $U$ and $V$ is a 2PL model (Birnbaum, 1968)

$$\log \frac{q_{h_U h_V, j}}{1 - q_{h_U h_V, j}} = \gamma_{Uj} u_{h_U} + \gamma_{Vj} v_{h_V} - \delta_j$$

where $q_{h_U h_V, j} = Pr(R_j = 1 | U = u_{h_U}, V = v_{h_V})$
identifiability constraints: $\gamma_{Vj} = 1, \delta_j = 0$ for a reference item

- $Y_j$ given $U$ is a Graded Response Model (GRM; Samejima, 1969)

$$\log \frac{p_{h_U, jy}}{1 - p_{h_U, jy}} = \alpha_j u_{h_U} - \beta_{jy}, \qquad y = 2,\dots,L$$

where $p_{h_U, jy} = Pr(Y_j \geq y | U = u_{h_U})$
identifiability constraints: $\alpha_j = 1, \beta_{j1} = 0$ for a reference item





## Model: likelihood inference

- We fit the proposed multidimensional LC-IRT model by maximizing the marginal likelihood using the EM algorithm (Dempster et al., 1977).
- We exploit the `R` package `MLCIRTwithin` (function `est_multi_poly_within`, which is devoted to the estimation of Multidimensional LC-IRT models in presence of within-item multidimensionality)

Bartolucci F., Bacci S., Gnaldi M. (2014). MultiLCIRT: An R package for multidimensional latent class item response models, *Computational Statistics and Data Analysis*, 71, 971–985.





## Selection of the number of latent classes

| $k_U$ | $k_V$ | $\hat{\ell}$ | # par | BIC |
|---|---|---|---|---|
| 2 | 2 | -6520.37 | 208 | 14446.41 |
| 2 | 3 | -6505.63 | 217 | 14477.76 |
| 3 | 2 | -6387.18 | 217 | 14240.87 |
| 3 | 3 | -6364.32 | 226 | 14255.96 |
| 4 | 2 | -6338.27 | 226 | 14203.86 |
| 4 | 3 | -6325.72 | 235 | 14239.58 |
| 5 | 2 | -6323.84 | 235 | 14235.84 |
| 5 | 3 | -6304.16 | 244 | 14257.30 |

- On the basis of BIC we select $k_U = 4$ latent classes for $U$ and $k_V = 2$ latent classes for $V$
- In order to check for local maxima, we repeat the model estimation process for different random starting values of the parameters

## Testing the ignorability of the missing data mechanism

- If a student decides not to take an exam in the considered year ($R_j = 0$) then the exam result $Y_j$ is missing: likely this is not ignorable
- In our model the ability $U$ affects both the decision to take an exam $R_j$ and the result $Y_j$
  $\rightarrow$ the missing data mechanism is not ignorable
- We test the ignorability assumption comparing our multidimensional LC-IRT model with a restricted model where the decision to take an exam $R_j$ does not depend on the ability $U$, i.e.

$$\gamma_{Uj} = 0, \qquad j = 1, \ldots, 24$$

- $LRT = 2 \times (6533.720 - 6338.268) = 390.904$, with 24 degrees of freedom yielding a very low $p$-value $\Rightarrow$ we proceed with the proposed multidimensional LC-IRT model accounting for the non-ignorable missing mechanism





## Estimated parameters

The selected model with **4** latent classes for the ability $U$ and **2** latent classes for the preference $V$ has 226 parameters:

- discrimination parameters for the effects of the latent variables
  - latent ability $U$ on exam result $Y_j$ ($\hat{\alpha}_j^*$)
  - latent ability $U$ on exam enrollment $R_j$ ($\hat{\gamma}_{1j}^*$)
  - latent preference $V$ on exam enrollment $R_j$ ($\hat{\gamma}_{2j}^*$)
- difficulty parameters shifting the distributions of $Y_j$ and $R_j$
  - higher $\hat{\delta}_j^* \rightarrow$ lower probability to take the exam
  - higher $\hat{\beta}_{jy}^* \rightarrow$ lower probability of a good result
- latent structure parameters
  - support points of $U$ ($u_{h_U}^*$) and support points of $V$ ($v_{h_V}^*$)
  - estimated coefficients of the multinomial logit model for the probabilities of $U$ ($\phi_{h_U}$) and $V$ ($\psi_{h_V}$)

The asterisk denotes standardization: for ease of interpretation, the support points have been standardized so that the latent variables have mean 0 and standard deviation 1, and the discrimination and difficulty parameters have been transformed accordingly.





## Estimated discrimination parameters for the exam result $Y_j$

- The exam results $Y_j$ are affected by the latent ability $U$ through the discrimination parameters $\hat{\alpha}_j^*$
- All those parameters are significantly different from zero $\rightarrow$ All the exams contribute to measure the latent ability $U$
- Exams with higher discrimination are more sensitive to variations in student ability (Accounting, Mathematics, and Statistics)
- There are differences across groups of the same course, especially for Law and Management





## Estimated discrimination parameters for the indicator of taking the exam $R_j$

- The response indicators $R_j$ (=1 if the student takes the exam in the considered year) are affected
  - by the latent ability $U$ through the discrimination parameters $\hat{\gamma}_{1j}^*$ and
  - by the latent preference $V$ through the discrimination parameters $\hat{\gamma}_{2j}^*$

  (this dependence on two latent variables is known as *within-item multidimensionality*, e.g. Adams, Wilson and Wang, 1997)
- The student latent ability $U$ significantly affects the enrollment for most exams: this provides evidence that the enrollment process generating missing exam results is not ignorable (as confirmed by the likelihood-ratio test)
- The student latent preference $V$ has a positive effect for Mathematics and Statistics, and negative for Law; thus $V$ can be interpreted as the preference of the student to take exams in *quantitative subjects* as opposed to exams in qualitative subjects (the effect is significant for about one-third of the items)
- The indicators of taking the exam $R_j$ are affected more by $U$ than by $V$, namely $|\hat{\gamma}_{1j}^*| > |\hat{\gamma}_{2j}^*|$, with the notable exception of Mathematics

## From difficulty parameters to predicted probabilities

- The estimates of the difficulty parameters are not easily interpretable
- It is more interesting to look at the predicted probabilities for a student with some hypothetical values of latent ability $U$ and latent preference $V$ (e.g. the mean values $U = V = 0$)
- The predicted probabilities vary with the degree program (*Economics* or *Business*) and the group (A-C, D-L, M-P, Q-Z)
  - We note a large variability among courses and, in some cases, also across groups of the same course for both the enrollment $R_j$ and the exam result $Y_j$
  - For the majority of courses, the most likely result is a failure and the modal grade of passed exams is 18-21
- To see how the probability to obtain an exam result depends on the student ability, we move the value of $U$ (we find that the result is highly influenced by the ability)





## Estimates for latent ability $U$ and latent preference $V$

Standardized estimated support points with corresponding average probabilities

| | Ability $U$ latent class $h_U$ | | | | Preference $V$ latent class $h_V$ | |
|---|---|---|---|---|---|---|
| | $h_U = 1$ | $h_U = 2$ | $h_U = 3$ | $h_U = 4$ | $h_V = 1$ | $h_V = 2$ |
| Support points $(u^*_{h_U}, v^*_{h_V})$ | -1.485 | -0.129 | 0.784 | 1.937 | -0.949 | 1.054 |
| Average probs $(\lambda_{h_U}, \overline{\pi}_{h_V})$ | 0.228 | 0.395 | 0.294 | 0.083 | 0.526 | 0.474 |

Estimated coefficients of the multinomial logit models for the probabilities of the support points

| | Model for ability $U$ | | | Model for preference $V$ |
|---|---|---|---|---|
| | $\hat{\phi}_{11j}$ | $\hat{\phi}_{12j}$ | $\hat{\phi}_{13j}$ | $\hat{\psi}_{11j}$ |
| Constant | 0.748 | 0.568 | -1.956 | -0.869 |
| Degree Economics ($j = 1$) | -0.434 | -0.030 | 0.045 | 0.716 |
| Female ($j = 2$) | 0.434 | 0.059 | -0.487 | -0.147 |
| HS grade ($j = 3$) | 0.014 | 0.118 | 0.265 | -0.020 |
| HS technical | – | – | – | – |
| HS humanities ($j = 4$) | 0.138 | 0.148 | 0.691 | -0.240 |
| HS scientific ($j = 5$) | -0.061 | 1.021 | 2.219 | 1.963 |
| HS other ($j = 6$) | -0.163 | -0.163 | -0.282 | -0.336 |
| Late enrollment ($j = 7$) | -0.191 | -1.415 | -1.834 | -0.744 |

Parameters in red have $p$-value$< 0.05$.





## Testing differences among the groups of a course

Likelihood-ratio tests comparing, separately for each course, the *full model* with the *restricted model with groups collapsed for the course under consideration*

| Model | $\hat{\ell}$ | # par | Deviance | df | $p$-value |
|---|---|---|---|---|---|
| Full model | -6336.114 | 226 | – | – | – |
| Collapsing Business | -6389.587 | 202 | 106.947 | 24 | 0.000 |
| Collapsing Mathematics | -6349.616 | 202 | 27.004 | 24 | 0.304 |
| Collapsing Law | -6397.061 | 202 | 121.894 | 24 | 0.000 |
| Collapsing Management | -6464.912 | 202 | 257.596 | 24 | 0.000 |
| Collapsing Economics | -6411.623 | 202 | 151.017 | 24 | 0.000 |
| Collapsing Statistics | -6358.218 | 202 | 44.208 | 24 | 0.007 |

- The likelihood ratio tests reveal significant differences among the groups for nearly all the courses (teacher effect)
- The only exception is represented by Mathematics





## Final remarks

We applied a novel multidimensional LC-IRT model to the analysis of first-year exams of university students:

- The discrete nature of the latent variables ensures flexibility and yields a clustering of students into homogenous classes
- The student ability is measured not only by the exam results of taken exams, but also by the indicators of taking the exams, thus accounting for the decisions to take or not to take any given exam in the considered year
  - in other words, the mechanism generating the missing exam results is treated as non-ignorable (and we found evidence it is non-ignorable)
  - the decision to take an exam is affected also by a specific latent variable which can be interpreted as the preference of the student to take exams in *quantitative subjects* as opposed to exams in qualitative subjects
  - modelling the indicators of taking the exams is relevant in the Italian university system, where many students do not take all the compulsory exams in the expected period - revealing the patterns is essential for corrective actions

# Final remarks /cont.

- For some courses we found noteworthy differences among groups (i.e. classes held by different teachers), pointing out a teacher effect on both the probability of taking the exam and the result - this raises a fairness issue
- The structural part of the model relates class membership (ability) to observed characteristics: the probability to belong to classes of greater ability is higher for
  - students from scientific high schools
  - students with a good school grade
  - students immediately enrolling to the university after the end of high school

  This information can be used by potential freshmen and by the university management for planning guidance and tutoring activities
- The proposed LC-IRT model is suitable for a wide range of applications characterized by ordinal items with non-ignorable missing item responses, e.g. in achievement tests, customer satisfaction and quality of life



Thanks for your attention!

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# Estimated item discrimination parameters

latent ability $U$ on exam result $Y$ (scaled parameters $\hat{\alpha}_j^*$)
latent ability $U$ on exam enrollment $R$ (scaled parameters $\hat{\gamma}_{1j}^*$)
latent preference $V$ on exam enrollment $R$ (scaled parameters $\hat{\gamma}_{2j}^*$)

| Item | | $U \to Y_j$ | | | $U \to R_j$ | | | $V \to R_j$ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Course | Group | $\hat{\alpha}_j^*$ | $\hat{se}_{\alpha_j^*}$ | $p$-value | $\hat{\gamma}_{Uj}^*$ | $\hat{se}_{\gamma_{Uj}^*}$ | $p$-value | $\hat{\gamma}_{Vj}^*$ | $\hat{se}_{\gamma_{Vj}^*}$ | $p$-value |
| Account | A-C | 2.127 | 0.285 | < 0.001 | 0.904 | 0.375 | 0.016 | 0.401 | 0.274 | 0.144 |
| | D-L | **1.795** | 0.259 | < 0.001 | 0.535 | 0.349 | 0.125 | 0.594 | 0.433 | 0.170 |
| | M-P | 2.527 | 0.380 | < 0.001 | 1.172 | 0.533 | 0.028 | -0.503 | 0.525 | 0.338 |
| | Q-Z | 2.337 | 0.357 | < 0.001 | 0.433 | 0.318 | 0.173 | 0.522 | 0.311 | 0.093 |
| Math | A-C | 2.241 | 0.410 | < 0.001 | 1.918 | 0.893 | 0.032 | 2.448 | 1.176 | 0.037 |
| | D-L | 2.134 | 0.386 | < 0.001 | 1.278 | 0.440 | 0.004 | 2.251 | 0.772 | 0.004 |
| | M-P | **1.731** | 0.402 | < 0.001 | 1.700 | 0.503 | 0.001 | 1.782 | 0.587 | 0.002 |
| | Q-Z | 2.963 | 0.707 | < 0.001 | 5.050 | 4.605 | 0.273 | 6.783 | 5.266 | 0.198 |
| Law | A-C | 1.849 | 0.427 | < 0.001 | 0.903 | 0.196 | < 0.001 | -0.380 | 0.221 | 0.085 |
| | D-L | 3.016 | 0.506 | < 0.001 | 1.303 | 0.254 | < 0.001 | -0.390 | 0.267 | 0.144 |
| | M-P | **1.391** | 0.306 | < 0.001 | 1.144 | 0.267 | < 0.001 | -0.804 | 0.314 | 0.011 |
| | Q-Z | 1.783 | 0.425 | < 0.001 | 1.033 | 0.241 | < 0.001 | -0.279 | 0.214 | 0.192 |
| Manag | Bus A-L | 2.990 | 0.530 | < 0.001 | 2.287 | 0.389 | < 0.001 | 0.675 | 0.315 | 0.032 |
| | Bus M-Z | **3.163** | 0.484 | < 0.001 | 1.339 | 0.249 | < 0.001 | -0.304 | 0.221 | 0.169 |
| | Eco A-L | 1.976 | 0.389 | < 0.001 | 1.106 | 0.250 | < 0.001 | 0.539 | 0.296 | 0.068 |
| | Eco M-Z | **0.662** | 0.306 | 0.030 | 2.212 | 0.500 | < 0.001 | 0.605 | 0.455 | 0.184 |
| MicroEcon | A-C | 1.249 | 0.325 | < 0.001 | 1.429 | 0.247 | < 0.001 | 0.310 | 0.238 | 0.193 |
| | D-L | 1.130 | 0.402 | 0.005 | 3.114 | 0.598 | < 0.001 | 0.450 | 0.319 | 0.159 |
| | M-P | 1.822 | 0.366 | < 0.001 | 1.889 | 0.353 | < 0.001 | -0.447 | 0.294 | 0.128 |
| | Q-Z | **2.350** | 0.588 | < 0.001 | 2.202 | 0.440 | < 0.001 | 0.926 | 0.282 | 0.001 |
| Statistics | A-C | 2.787 | 0.445 | < 0.001 | 2.333 | 0.466 | < 0.001 | 0.946 | 0.387 | 0.014 |
| | D-L | 2.496 | 0.389 | < 0.001 | 1.567 | 0.290 | < 0.001 | 0.808 | 0.295 | 0.006 |
| | M-P | 2.867 | 0.497 | < 0.001 | 1.772 | 0.319 | < 0.001 | 0.104 | 0.292 | 0.722 |
| | Q-Z | 2.258 | 0.468 | < 0.001 | 1.998 | 0.469 | < 0.001 | 1.020 | 0.347 | 0.003 |



# Predicted probabilities of enrollment $R$

Probabilities at some values of latent ability $U$ and latent preference $V$.

| Item | | $P(R=1 \mid U=u, V=v)$ | | | | | | Range | |
|---|---|---|---|---|---|---|---|---|---|
| Course | Class | $u$ / $v$ | 0 / 0 | $-\sigma_U$ / 0 | $+\sigma_U$ / 0 | 0 / $-\sigma_V$ | 0 / $+\sigma_V$ | $\pm\sigma_U$ | $\pm\sigma_V$ |
| Accounting | A-C | | 0.96 | 0.91 | 0.98 | 0.94 | 0.98 | 0.07 | 0.03 |
| | D-L | | 0.95 | 0.92 | 0.97 | 0.92 | 0.97 | 0.05 | 0.06 |
| | M-P | | 0.97 | 0.92 | 0.99 | 0.98 | 0.96 | 0.07 | -0.03 |
| | Q-Z | | 0.91 | 0.87 | 0.94 | 0.86 | 0.95 | 0.07 | 0.09 |
| Mathematics | A-C | | 0.78 | 0.37 | 0.96 | 0.27 | 0.97 | 0.58 | 0.72 |
| | D-L | | 0.78 | 0.50 | 0.93 | 0.28 | 0.97 | 0.42 | 0.69 |
| | M-P | | 0.77 | 0.38 | 0.95 | 0.36 | 0.95 | 0.57 | 0.59 |
| | Q-Z | | 0.87 | 0.10 | 1.00 | 0.02 | 1.00 | 0.89 | 0.99 |
| Law | A-C | | 0.32 | 0.16 | 0.54 | 0.41 | 0.25 | 0.38 | -0.17 |
| | D-L | | 0.51 | 0.22 | 0.79 | 0.60 | 0.41 | 0.57 | -0.19 |
| | M-P | | 0.58 | 0.31 | 0.81 | 0.76 | 0.38 | 0.51 | -0.37 |
| | Q-Z | | 0.56 | 0.31 | 0.78 | 0.63 | 0.49 | 0.47 | -0.14 |
| Management | Bus A-L | | 0.88 | 0.43 | 0.99 | 0.79 | 0.94 | 0.56 | 0.15 |
| | Bus M-Z | | 0.79 | 0.50 | 0.94 | 0.84 | 0.74 | 0.44 | -0.10 |
| | Eco A-L | | 0.64 | 0.37 | 0.85 | 0.51 | 0.76 | 0.47 | 0.25 |
| | Eco M-Z | | 0.88 | 0.45 | 0.99 | 0.81 | 0.93 | 0.53 | 0.13 |
| MicroEcon | A-C | | 0.27 | 0.08 | 0.60 | 0.21 | 0.34 | 0.52 | 0.13 |
| | D-L | | 0.14 | 0.01 | 0.79 | 0.10 | 0.21 | 0.78 | 0.11 |
| | M-P | | 0.60 | 0.18 | 0.91 | 0.70 | 0.49 | 0.72 | -0.21 |
| | Q-Z | | 0.53 | 0.11 | 0.91 | 0.31 | 0.74 | 0.80 | 0.43 |
| Statistics | A-C | | 0.85 | 0.36 | 0.98 | 0.69 | 0.94 | 0.63 | 0.25 |
| | D-L | | 0.68 | 0.31 | 0.91 | 0.49 | 0.83 | 0.60 | 0.34 |
| | M-P | | 0.69 | 0.27 | 0.93 | 0.67 | 0.71 | 0.66 | 0.04 |
| | Q-Z | | 0.87 | 0.47 | 0.98 | 0.70 | 0.95 | 0.51 | 0.25 |

# Predicted probabilities of exam result $Y$

Predicted probabilities at some values of latent ability $U$

| Item | | $P(Y = y_k \mid U = 0)$ | | | | | Success rate $P(Y > 0 \mid U)$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Course | Class | 0 | 1 | 2 | 3 | 4 | $U = u$ | | | |
| | | Failed | 18-21 | 22-24 | 25-27 | 28-30 | $-\sigma_U$ | 0 | $+\sigma_U$ | range |
| Accounting | A-C | 0.35 | 0.33 | 0.17 | 0.12 | 0.03 | 0.18 | 0.65 | 0.94 | 0.76 |
| | D-L | 0.65 | 0.17 | 0.13 | 0.05 | 0.01 | 0.08 | 0.35 | 0.77 | 0.68 |
| | M-P | 0.38 | 0.22 | 0.30 | 0.09 | 0.01 | 0.12 | 0.62 | 0.95 | 0.84 |
| | Q-Z | 0.14 | 0.29 | 0.35 | 0.18 | 0.03 | 0.37 | 0.86 | 0.98 | 0.61 |
| Mathematics | A-C | 0.84 | 0.11 | 0.03 | 0.01 | 0.01 | 0.02 | 0.16 | 0.64 | 0.62 |
| | D-L | 0.80 | 0.14 | 0.04 | 0.02 | 0.00 | 0.03 | 0.20 | 0.67 | 0.65 |
| | M-P | 0.87 | 0.07 | 0.04 | 0.02 | 0.01 | 0.03 | 0.13 | 0.45 | 0.43 |
| | Q-Z | 0.91 | 0.09 | 0.00 | 0.00 | 0.00 | 0.01 | 0.09 | 0.67 | 0.66 |
| Law | A-C | 0.82 | 0.11 | 0.06 | 0.01 | 0.00 | 0.03 | 0.18 | 0.58 | 0.54 |
| | D-L | 0.53 | 0.17 | 0.24 | 0.06 | 0.01 | 0.04 | 0.47 | 0.95 | 0.91 |
| | M-P | 0.71 | 0.14 | 0.04 | 0.09 | 0.02 | 0.09 | 0.29 | 0.62 | 0.53 |
| | Q-Z | 0.48 | 0.26 | 0.18 | 0.07 | 0.01 | 0.15 | 0.52 | 0.87 | 0.71 |
| Management | Bus A-L | 0.22 | 0.19 | 0.35 | 0.18 | 0.06 | 0.15 | 0.78 | 0.99 | 0.83 |
| | Bus M-Z | 0.76 | 0.06 | 0.13 | 0.04 | 0.01 | 0.01 | 0.24 | 0.88 | 0.87 |
| | Eco A-L | 0.38 | 0.20 | 0.21 | 0.16 | 0.04 | 0.18 | 0.62 | 0.92 | 0.74 |
| | Eco M-Z | 0.10 | 0.03 | 0.09 | 0.57 | 0.21 | 0.82 | 0.90 | 0.95 | 0.12 |
| MicroEcon | A-C | 0.71 | 0.12 | 0.10 | 0.05 | 0.02 | 0.11 | 0.29 | 0.59 | 0.48 |
| | D-L | 0.81 | 0.05 | 0.05 | 0.06 | 0.04 | 0.07 | 0.19 | 0.42 | 0.35 |
| | M-P | 0.36 | 0.16 | 0.16 | 0.25 | 0.08 | 0.23 | 0.64 | 0.92 | 0.69 |
| | Q-Z | 0.65 | 0.12 | 0.07 | 0.08 | 0.09 | 0.05 | 0.35 | 0.85 | 0.80 |
| Statistics | A-C | 0.50 | 0.29 | 0.12 | 0.06 | 0.03 | 0.06 | 0.50 | 0.94 | 0.88 |
| | D-L | 0.55 | 0.23 | 0.16 | 0.04 | 0.02 | 0.06 | 0.45 | 0.91 | 0.84 |
| | M-P | 0.67 | 0.20 | 0.08 | 0.04 | 0.02 | 0.03 | 0.33 | 0.90 | 0.87 |
| | Q-Z | 0.62 | 0.20 | 0.06 | 0.07 | 0.05 | 0.06 | 0.38 | 0.85 | 0.79 |